evolutionary dynamics of collective action

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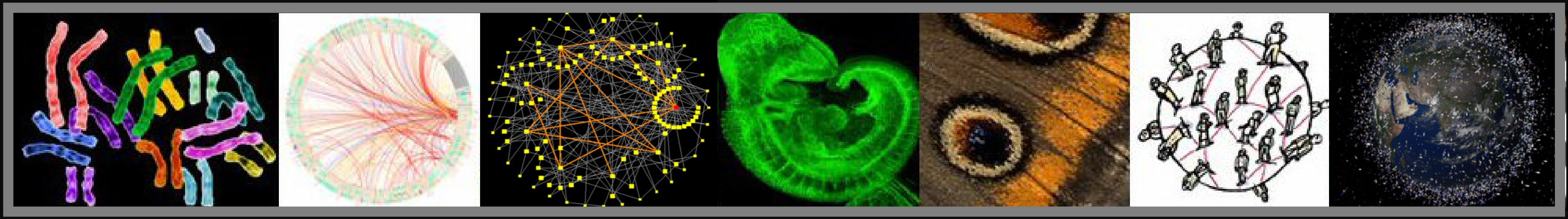
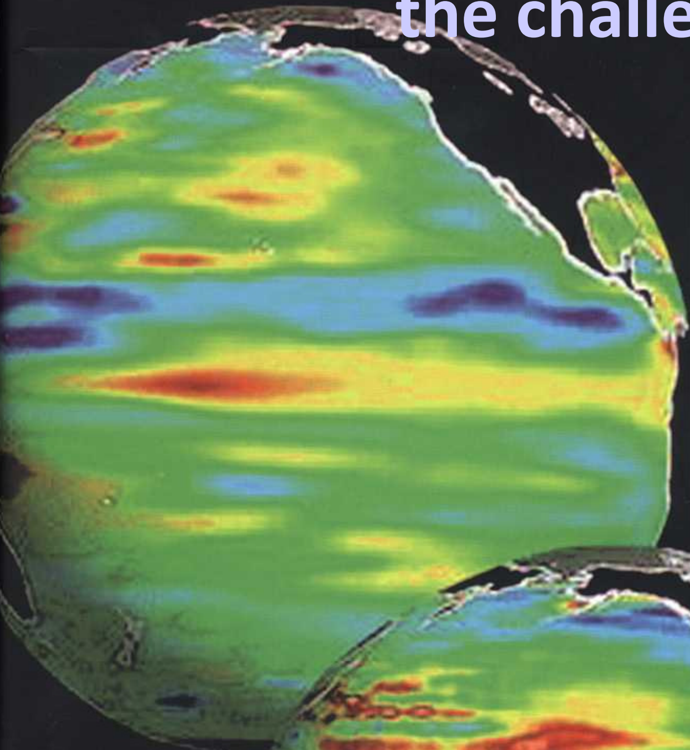
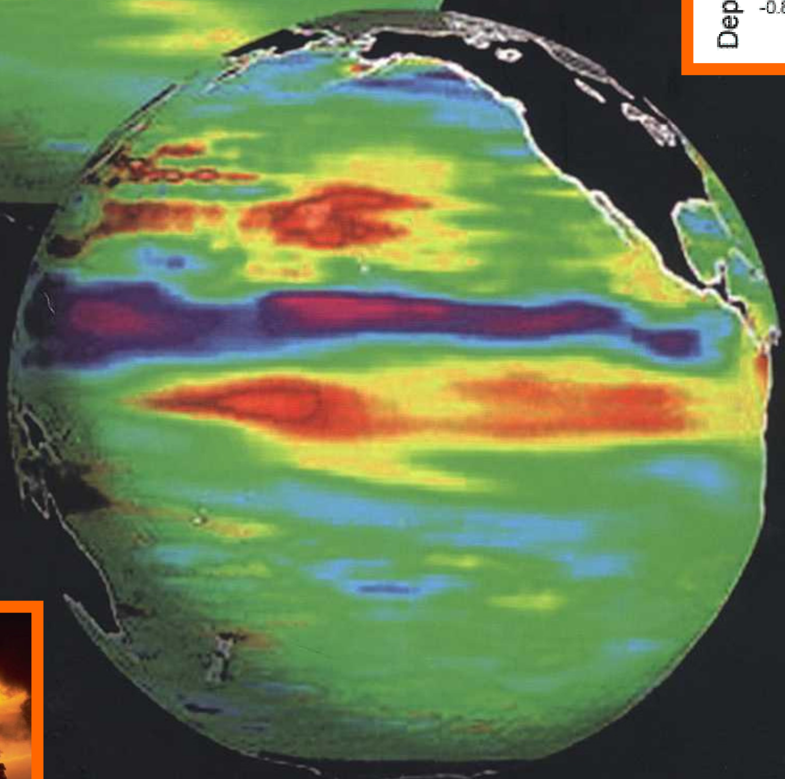
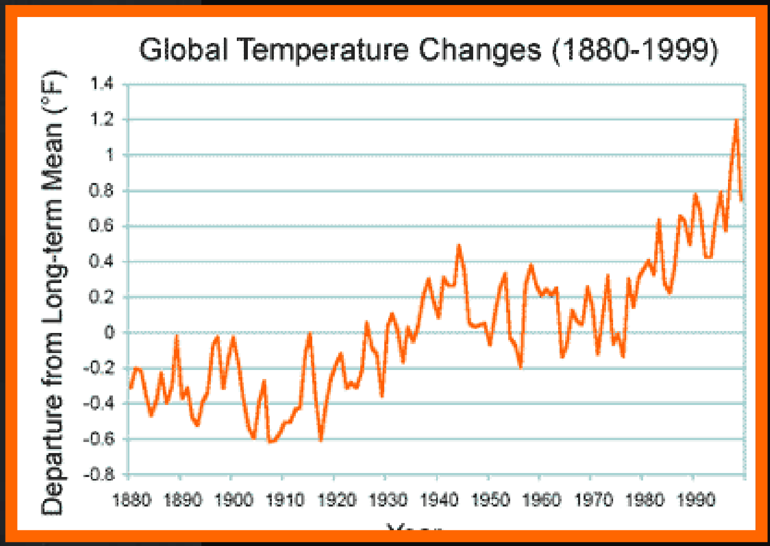


warwick - UK, 27-apr-2009

the challenge of minimizing climate change







saving the climate requires …

cooperation !

the challenge of minimizing climate change

Cooperation between individuals

Cooperation between countries

	per capita CO_2 emmisions
france / sweden	x
UK / japan	2x
USA	3x

cooperation

cooperation is on the basis of some of the
major transitions in evolution

[Maynard-Smith & Szathmáry, *The major transitions in evolution*, OUP95]

cooperation is essential for the evolution of reproductive entities

genes *cooperate* to form cells
cells *cooperate* to form multi-cellular organisms
individuals *cooperate* to form groups and societies
human culture is a *cooperative* process.

[Nowak, *Evolutionary Dynamics: Exploring the Equations of Life*, HUP06]

how to formalize cooperation ?

Donor
Pays a cost c

→

Receiver
Receives a benefit b

b>c

Why should we cooperate?

if natural selection is based on competition, how can it lead to cooperation ?

cooperation & climate change

Cooperation between individuals

Cooperation between countries

the cooperation we need to consider involves collective action

→ public goods games (N-person games)

	per capita CO_2 emissions
france / sweden	x
UK / japan	2x
USA	3x

preserving the global climate is the biggest public goods game ever

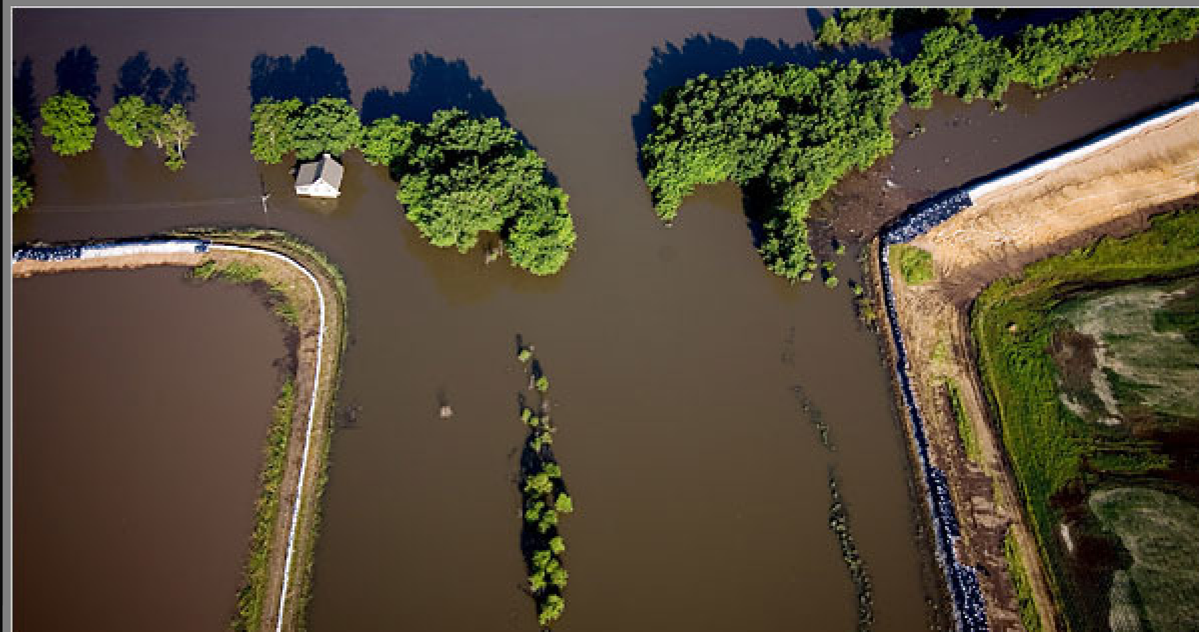
cooperation among humans

- **collective action to shelter & protect**

why do people contribute to public goods?

cooperation among humans

- collective action to shelter & protect



Natural Disaster Assistance for Missouri Citizens
How to Construct a Sandbag Emergency Levee

cooperation among humans

- **collective action to hunt & nourish**

  involves the coordinated action of many. . .

why do people contribute to public goods?

cooperation among humans

- water sharing
- tax paying and social welfare
- stabilizing the Earth's climate

why do people contribute to public goods?

cooperation among humans

- open source projects …

why do people contribute to public goods?

a game experiment on climate change

[Milinski *et al.*, *PNAS* 195 (2008) 2291

6 players, 10 rounds

each player : 40 €

contribution in each round : 0 (selfish), 2 (fair) or 4 (altruistic)

cost for saving the planet : 120 €

if Σ contributions ≥ 120 €, *planet is saved and each gets away with money left*

if Σ contributions < 120 €, *planet is saved with 10% prob., else all loose everything*

	per capita CO_2 emissions	strategy
france / sweden	x	*altruistic*
UK / japan	2x	*fair*
USA	3x	*selfish*

a game experiment on climate change

[Milinski *et al.*, *PNAS* 195 (2008) 2291

NASH equilibrium : each player contributes 2€ per round

RESULTS :

