AMERICAN MATHEMATICAL SOCIETY
MathSciNet Mathematical Reviews on the Web

Previous | Up | Next Article

Citations
From References: 0
From Reviews: 0

MR1983590 (2005e:76013) 76B03 (35Q30 35Q35)

Shnirelman, A. (4-HULL)

Weak solutions of incompressible Euler equations. (English summary)

Handbook of mathematical fluid dynamics, Vol. II, 87–116, *North-Holland, Amsterdam*, 2003.

This article contains two examples of weak solutions. The first example is a simplification and clarification of a famous example constructed by V. Scheffer [J. Geom. Anal. **3** (1993), no. 4, 343–401; MR1231007 (94h:35215)]. Scheffer's example is an unbounded and discontinuous vector field $u(x,t)\in L^2(\mathbb{R}^2\times\mathbb{R})$, satisfying the usual identities (obtained from the Euler equations by integration by parts) for all test-functions v,φ, and such that $u(x,t)\equiv 0$ if $|x|^2+|t|^2>1$. The present author constructs a simplified example of a weak solution $u(x,t)$ on a 2-d torus $\mathbb{T}^2$, such that $u(x,t)\equiv 0$ for $|t|>C$. Thus, the zero solution is not unique in the class of all weak solutions; the same is true for all smooth solutions as well (but he does not know whether any $weak$ solution is nonunique).

The second example, having nothing in common with the previous one, is a weak solution $u(x,t)$ on a 3-d torus $\mathbb{T}^3$, whose kinetic energy $E(t)=\int_{\mathbb{T}^3}\frac{1}{2}|u(x,t)|^2dx$ decreases monotonically in time. Such behavior is characteristic for a highly turbulent flow in the absence of external forces (i.e., in case of decaying turbulence). But other properties of this solution are different from what we can anticipate for turbulent flows; so its physical meaning is doubtful.

Also in this work the author discusses the energy balance in weak solutions. It is well known that for regular solutions of the Euler equations kinetic energy is constant. For weak solutions the energy is constant if they are not very singular. (The borderline is at the regularity about the Hölder class $C^{1/3}$. This was conjectured by L. Onsager [Nuovo Cimento (9) **6** (1949), Supplemento, no. 2(Convegno Internazionale di Meccanica Statistica), 279–287; MR0036116 (12,60f)] and proved by P. Constantin, W. E and E. S. Titi [Comm. Math. Phys. **165** (1994), no. 1, 207–209; MR1298949 (96e:76025)] and G. L. Eyink [Phys. D **78** (1994), no. 3-4, 222–240; MR1302409 (95m:76020)].) But for less regular weak solutions the energy is no longer constant. Therefore, the energy change is connected with irregularities of the velocity field. J. Duchon and R. Robert [Nonlinearity **13** (2000), no. 1, 249–255; MR1734632 (2001c:76032)] have found an explicit formula expressing the local rate of energy dissipation (or production) due to irregularities of the velocity field. The author checks that in his example of a weak solution with decreasing energy the local rate of energy dissipation is positive. Thus, not only does the total energy decrease, but it decreases locally, too.

{For the entire collection see MR1983587 (2004a:76002)}

Reviewed by *A. Alexandrou Himonas*

© *Copyright American Mathematical Society 2005, 2009*