Environmental Limits to growth: instabilities and dangerous rates of change

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School of Engineering, Computing and Mathematics
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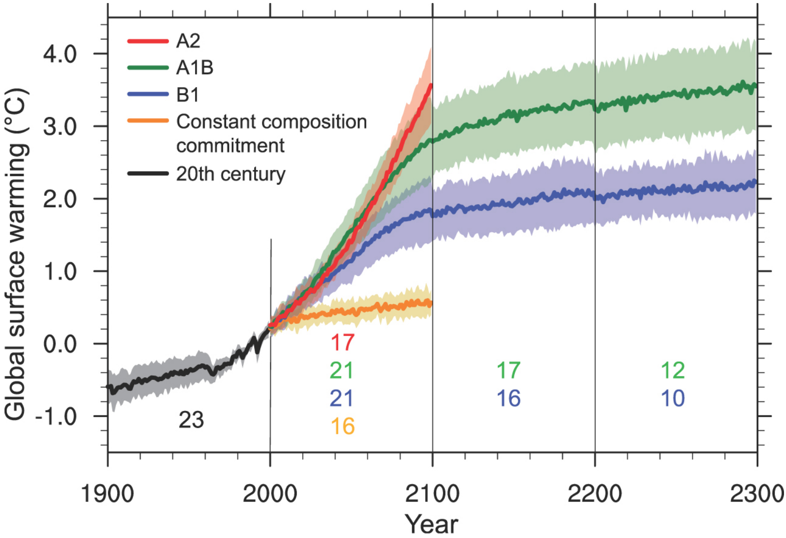
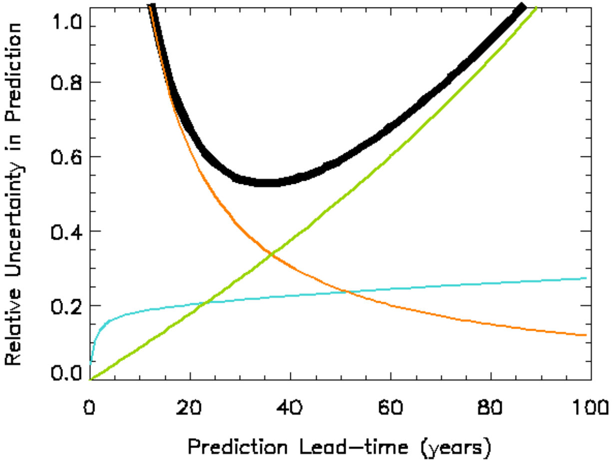
Mathematics and climate change mitigation, Warwick Mathematics Institute, 27 April 2009