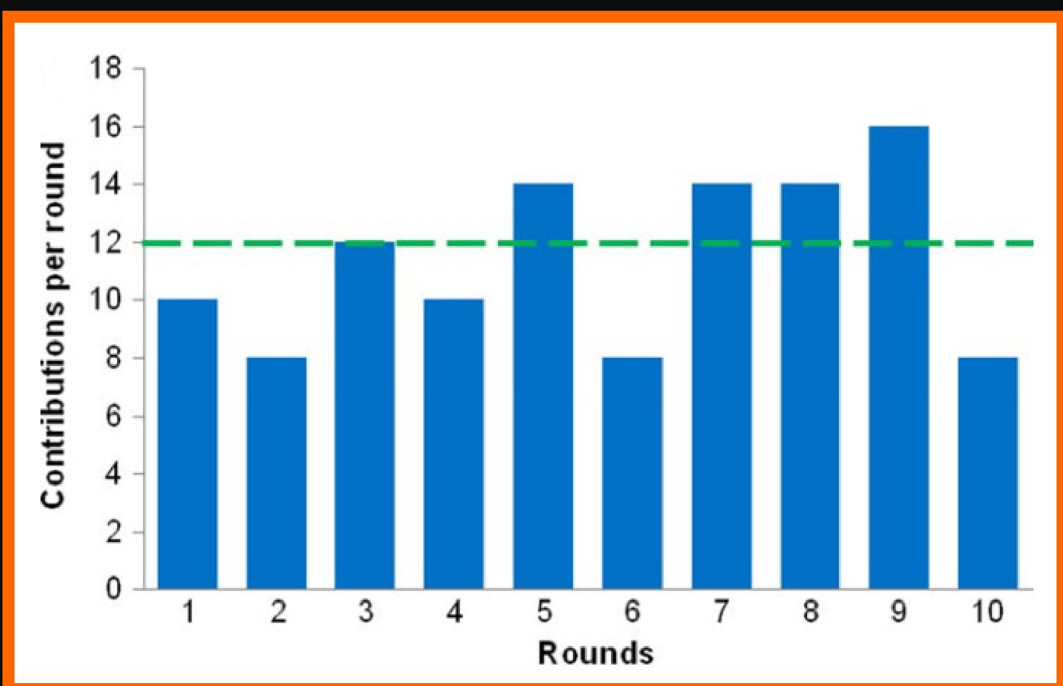
50% of times planet was saved !!!

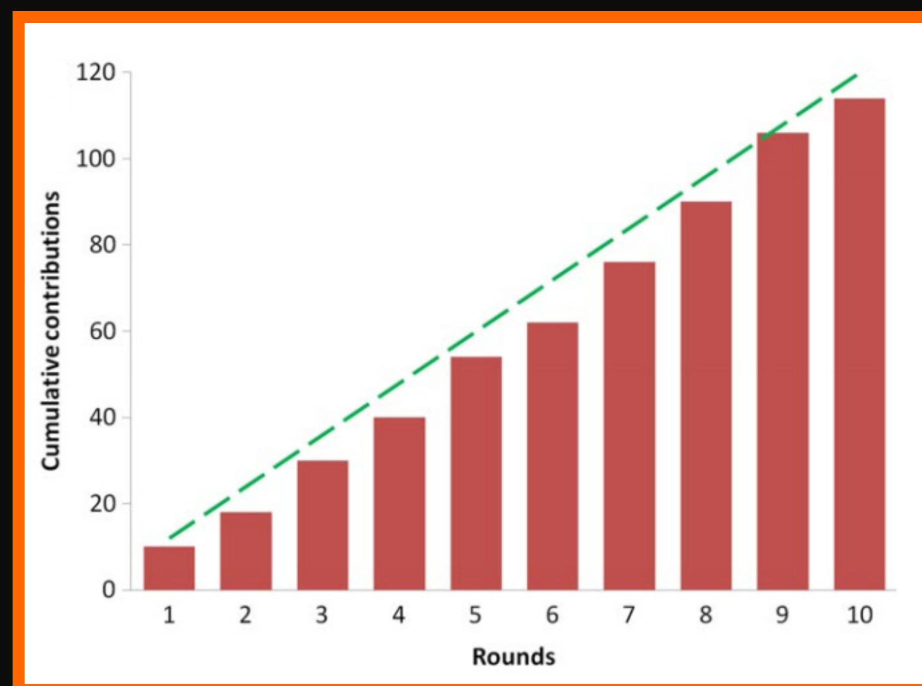
50% of times average contribution = 113 € < 120 €

example of a failed attempt :

did altruists feel they had contributed enough ?

what was in the mind of the free riders ?





a game experiment on climate change

[Milinski *et al.*, *PNAS* 195 (2008) 2291

if :

i. planet is saved with *prob* = 50% → 1 /10 reached 120€
ii. planet is saved with *prob* = 90% → 0 /10 reached 120€

still :

i. <contribution> = 92€
ii. <contribution> = 73€

these experiments portray, once more, among other things, the *bounded rationality* of human participants.

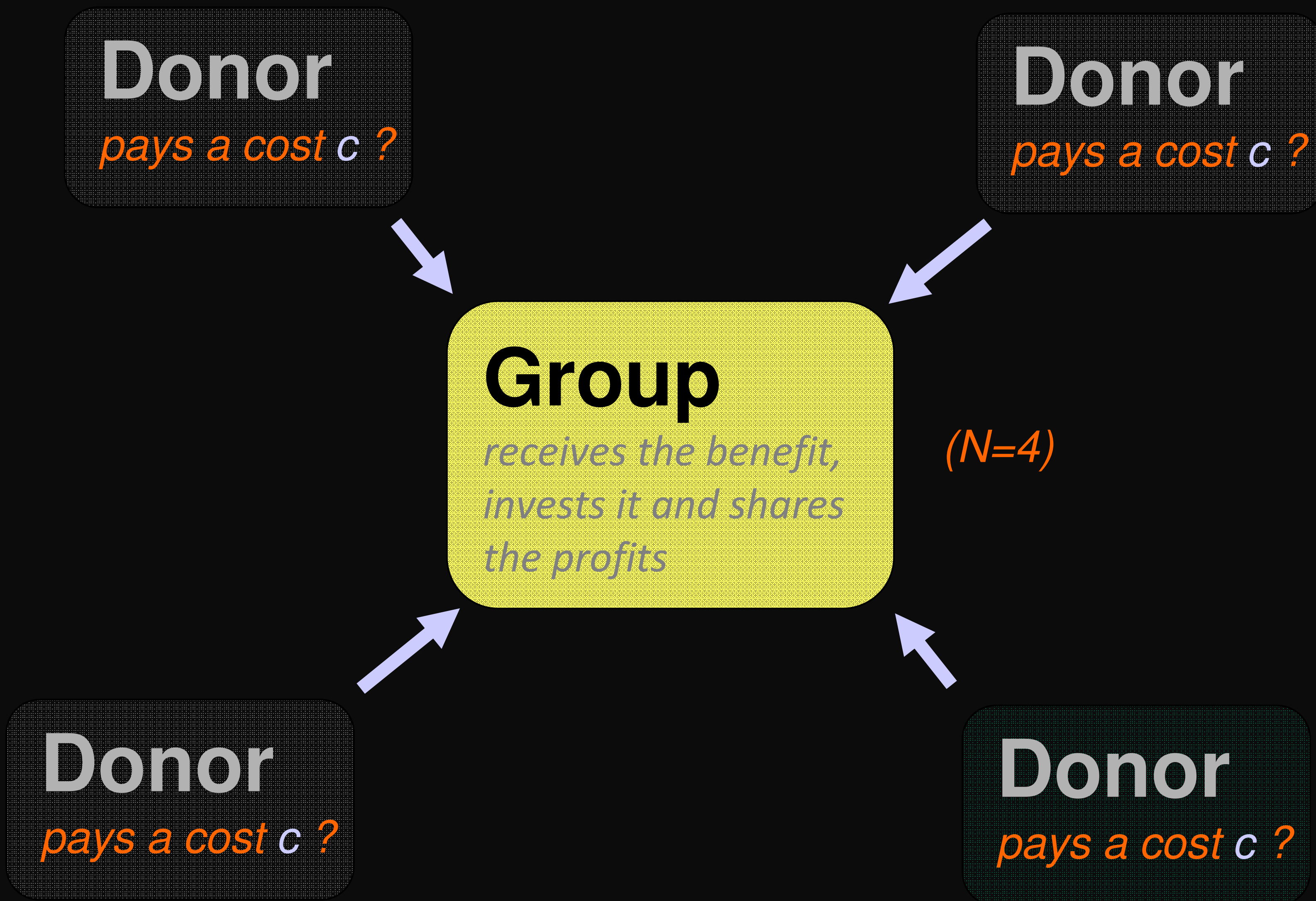
general mathematical tools

Evolutionary Game Theory & the appropriate Game
used by many scientists from diverse disciplines

Specifically

- **Evolution** – ***REPLICATOR DYNAMICS***
- **Games** – ***N-person prisoner's dilemma***
  . . .
- **Communities** – ***HOMOGENEOUS ANSATZ :***
  WELL-MIXED populations

N-person Prisoner's dilemma
or *Public Goods Game*



"RATIONAL" GOAL :
each maximizes own's payoff !

public goods games

N-person Prisoner's Dilemma

-2 types of players:

Cooperators **contribute an amount** c **(cost) to a public good**

Defectors **do not contribute.**

The total contribution is multiplied by *F* and equally distributed among all individuals in the group (of size *N*); then

$$P_D = \frac{Fcn_C}{N}$$

(c=1)

$$P_C = P_D - c$$

assuming someone *contributes*,

it is always best to defect !

evolutionary game theory

- populations are *infinite*; there is a fraction x_C of **Cs** & x_D (= 1- x_C) of **Ds**

- populations are *well-mixed* ; everybody is **equally likely** to interact with everybody else (*mean field*); fitness is obtained by averaging over all possible groups of size N; hence, ALL **Cs** have the same fitness & also ALL **Ds** have the same fitness (*mean field*)

- evolution → *replicator dynamics* : strategies' evolution follow the gradient of natural selection determined by relative fitness

replicator equation

$$\begin{cases} \dot{x}_C = x_C(f_C(\vec{x}) - \phi) \\ \dot{x}_D = x_D(f_D(\vec{x}) - \phi) \end{cases}$$

those strategies whose fitness (reproductive success) exceeds the average fitness Φ of the population will increase in frequency; those that don't will decline.

for *N*-person games (Public Goods Games), we have

$$f_D(x) = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} P_D(k)$$

$$f_C(x) = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} P_C(k+1)$$

replicator equation

$$x_C + x_D = 1 \rightarrow x \equiv x_C \Rightarrow x_D = 1 - x \rightarrow$$ **1 equation !**

