UNIQUE NON-INTEGER BASE EXPANSIONS AND BIFURCATION SET

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ABSTRACT. Non-integer base expansions have received much attention since the pioneering papers of Rényi [6] and Parry [5]. It has applications in ergodic theory, fractal geometry, number theory, symbolic dynamics and so on. In this talk I will present some recent progress on unique non-integer base expansions based on the papers [1, 2, 3, 4]. To be more precise, for $q \in (1, 2)$ the sequence $(x_i) = x_1 x_2 \cdots$ of zeros and ones is called a q-expansio of x if

$$x = \frac{x_1}{q} + \frac{x_2}{q^2} + \cdots .$$

Then x has a q-expansion if and only if $x \in [0, 1/(q-1)]$. Sidorov showed in [7] that Lebesgue almost every $x \in [0, 1/(q-1)]$ has a continuum of q-expansions. On the other hand, there are also many numbers in $[0, 1/(q-1)]$ having a unique q-expansion. For $q \in (1, 2)$ we set

$$\mathcal{U}_q := \left\{ x \in \big[0, \frac{1}{q-1}\big] : x \text{ has a unique } q\text{-expansion} \right\}.$$

Then $\mathcal{U}_q$ is a Lebesgue null set for all $q \in (1, 2)$. In this talk we will describe the fractal properties of the univoque sets $\mathcal{U}_q$ with the parameter $q \in (1, 2)$. We will also describe some interesing properties of the correspooning bifurcation set in the parameter space.

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