THE CAUCHY PROBLEM FOR DISSIPATIVE HÖLDER EULER FLOWS

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We address the Cauchy problem for the incompressible Euler equations in a periodic setting. Our result aims at showing that, below the Onsager's critical regularity of Hölder $1/3$ in space, the Euler equations are ill-posed, and the kind of non-uniqueness one obtains is an instance of an *h-principle* phenomenon. Basing on the estimates developed by Buckmaster, De Lellis, Isett and Székelyhidi in [1], we prove [2, 3] the existence of infinitely many Hölder $1/5-\varepsilon$ initial data, each one admitting infinitely many Hölder $1/5-\varepsilon$ solutions with preassigned total kinetic energy. Moreover, we prove that the set of non-uniqueness initial data so constructed is dense among L^2 solenoidal vector fields. This second step requires a new set of ideas which have been recently used to prove the full Onsager's conjecture, namely non-uniqueness of Euler solutions up to exponent $1/3-\varepsilon$ [4].

References

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