$$\dot{x} = x(1-x)\underbrace{\left[f_C(x) - f_D(x)\right]}_{\Delta(x)}$$

equilibria of the replicator equation

$$x = 0 \vee x = 1 \vee \Delta(x) = 0$$

$$\Delta(0) < 0 \Rightarrow x = 0 \;\; is\ stable$$

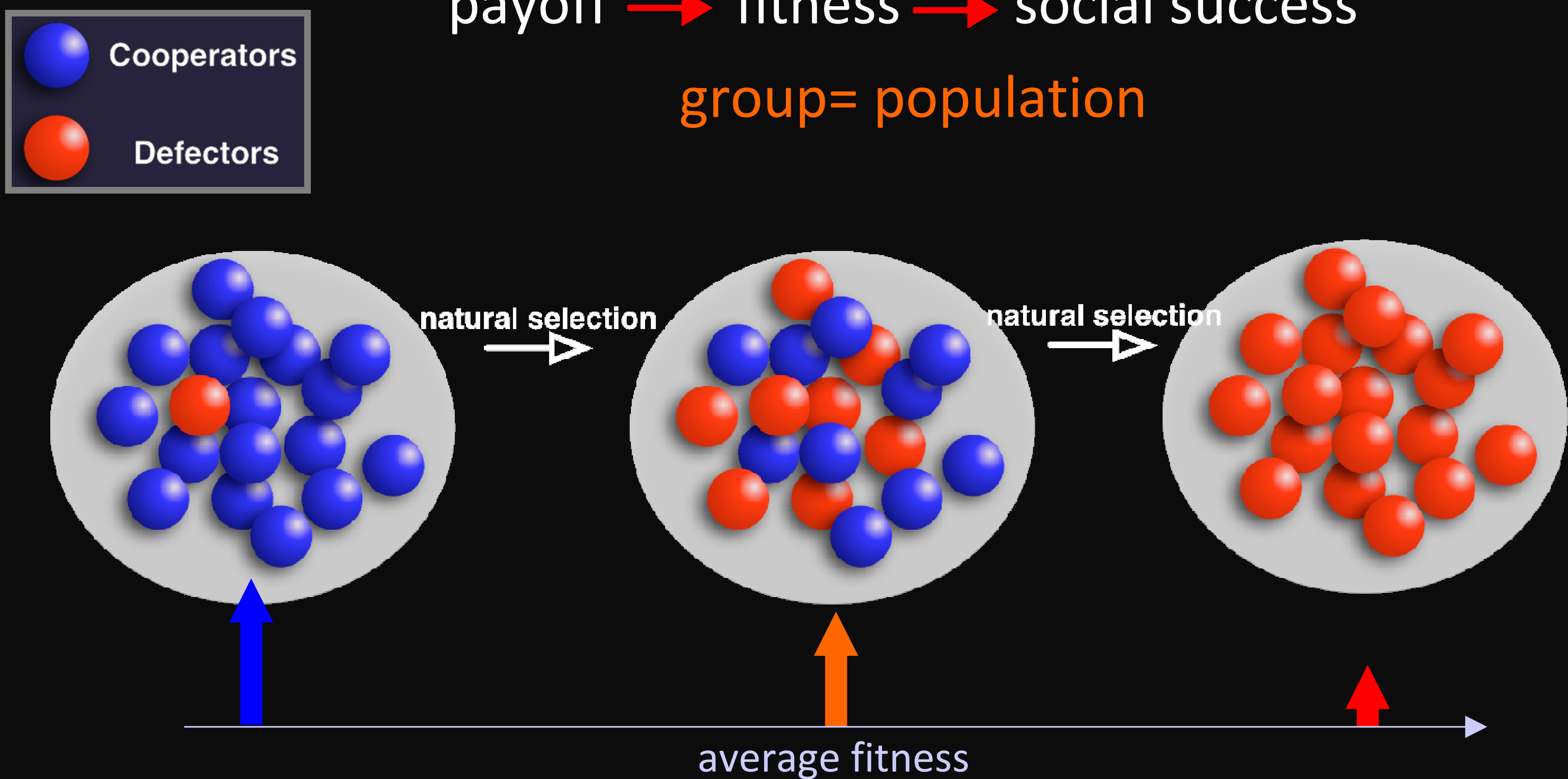
$$\Delta(1) > 0 \Rightarrow x = 1 \;\; is\ stable$$

$$x^* \in \left]0,1\right[: \Delta(x^*) = 0 ; \Delta'(x^*) < 0 \Rightarrow stable\, ;\, else\;\; unstable$$

evolutionary game theory

payoff → fitness → social success

group= population



natural selection favors defection!

natural selection leads to the tragedy of the commons!

***N*-person dilemma** : despite **groups of cooperators** being better-off than **groups of defectors**, individual "rational choice" leads to ***full* defection**

1

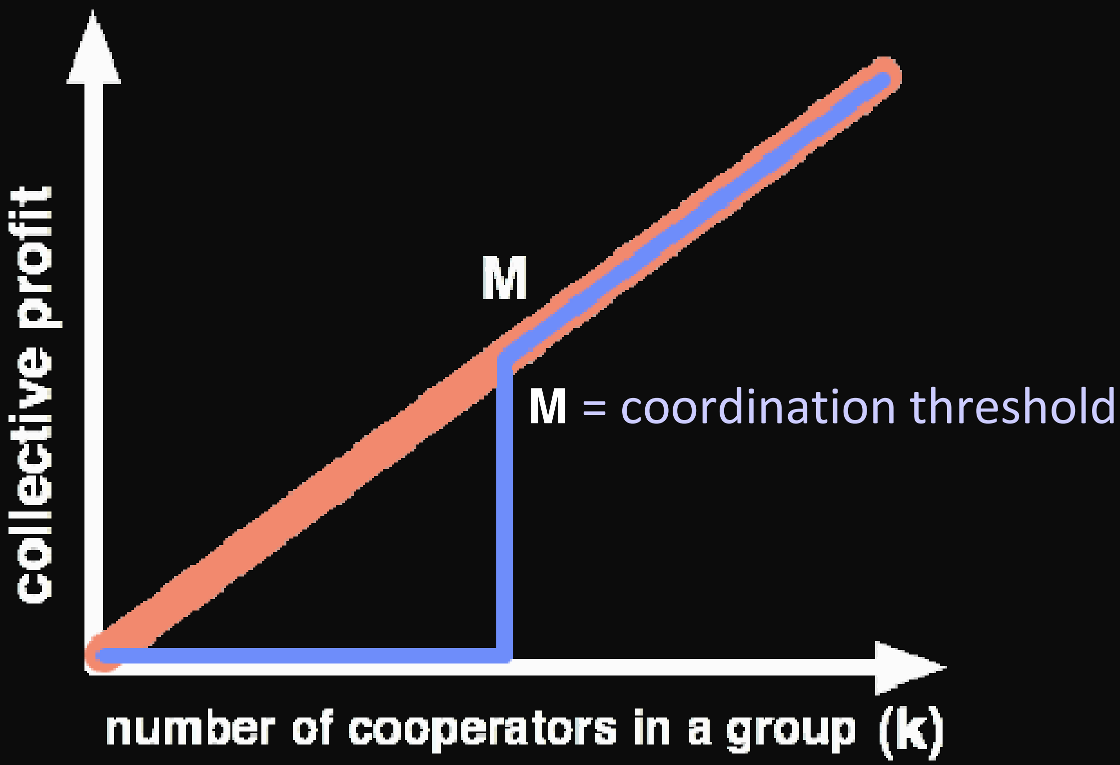
features of (human) collective action beyond conventional EGT

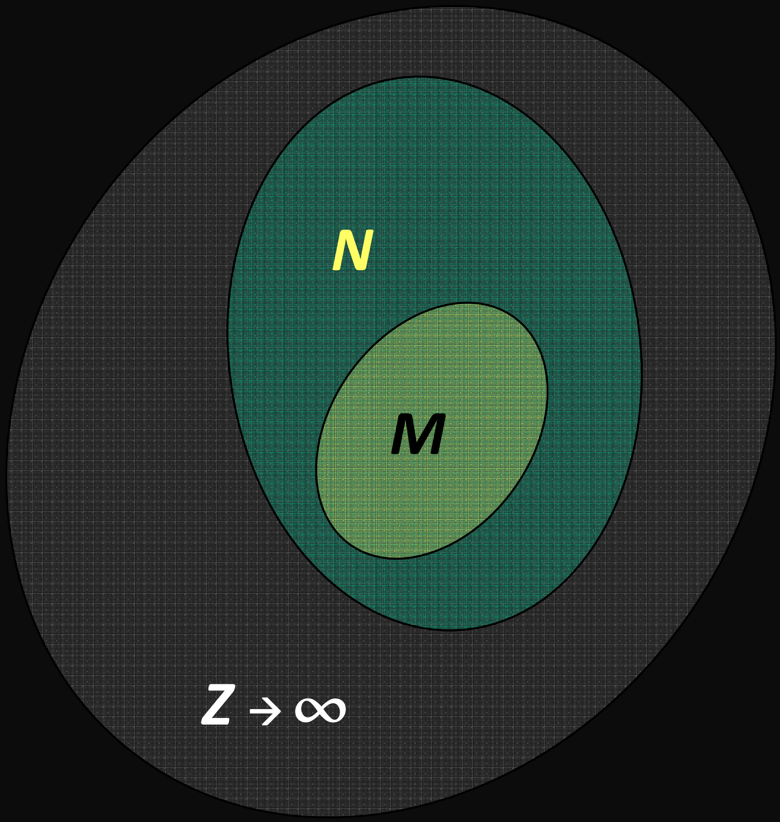
coordination of collective action is often mandatory

often the collective benefit is only achieved above a certain number of contributions.

moreover, real populations are FINITE !!!

N-person Coordination game





what happens when we require partial coordination before achieving collective action ?

N-person Coordination game

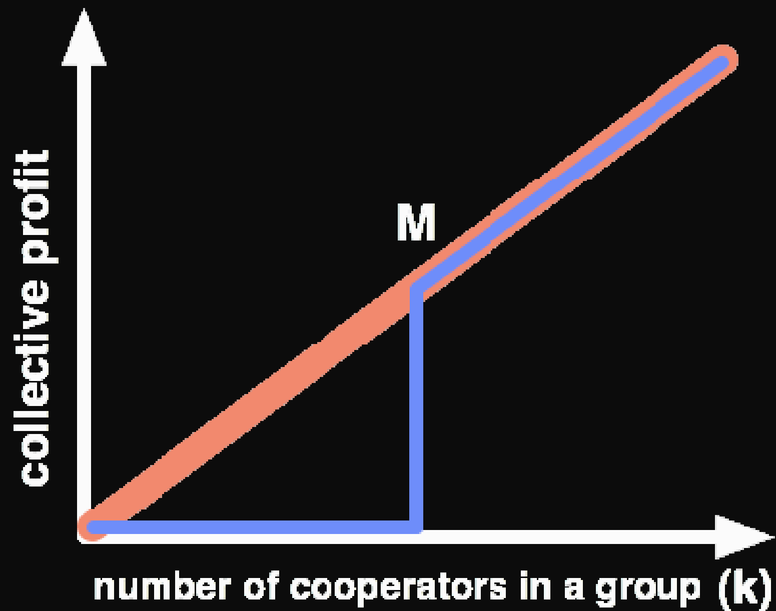
N-person Prisoner's Dilemma with coordination requirements

Cooperators contribute an amount c (cost) to a public good
Defectors do not contribute.

the total contribution (*ck*) is multiplied by *F* and equally distributed among all individuals in the group (of size *N*), only if threshold number of Cs is exceeded !!!

