RENORMALIZATION FOR CRITICAL CIRCLE MAPS WITH NON-INTEGER EXPONENTS

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An analytic critical circle map with critical exponent α is a homeomorphism of the circle $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ that is analytic everywhere in $\mathbb{T}$ except possibly at one point at which in some local chart it can be represented as $x \mapsto \psi(x|x|^{\alpha-1})$, for some analytic diffeomorphism ψ. We construct a renormalization operator which acts on analytic critical circle maps whose critical exponent α is not necessarily an odd integer $2n + 1$, $n \in \mathbb{N}$. When $\alpha = 2n + 1$, our definition generalizes the cylinder renormalization operator previously constructed by Yampolsky. In the case when α is close to an odd integer, we prove hyperbolicity of renormalization for maps of bounded type. We use it to prove universality and $C^{1+\beta}$-rigidity for such maps. The universality phenomenon here is analogous to the Feigenbaum-Coullet-Tresser universality in one parameter families of unimodal maps. This is a joint work with Michael Yampolsky.