TILING SPACES, QUASICRYSTALS AND ERGODIC THEORY

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Given a finite collection $\mathcal{P}$ of polytopes in Euclidean space $\mathbb{R}^n$, one can construct a related space $T(\mathcal{P})$ consisting of tilings formed from translates of elements of $\mathcal{P}$. This tiling space $T(\mathcal{P})$ admits a natural topological structure with an associated continuous translation action. We will explore the ergodic properties of this action and discuss how these can be identified with spectral properties of related quasicrystals. I will discuss recent joint work with Hunton, where we show how the homological structure of $T(\mathcal{P})$ can be used to construct topological invariants that at the same time carry information about the finite measures that are invariant under a related action on a transversal. Then we consider an example from recent work with Sadun that illustrates how the cohomology of $T(\mathcal{P})$ in dimension two can have surprising behaviour that has important implications for the possible dynamics. In particular, this provides a counterexample to various conjectures about the behaviour of the Ruelle Sullivan map.