$$P_D = \frac{F}{N} ck\, H(k - M)$$

$$P_C = P_D - c$$



cooperators now can pay a *cost in vain*

N-person Coordination game

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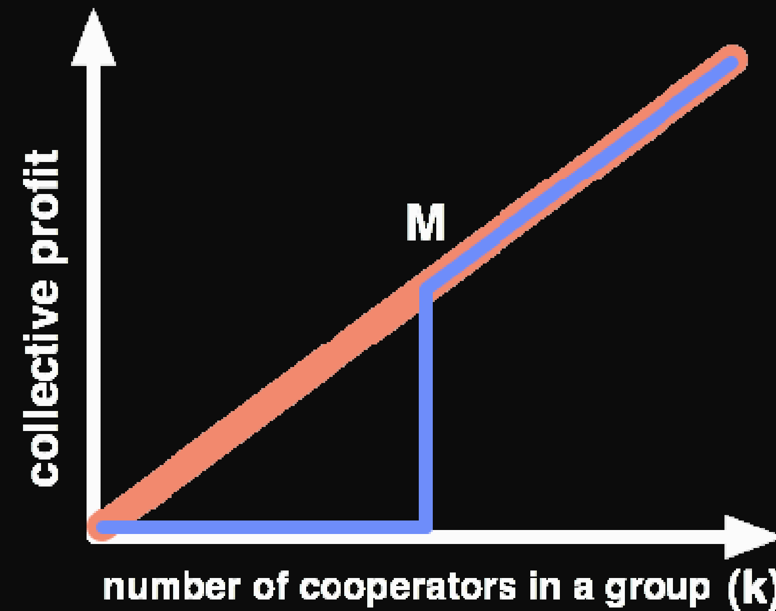
Heaviside step function



cooperators now can pay a *cost in vain*

N-person Coordination game
tricky yet *fun* math

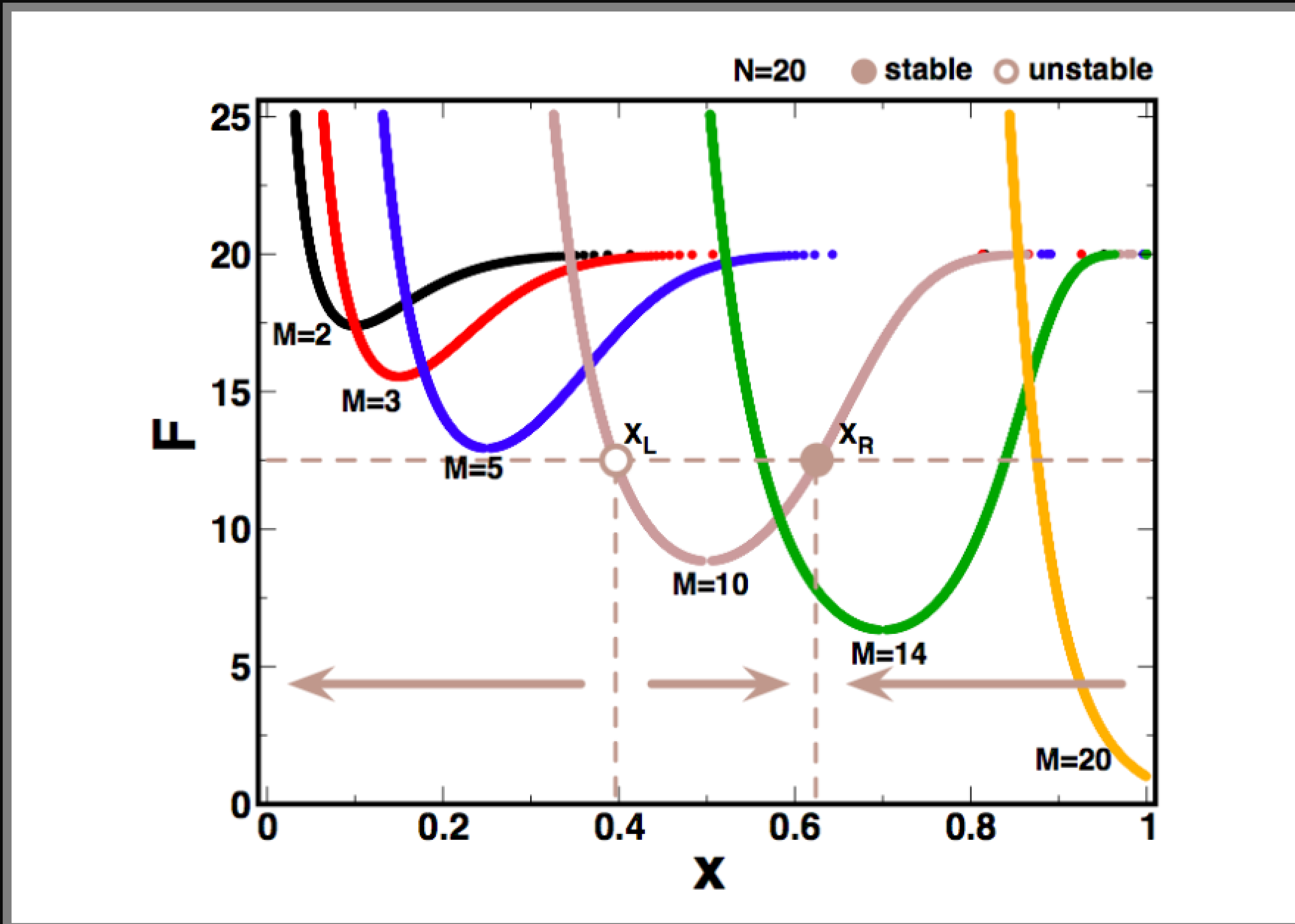
$$f_C - f_D \equiv Q(x) = -c\left[1 - \frac{F}{N}R(x)\right]$$

$$R(x) = x^{M-1}\left[\sum_{k=M}^{N-1}\binom{N-1}{k}x^{k-M+1}(1-x)^{N-1-k} + M\binom{N-1}{M-1}(1-x)^{N-M}\right]$$

$$\lambda^* = R(M/N)$$

	$F/N < \lambda^*$	$F/N = \lambda^*$	$\lambda^* < F/N \leq 1$	$1 < F/N$
stable	0	0	$0,\ x_R$	0,1
unstable	1	$M/N,\ 1$	$x_L,\ 1$	x_L

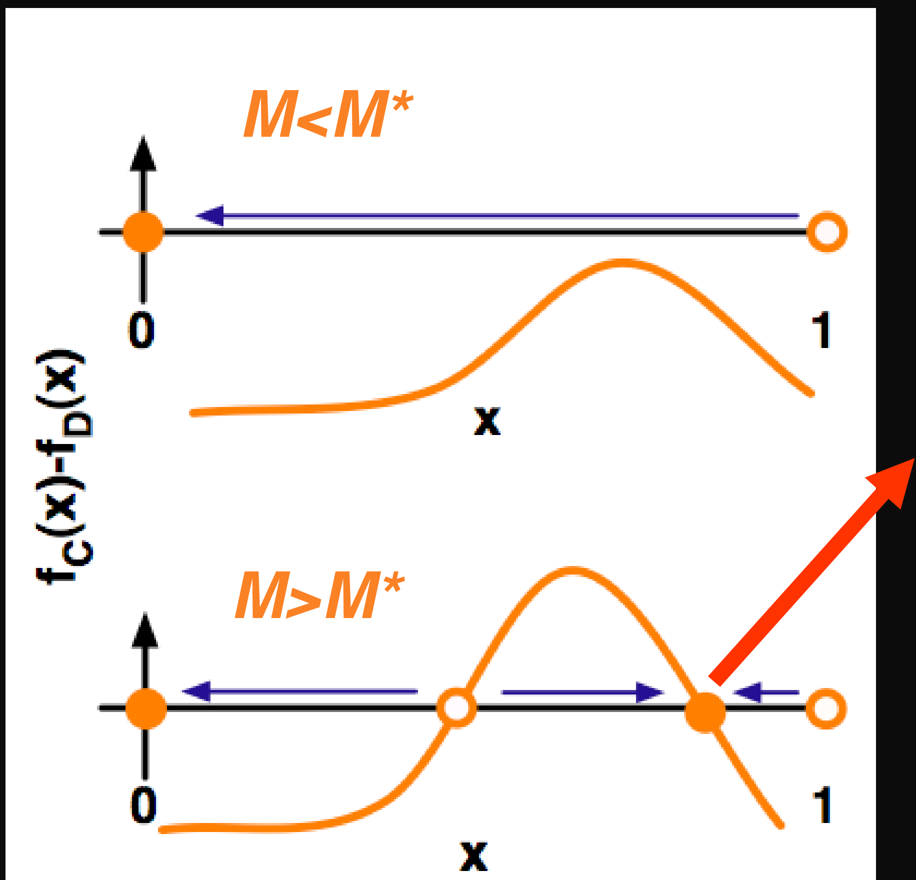
N-person Coordination game



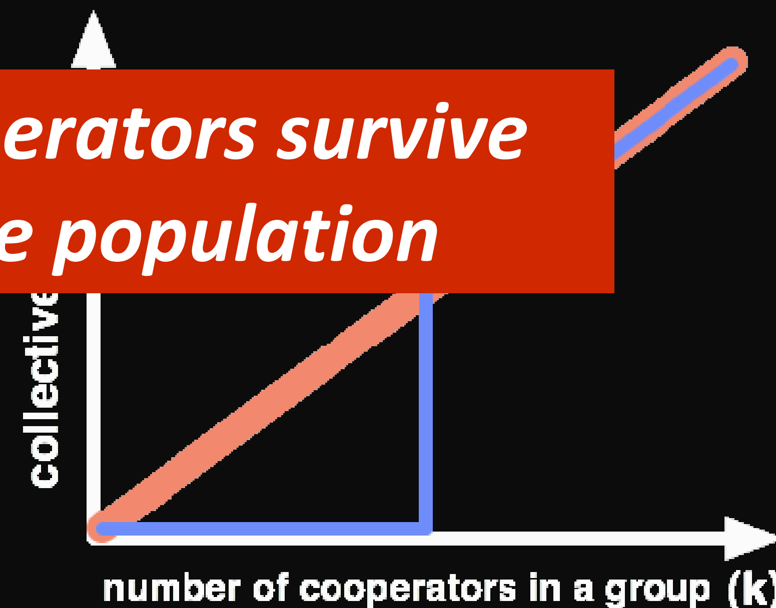
evolutionary dynamics in infinite pops

$f_C - f_D$ = ugly stuff, *but we can still find its roots analytically*

for each value of F (multiplication factor), there's a M* such that



cooperators survive in the population



x = fraction of cooperators

evolution in finite populations

yet, we know that populations are finite...

How to formalize evolutionary dynamics in finite populations?

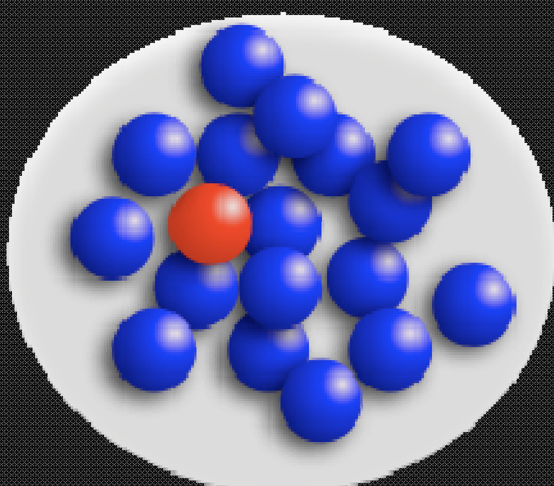
group sampling is no longer binomial but hypergeometric . . .

evolution in finite populations

yet, we know that populations are finite...

How to formalize evolutionary dynamics in finite populations?
group sampling is no longer binomial but hypergeometric . . .

Imagine the simplest form of social learning:



Imitate a random individual with a probability that grows with the payoff difference.