IPCC projections dominated by emissions uncertainty



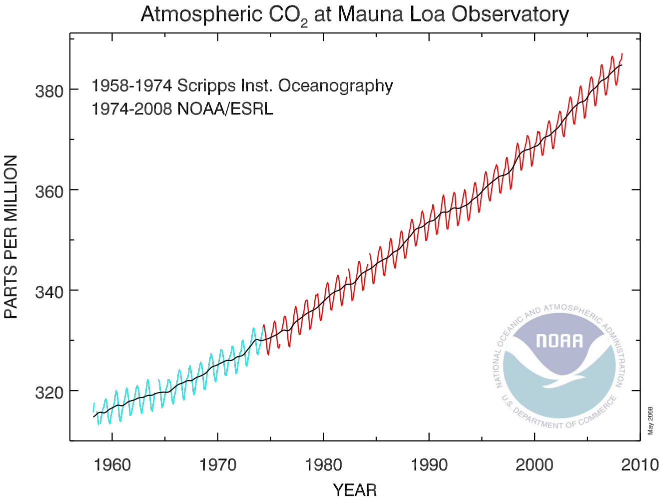
IPCC AR4 WG1 Fig 10.4

Long Term Uncertainty is Dominated by Emissions



Cox and Stephenson, Science, 2007

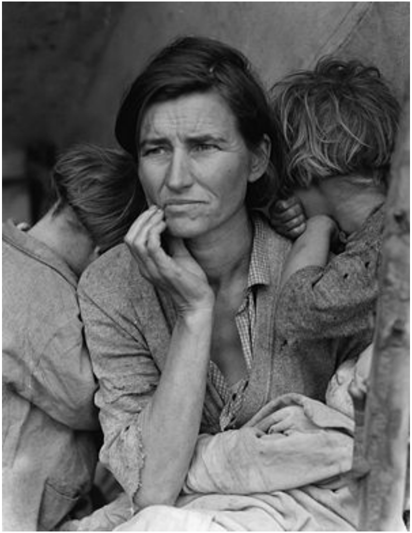
Emission paths are not Optional



Emissions Depend on Economy

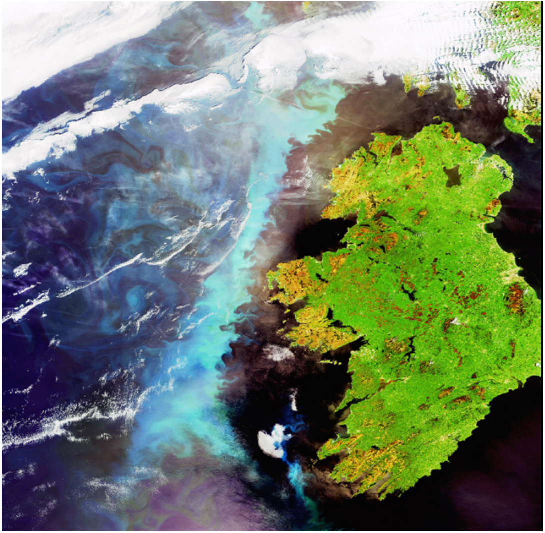
http://airlineworld.wordpress.com

Economy Depends on Climate



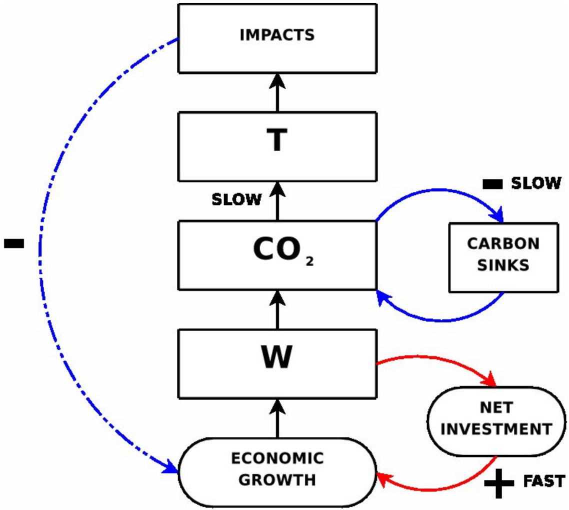
http://en.wikipedia.org/wiki/Florence_Owens_Thompson

Climate Depends on Emissions



http://www.awi.de/en/news/press_releases/

Fast positive feedback and slow negative feedback



3 Variable Model: Assumptions

$$\frac{\partial \hat{C}}{\partial \hat{t}} = \chi e^{-\mu \hat{t}} \hat{W} - \frac{1}{\tau_C} \left(\hat{C} - C_{PI} \right);$$

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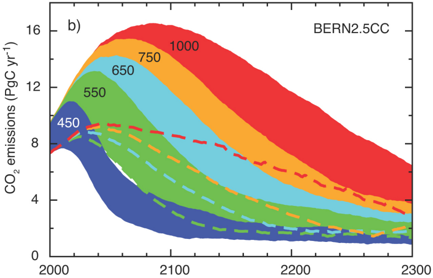

IPCC AR4 WG1 Fig 10.22

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3 Variable Model: Non-dimensional form

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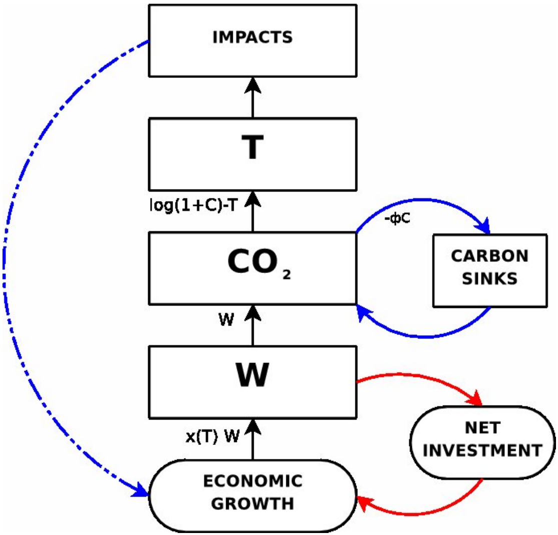
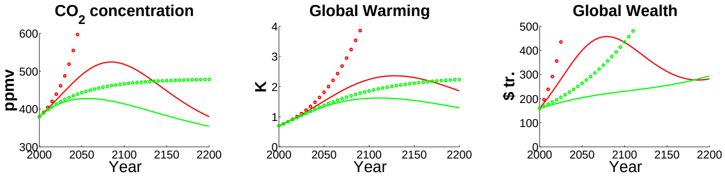
Fast positive feedback and slow negative feedback



