

INTRODUCTION TO THE FOCK SPACE WITH PROBABILISTIC APPLICATIONS

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ABSTRACT. The fermionic Fock space is a natural vector space whose basis is indexed by partitions of an integer, or equivalently Maya diagrams, sequences of 0-1 and normally ordered wedge products. The bosonic Fock space is a vector space of polynomials in infinitely many variables. Each of these spaces were initially considered by particle physicists as a model of a state space where two types of elementary particles interact, the bosons and the fermions.

The mathematical covering of these a priori unrelated spaces uses the language of representation theory. Fermionic Fock spaces are representations of a Clifford algebra whose generators correspond to adding or removing a particle in a given energy state, and bosonic Fock spaces are in the same way representations of a Weyl algebra. These spaces were shown to have a richer structure that we will discuss if time permits : they carry representations of many more algebras such as infinite rank matrix algebras, affine Kac-Moody algebras and quantum groups. Understanding the various relations between the actions of these various algebras on the Fock spaces has proven useful in many applications, and in particular in probability theory with Okounkov's calculation of the correlation functions of the Schur measure.

The goal of this talk is to define the Fock spaces and their relationship (boson-fermion correspondence) and to show how they can be used to compute quantities of interest in probabilistic particles systems.

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