QUASISPHERES AND EXPANDING THURSTON MAPS

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A quasisymmetric map is one that changes angles in a controlled way. As such they are generalizations of conformal maps and appear naturally in many areas, including Complex Analysis and Geometric group theory. A quasisphere is a metric sphere that is quasisymmetrically equivalent to the standard 2-sphere. An important open question is to give a characterization of quasispheres. This is closely related to Cannon's conjecture. This conjecture may be formulated as stipulating that a group that "behaves topologically" as a Kleinian group "is geometrically" such a group. Equivalently, it stipulates that the "boundary at infinity" of such groups is a quasisphere.

A Thurston map is a map that behaves "topologically" as a rational map, i.e., a branched covering of the 2-sphere that is postcritically finite. A question that is analog to Cannon's conjecture is whether a Thurston map "is" a rational map. This is answered by Thurston's classification of rational maps.

For Thurston maps that are expanding in a suitable sense, we may define "visual metrics". The map then is (topologically conjugate) to a rational map if and only if the sphere equipped with such a metric is a quasisphere. This talk is based on joint work with Mario Bonk.