

Test 3

- Write your full name and email on the first sheet
- Time: **50 minutes**
- Books, notes and calculators **are not allowed**

Problem 1 Determine $f(k, d)$, the smallest f such that for every set X of f distinct lattice points in \mathbb{R}^d (that is, points with integer coordinates) there are two points $\mathbf{x}, \mathbf{y} \in X$ such that the straight line segment $\{t\mathbf{x} + (1-t)\mathbf{y} : t \in [0, 1]\}$ connecting \mathbf{x} to \mathbf{y} contains at least k lattice points in its interior.

Problem 2 Evaluate

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n3^m + m3^n)}.$$