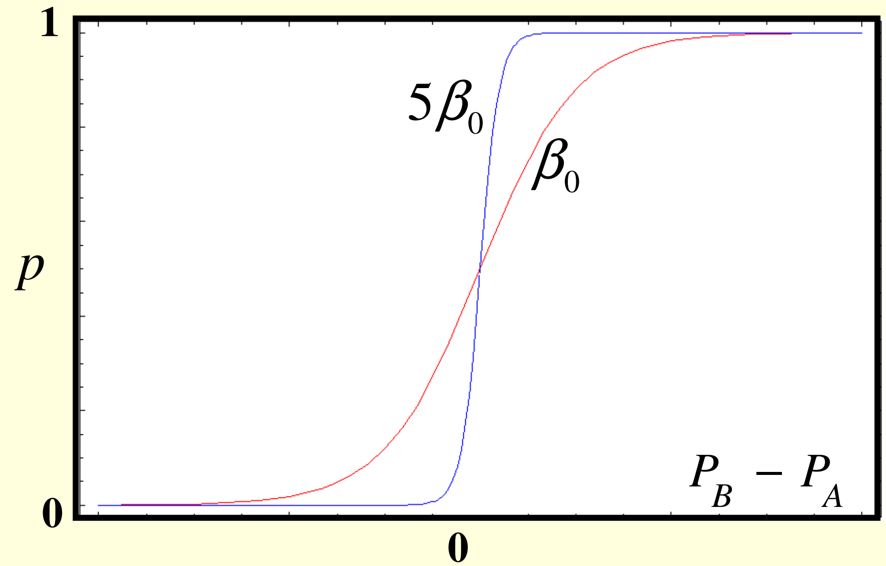
($N \to \infty$ ➡ replicator equation)

evolution in finite populations

at every "time"-step :
- randomly choose 2 individuals (A & B) from the population ;
- strategy of B replaces that of A with a prob given by

$$p = \left[1 + e^{-\beta(P_B - P_A)}\right]^{-1}$$

- P_A ***and*** P_B ***are the payoffs of A and B ;***
- β ***controls how smoothly the probability changes from 0 to 1 :***



$\beta << 1$:

Weak selection

Pacheco, Traulsen, Nowak, Phys Rev E (2006)

evolution in finite populations

at each time step we have a probability to increase and to decrease the number of cooperators, which is

(Z=population size, *j*=number of cooperators)

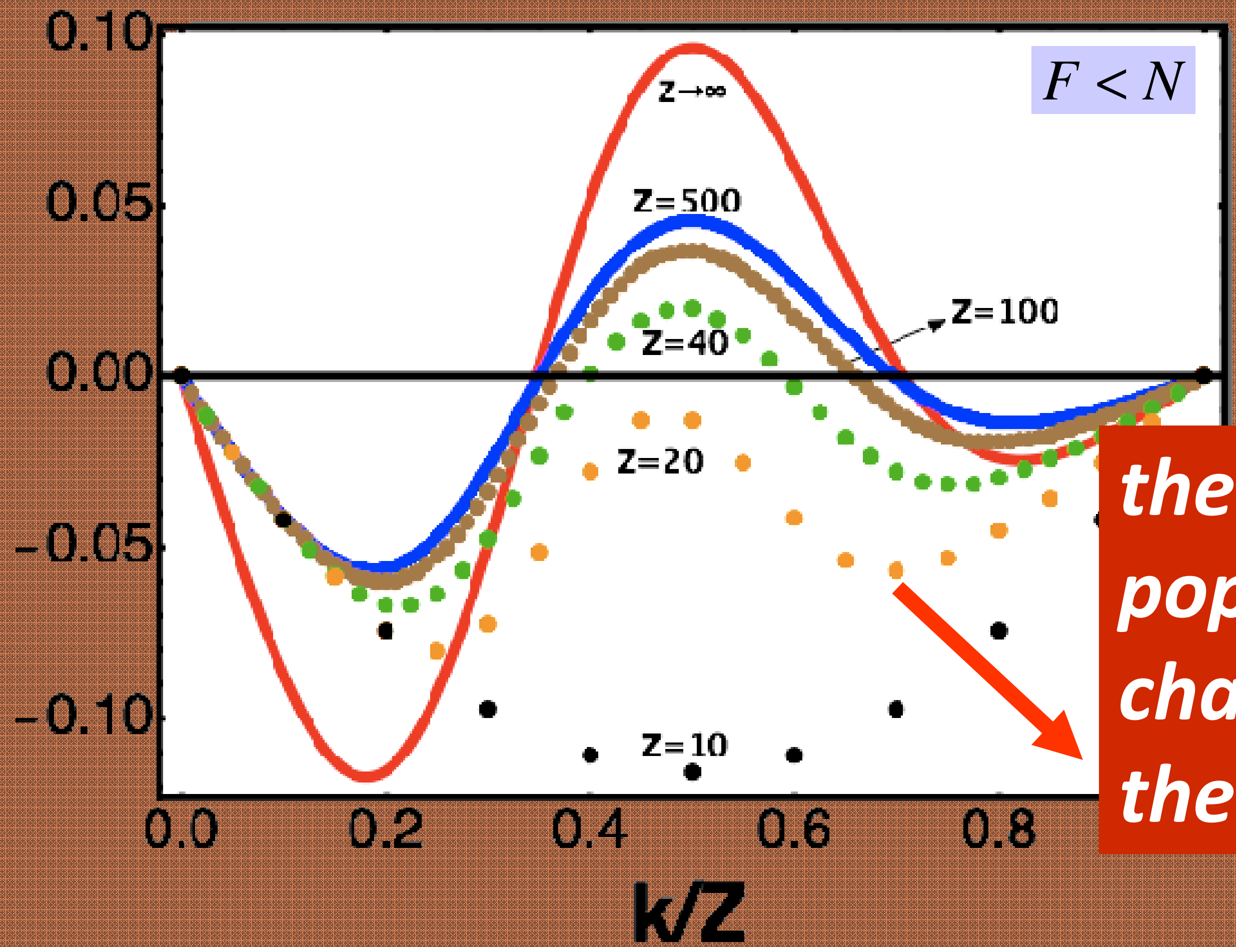
$$T_j^{\pm} = \frac{j}{Z} \frac{Z-j}{Z} \frac{1}{1+e^{\mp\beta(f_C(j)-f_D(j))}}$$

- $\frac{j}{Z}$: prob select C
- $\frac{Z-j}{Z}$: prob select D
- $\frac{1}{1+e^{\mp\beta(f_C(j)-f_D(j))}}$: take-over prob

the gradient of selection becomes . . .

$$g(k) \equiv T^+(k) - T^-(k) = \frac{k}{Z}\frac{Z-k}{Z}\tanh\left\{\frac{\beta}{2}\left[f_C(k) - f_D(k)\right]\right\}$$

evolution in finite populations



F = mult. Factor = 8
N =Group Size = 10
M =Threshold = 5
k = total nº of cooperators
Z = population size

the size of the population can change the nature of the game

conclusions (1)

when a minimum number of cooperators is needed to perform a collective task, a rich dynamics with multiple equilibria emerges

cooperation can be achieved if a minimum fraction of cooperators is present from the start

cooperation can become stable

whenever one takes into consideration that populations are finite, evolutionary dynamics can be profoundly affected.

Pacheco, Santos, Souza, Skyrms, Proc. Royal Society ***B*** 276 1655 (2009)

2

***well-mixed assumption is <u>unrealistic</u>* in large populations**

modern societies are regulated by complex networks of exchange and cooperation

individuals have different roles in the communities and may decide to cooperate depending on their social context

diversity of roles

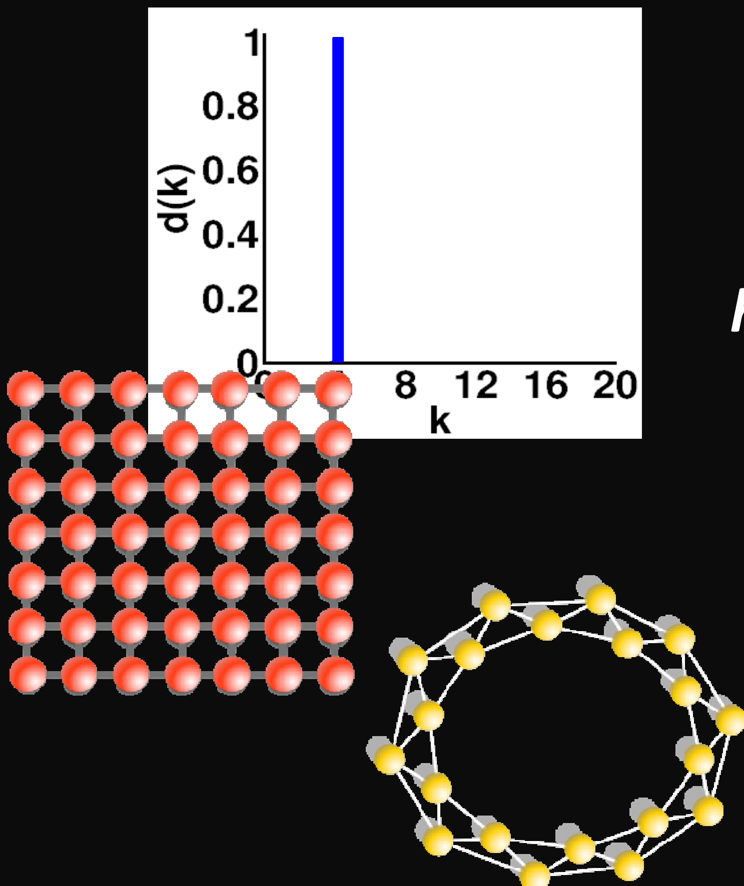
new input

(>1998) computer analysis of social, biological, chemical etc. interaction networks reveals existence of small-world effects and power-law distribution of connectivities.

most real world networks are extremely heterogeneous

a world of complex ties

regular graphs, lattices...

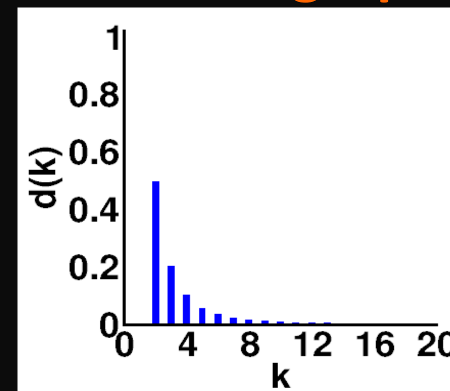


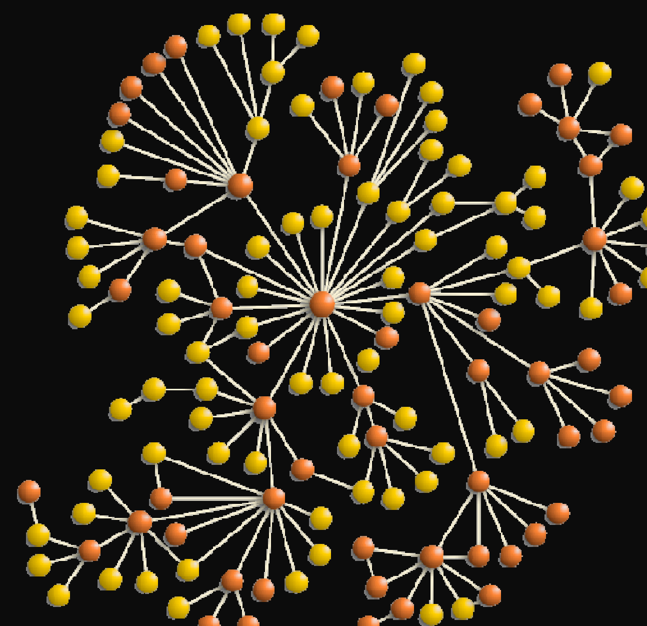
k = degree = number of connections

heterogeneity

+
small-world effects

scale-free graphs

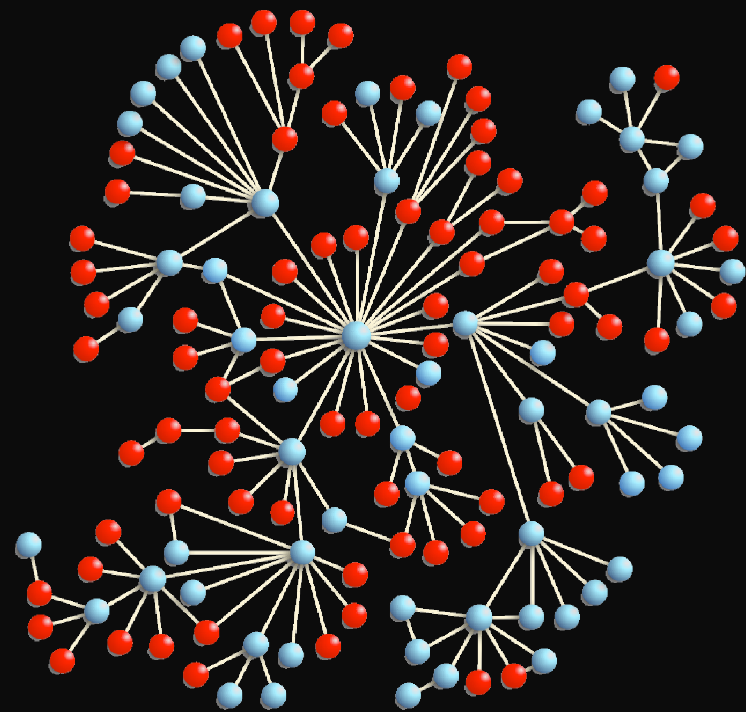




growth
+
preferential attachment

Barabási & Albert, Science 1999

Barabási & Albert scale-free small-worlds



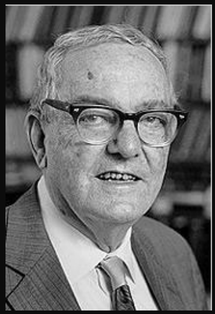
start with m_0 vertices; then :

(**growth**) : add one vertex at a time and create m edges connecting this vertex to m other existing vertices;

(**preferential attachment**): edge $i \to j$ ($j=1,...,m$) is chosen with *prob.*

$P_i = k_i / \Sigma_i k_i$ where k_i is the connectivity of vertex i.

preferential attachment is well-known . . .