3 Variable Model: Projections

Decarbonisation assumed to be 1%, $\tau_T = \tau_C = 50$ years, $\Delta T_{2*CO2} = 3$K, $\frac{\partial \hat{W}}{\partial \hat{t}} = \xi_0 \left(1 - 0.5\,\hat{T}\right) \hat{W}$



Background rate of economic growth $\xi_0 = 4\%$ (red) and 1% (green)

(In)stability of equilibria

$$\dot{C} = W - \phi C$$
$$\dot{T} = \log(1 + C) - T$$
$$\dot{W} = \mathrm{x}(T) W$$

Linearise, let $\Delta C = C - C_e$ etc

$$\begin{pmatrix} \dot{\Delta C} \\ \Delta T \\ \Delta W \end{pmatrix} = \begin{pmatrix} -\phi & 0 & 1 \\ \frac{1}{1+C_e} & -1 & 0 \\ 0 & \frac{\mathrm{dx}(T_e)}{\mathrm{d}T} W_e & \mathrm{x}(T_e) \end{pmatrix} \begin{pmatrix} \Delta C \\ \Delta T \\ \Delta W \end{pmatrix} + \text{ H.O.T}$$

Stability of Positive Equilibrium depends on pace of Expansion

Jacobian is

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Characteristic polynomial is

$$\mu\,(\mu+\phi)\,(\mu+1) - \frac{\frac{\mathrm{d}x(T_e)}{\mathrm{d}T} W_e}{1+C_e} = 0$$

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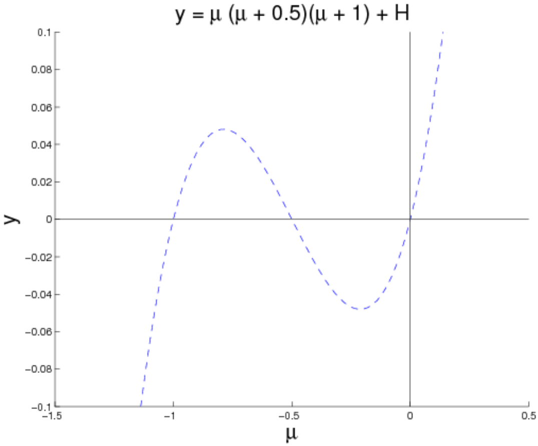
I.e.

$$\mu\left(\mu+\phi\right)\left(\mu+1\right) \underbrace{- \frac{\mathrm{d}x(T_e)}{\mathrm{d}T}\phi\left(1-e^{-T_e}\right)}_{h} = 0$$

Eigenvalues of Jacobian at positive equilibrium may have Negative or Positive Real Part



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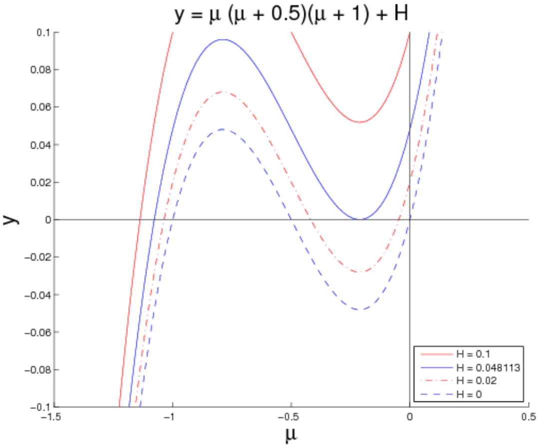

Stability in terms of dimensional variables

Unstable iff

$$-\frac{\Delta T_{2*CO2}}{\log 2}\frac{\mathrm{d}\hat{x}(\hat{T}_e)}{\mathrm{d}\hat{T}}\left(1-e^{-\hat{x}^{-1}(\mu)\log 2/\Delta T_{2*CO2}}\right) > \frac{1}{\tau_T}+\frac{1}{\tau_C}.$$

