Combinatorics Seminar

Friday March 15, 2013 at 2PM

Room B1.01

Peter Allen

(London School of Economics)

Embedding and counting in sparse graphs

There has been substantial interest in the last fifteen years in extremal graph theory 'relative to' sparse random or pseudorandom graphs: for instance, what fraction of the edges of the random graph $G_{n,p}$ must we delete in order to remove all triangles? This question is just Turán's theorem in the 'dense case' $p = 1$, but is non-trivial when $p \ll 1$.

Following the work of Conlon and Gowers, and Schacht, we can give a detailed answer to the above question not only for triangles but for any fixed graph H; more generally, we now (due to Conlon, Gowers, Samotij and Schacht) have a 'Sparse Counting Lemma' for random graphs complementing the existing 'Sparse Regularity Lemma' of Kohayakawa and independently Rödl. For pseudorandom graphs, however, we know much less. I will describe the current best 'Sparse Counting Lemma' for pseudorandom graphs, improving recent work of Conlon, Fox and Zhao (joint work with Julia Boettcher, Jozef Skokan and Maya Stein).

Classical extremal graph theory also deals with 'Dirac-type' problems where one asks for conditions on G to contain some large, or spanning, subgraph H (such as a Hamilton cycle). For these problems it is typically necessary to invoke the Szemerédi Regularity Lemma together with the Blow-up Lemma. In order to solve the corresponding problems relative to sparse graphs, one therefore requires a sparse version of the latter. The natural generalisation of the Blow-up Lemma to sparse graphs is however false. I will explain what a Blow-up Lemma is, why it fails for sparse graphs, and what we can prove (joint with Julia Boettcher, Hiep Han, Yoshiharu Kohayakawa and Yury Person).

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