Economics

rich get richer effect (Simon, 1955)

History of Science

cumulative advantage (Derek Price, 1965)

(the more you are cited the more likely you'll be cited again)

Sociology

Matthew effect (Merton 1968)

(if a Nobel laureate is the 1st author, all others are thought of as technicians)

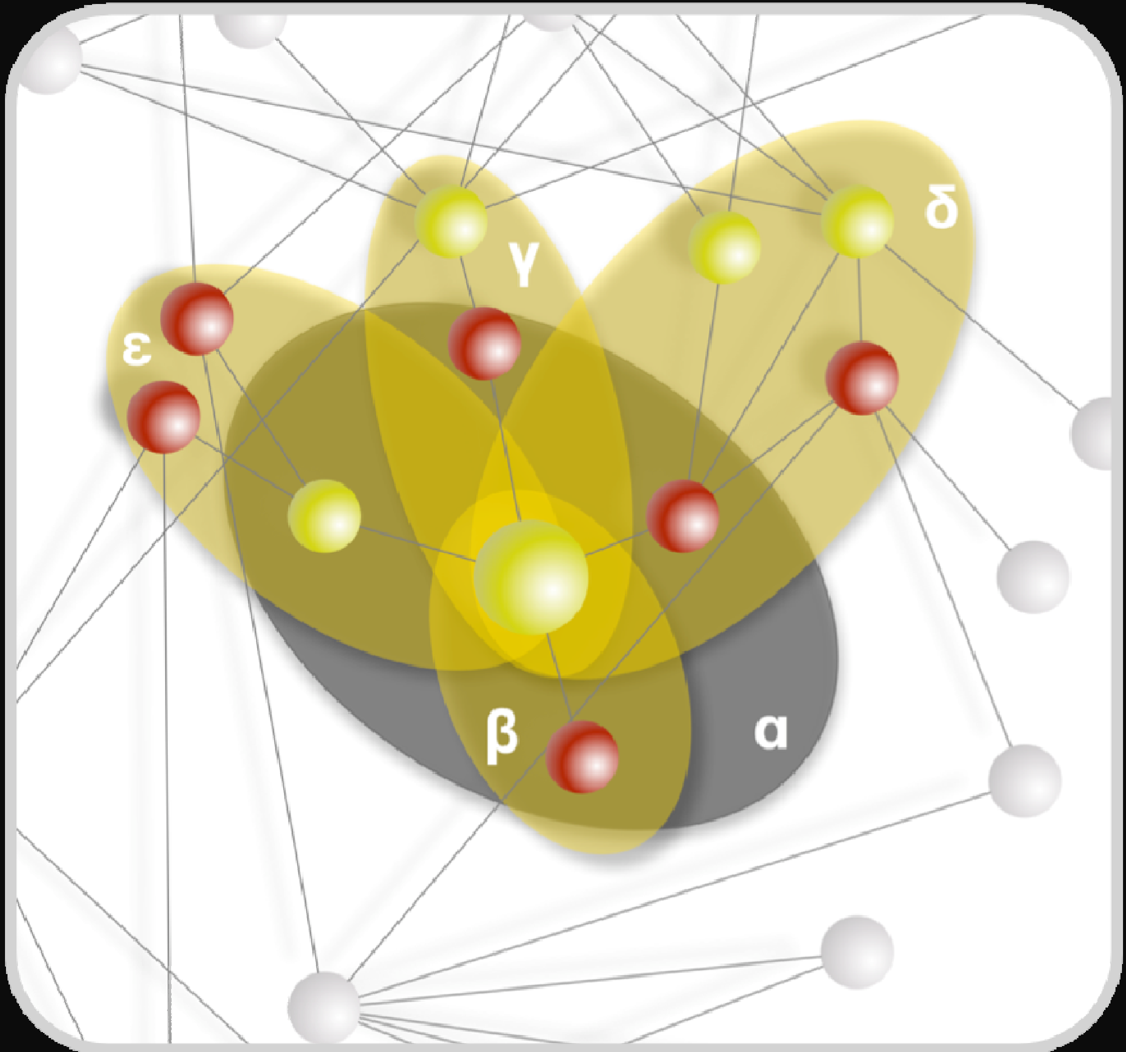
introducing diversity

Public Goods Games in structured populations

k = degree = number of connections

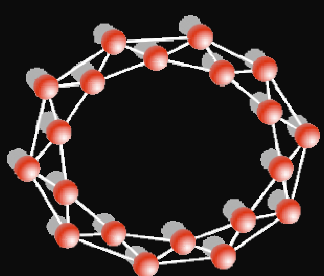
each individual plays k+1 public goods games

the size of each public good is defined by the connections of each central individual

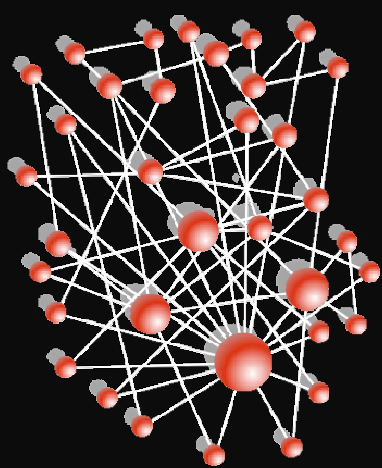


introducing diversity

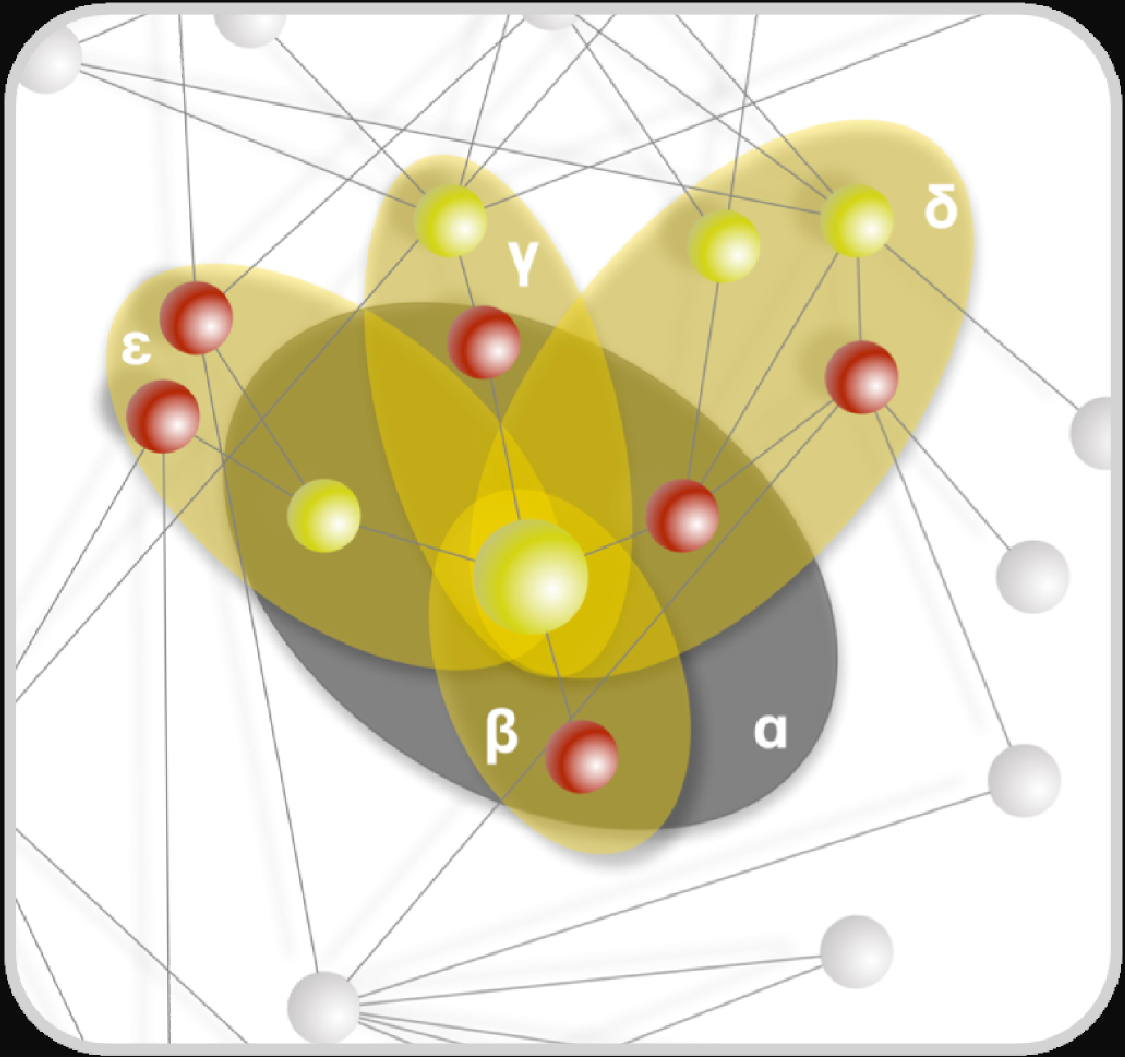
graph heterogeneity leads individuals to engage in different numbers of PGGs with different group sizes



homogeneous



heterogeneous



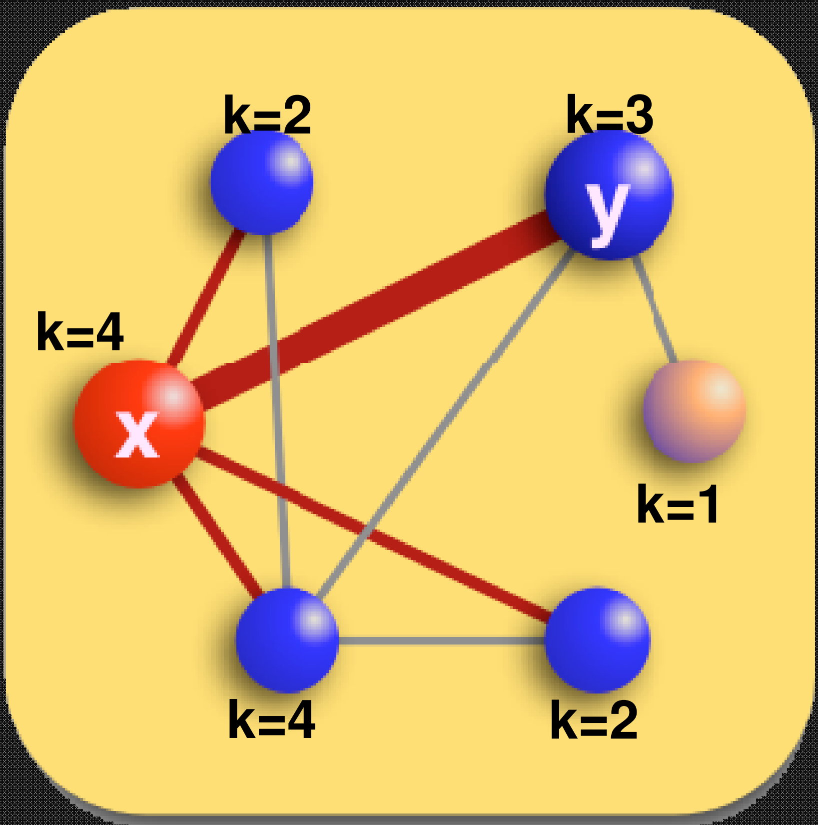
diversity (*symmetry breaking*) :
a new route to cooperation under collective action ?