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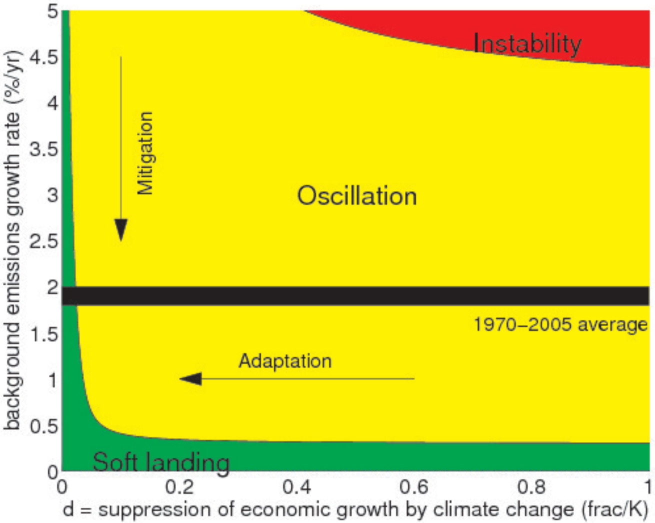
Soft-landing if

$$-\frac{\Delta T_{2*CO2}}{\log 2}\frac{\mathrm{d}\hat{x}(\hat{T}_e)}{\mathrm{d}\hat{T}}\left(1-e^{-\hat{x}^{-1}(\mu)\log 2/\Delta T_{2*CO2}}\right)<\frac{1}{8}\min\left(\frac{1}{\tau_T},\frac{1}{\tau_C}\right).$$

Example: $\frac{\partial \hat{W}}{\partial \hat{t}} = \xi_0 \left(1 - d\,\hat{T}\right) \hat{W}.$



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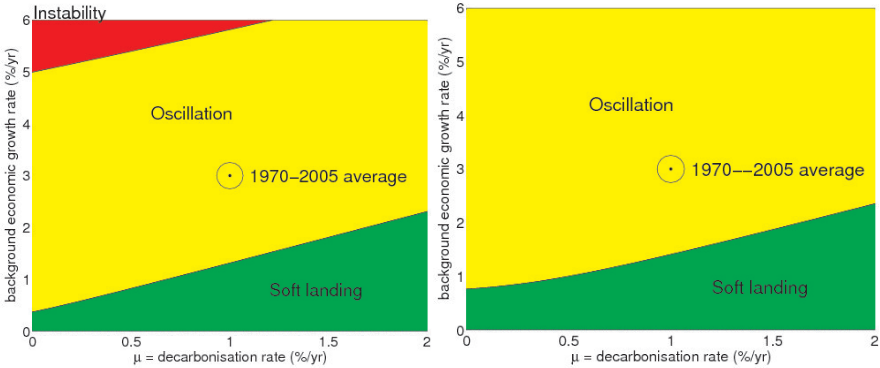
d = 0.5 (left); d = 0.1 (right)



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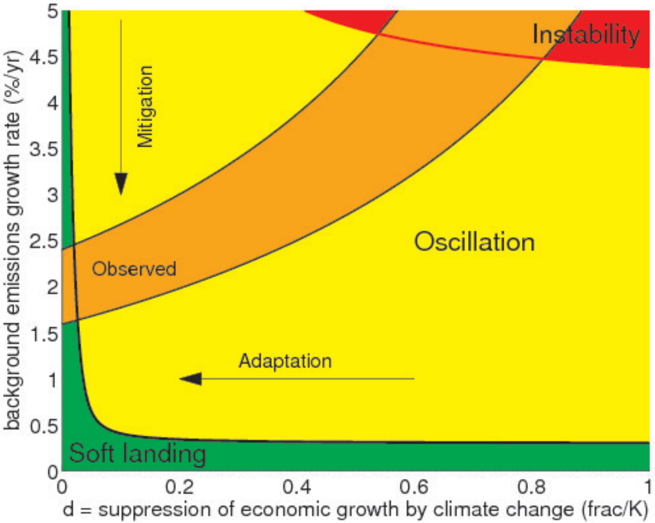

Summary

- Coupled climate-economy dynamics are very different to uncoupled dynamics.
- Under the current model assumptions, an equilibrium is possible in climate and emissions, with economic growth equal to the decarbonisation rate. Thus mitigation is necessary for sustainable growth.
- Decarbonisation alone does not enable a soft-landing on the equilibrium. Adaptation and an economic slow-down is required.

Thank you!


köszönöm
תודה!
děkuji
mahalo
고맙습니다
thank you
merci
谢谢
danke
Ευχαριστώ
شكرا
どうもありがとう
gracias