Rotation sets for beta-shifts and torus homeomorphisms

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Given $\beta > 0$, Renyi's beta-shift Z_β encodes the collection of expansions base β of all $x \in [0,1]$. The *digit frequency vector* records the relative frequencies of the various digits in the beta-expansion of a given x, and the set of all such vectors for a given beta is its *digit frequency set*. We show that this set is always compact, convex, $k-$dimensional (where $k \leq \beta \leq k+1$), and it varies continuously with β. When $k = 2$ we give a complete description of the family of the digit frequency sets: roughly, it looks like nested convex-set valued devil's staircases. Considering the family of sets with the Hausdorff topology, the typical frequency set has countably infinite vertices with a single, completely irrational limit vertex. We then discuss how these results yield a near complete understanding of the rotation sets and their bifurcations of a family of two-torus homeomorphisms.