***cooperators* will increase their fitness to the extent they succeed in maximizing their amount of cooperative interactions per generation.**

***defectors* will also increase their fitness by exploiting more cooperators per generation.**

WHO – defectors or cooperators – will be able to profit from diversity ?

evolution on graphs

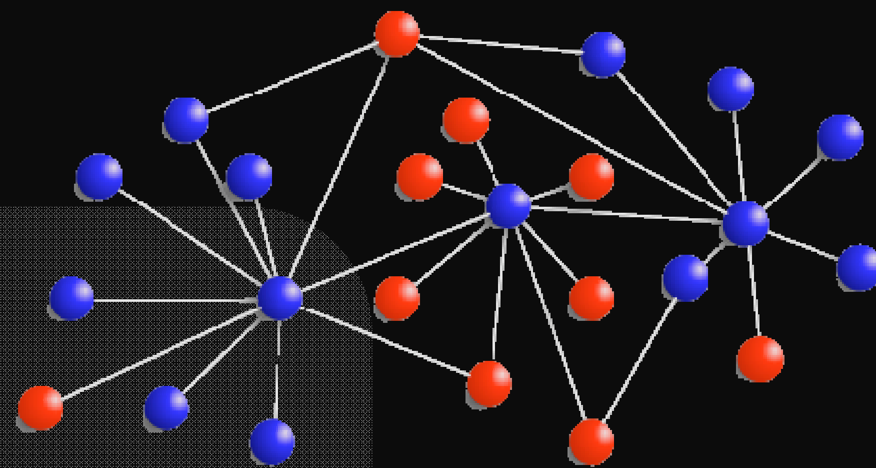


Imitate a random neighbor with a probability that increases with the payoff difference.

$$p = \left(f_y - f_x\right) / M$$

F. C. Santos & J. M. Pacheco, Phys. Rev. Lett. 95 (2005) 098104.

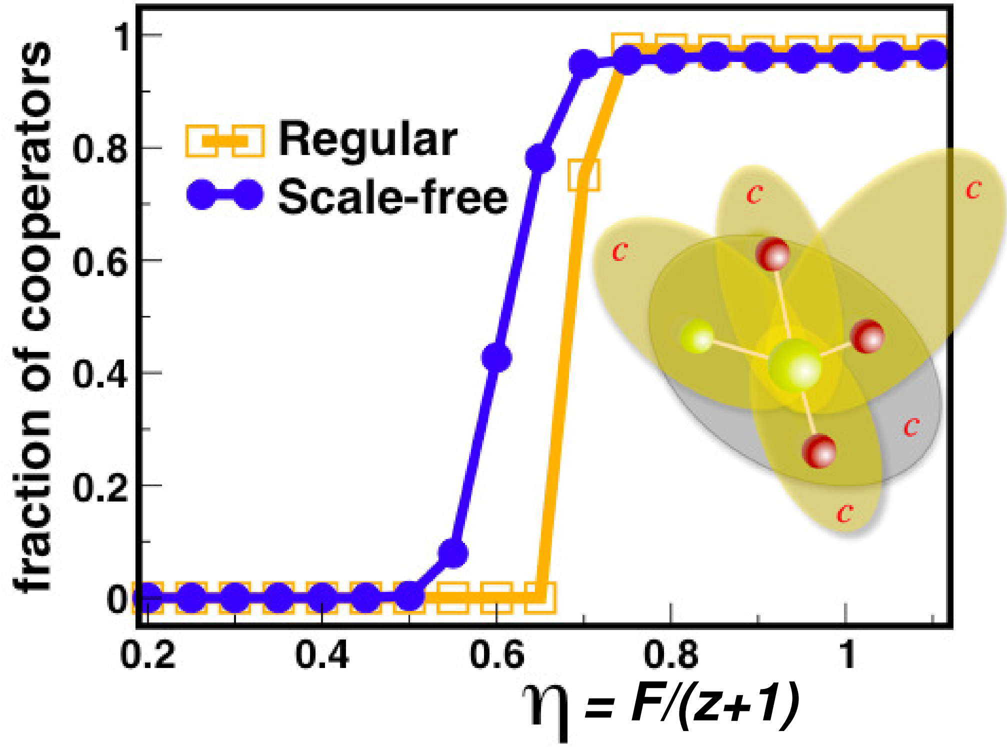
simulations on graphs

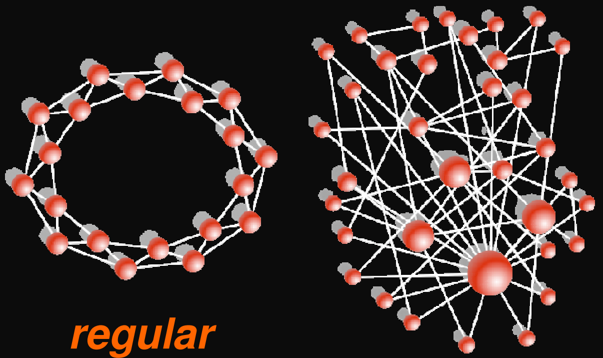


setup:

- Community = static graph
- 50% of Ds and 50% of Cs.
- evolve for 10^6 generations
- run many simulations for one graph.
- run many realizations for same class of graphs.

results





z = average degree = 4

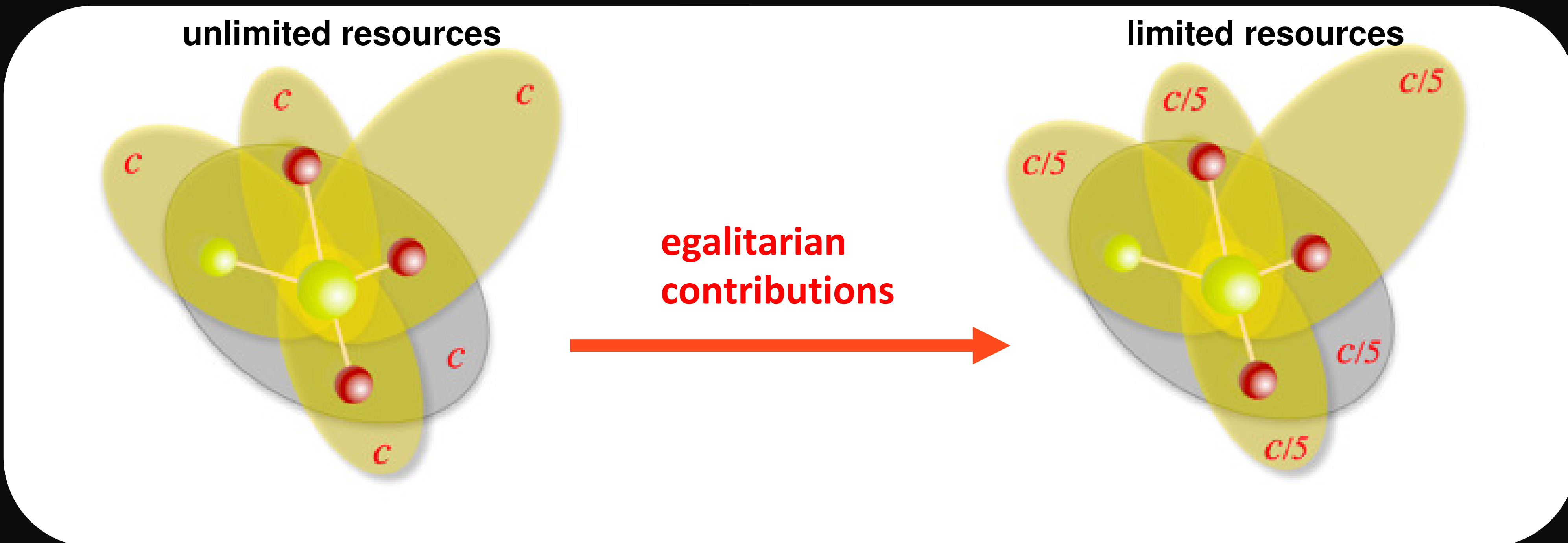
z+1 = average group size

population size = 1000

diversity in the number and size of public goods games enhances cooperation

contributive schemes

diversity creates new *possibilities* . . .



Fixed cost per game

contribution proportional to degree

Fixed cost per individual

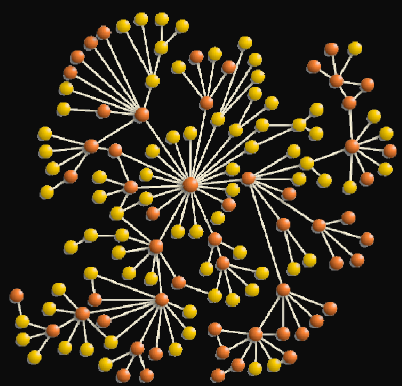
equal contribution independent of degree

real world situations will fall between these 2 limits

(P. Rubin, Darwinian Politics, RUP, 2002)

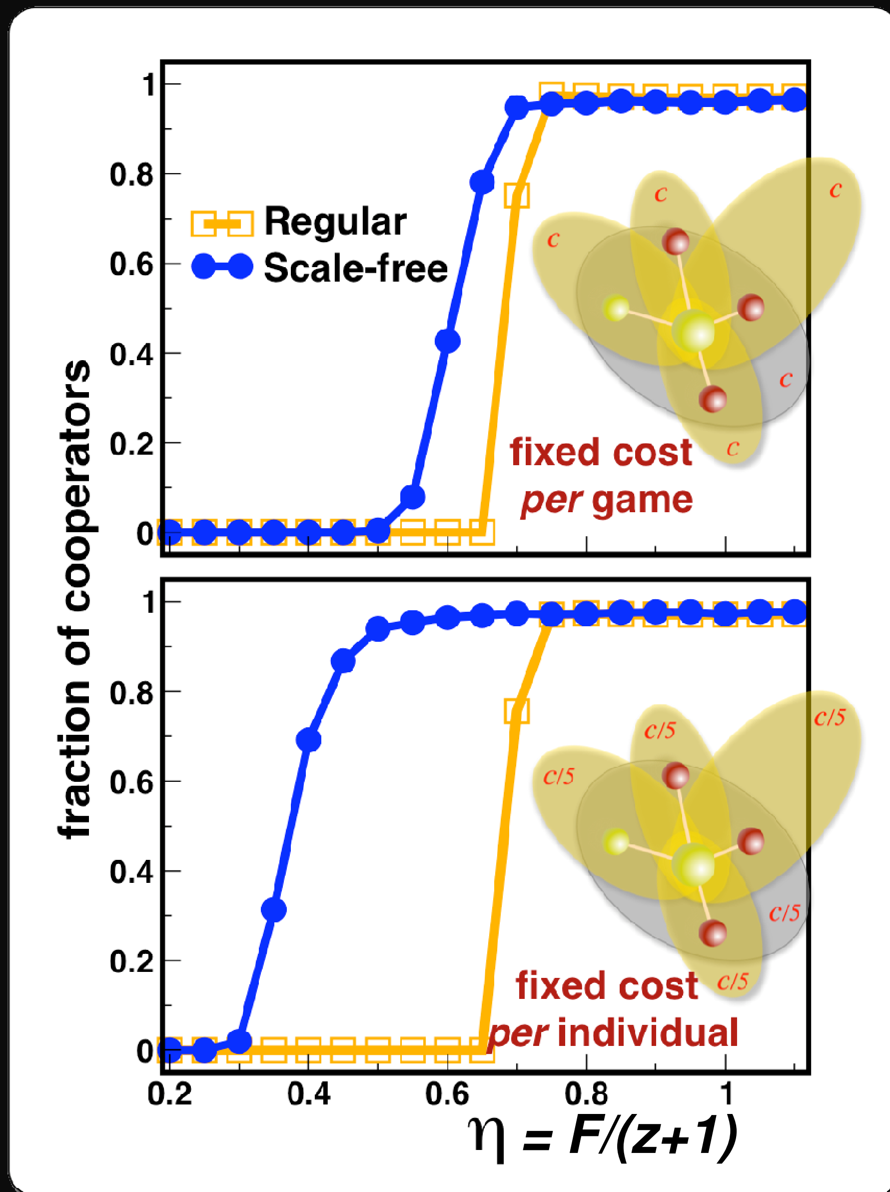
results

fixed cost per game × fixed cost per individual

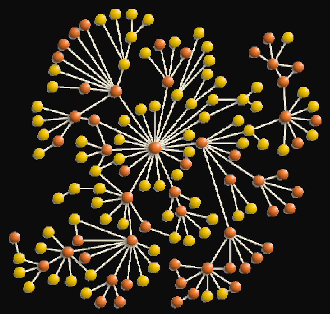


cooperation dominates whenever individuals contribute a fixed amount.

whenever
the act of giving is more important than the amount given,
cooperation prevails

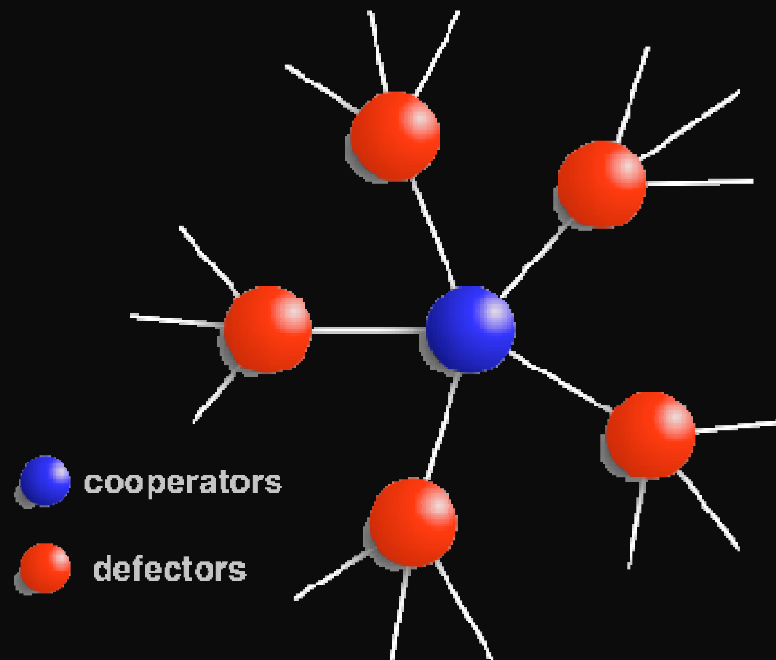


cooperation on the star(s)



the most disadvantageous situation for a cooperator:

star of size Z
1 hub (C)
Z-1 leaves (Ds)
every leaf has k-1 links (Ds)



the cooperator becomes advantageous whenever:

$$F > \frac{k}{1-2/Z}$$

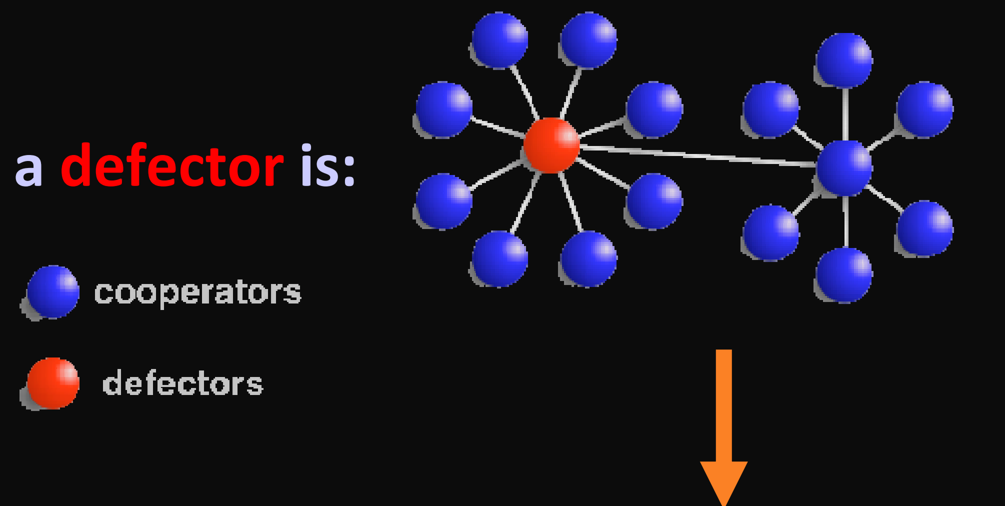
1) *cooperators can become advantageous in highly connected nodes!!*
2) *the critical multiplication factor F decreases with increasing Z and with decreasing k (k=Z -> regular network)*

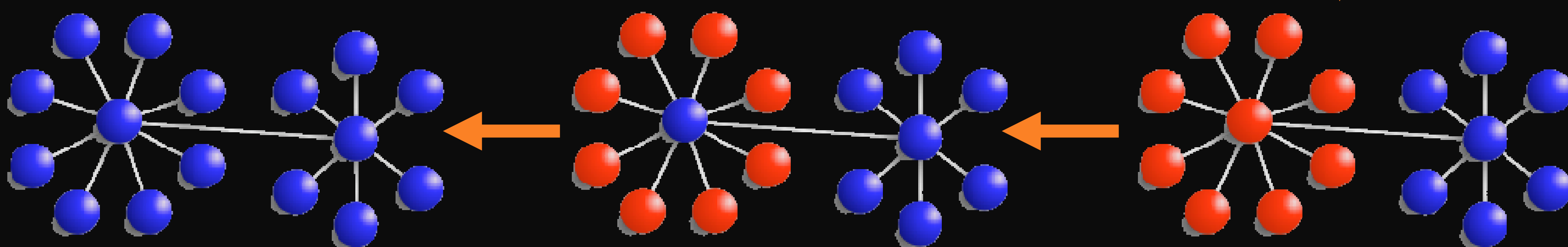
cooperation on the star(s)

cooperators can dominate if they invade the hubs...

but, how can they get there?

the most advantageous situation for a defector is:

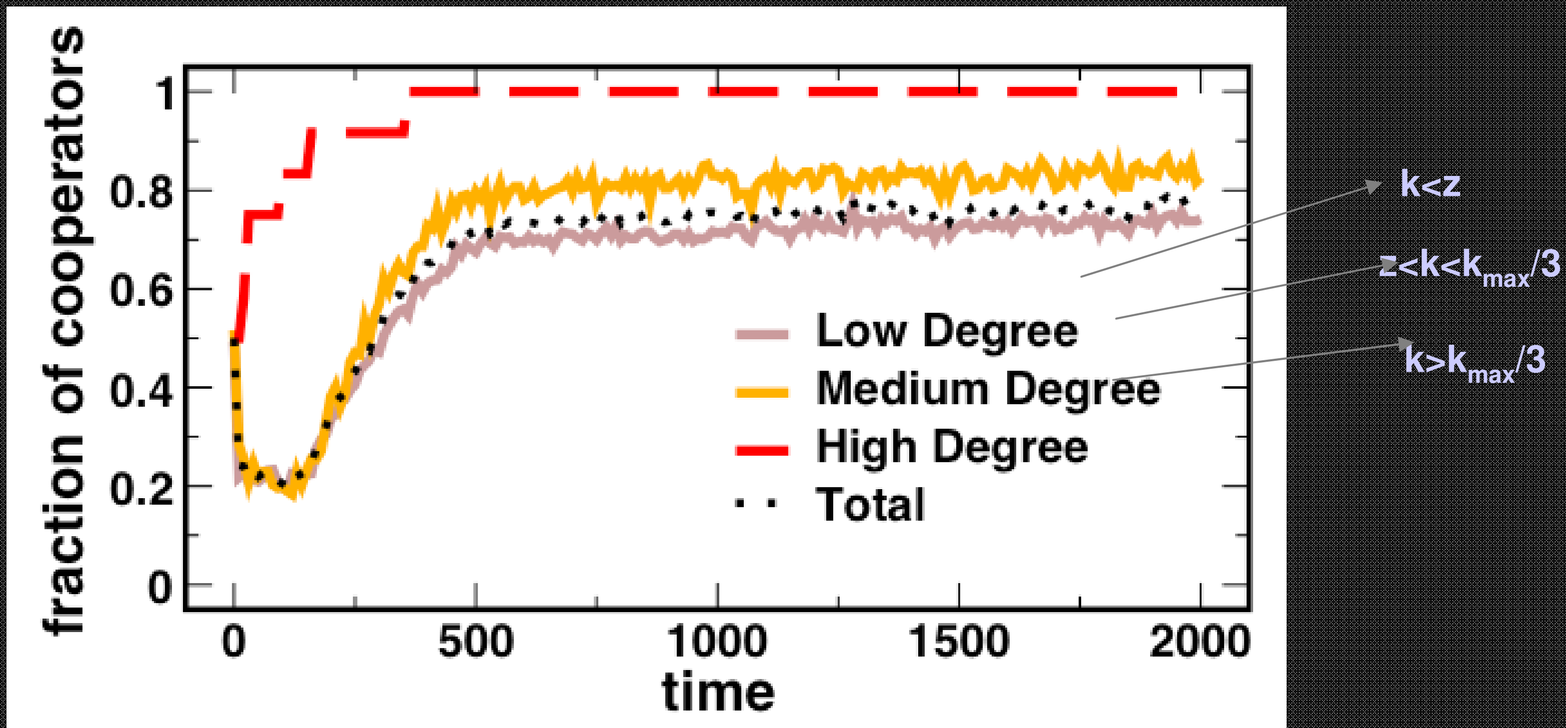




defectors are victims of their own success !!

evolutionary dynamics of the marginally & centrally connected

highly connected individuals are the ones who quickly become cooperators:



conclusions

***diversity** paves the way for cooperators to explore **their self-sustaining** interaction nature and **outperform** defectors.*

the effect is enhanced whenever individuals contribute the same amount irrespective of the number of PGGs they engage (contributions depending on social context)

results may explain the emergence of cooperation when participation is compulsory and in the absence of mechanisms based on individual reputation or punishment.

Francisco C. Santos, Marta D. Santos and Jorge M. Pacheco, Nature 454, 213-216 (2008).