What is Emergence?

R.S.MacKay
Mathematics Institute and
Centre for Complexity Science,
University of Warwick

Outline

1. Many views
2. My view
3. Research Programme

1. Many views, eg Wikipedia

- “Emergence is the way complex systems and patterns arise out of a multiplicity of relatively simple interactions”
- A termite “cathedral” mound produced by a termite colony: a classic example of emergence in nature.

Whole > Sum of Parts

- Aristotle: “The whole is something over and above its parts, and not just the sum of them all”
- PW Anderson, 1972: “We can now see that the whole becomes not merely more, but very different from the sum of its parts”.

Weak v Strong emergence

- Chalmers, 2006: A high-level phenomenon is weakly/strongly emergent with respect to a low-level domain when it arises from the latter but truths concerning it are

unexpected given the principles governing /

not deducible even in principle from truths in the low-level domain.

2. My view

- What emerges from a spatially extended dynamical system is a “space-time phase”: probability distributions over space-time histories that arise from typical initial probabilities in the distant past.
- Amount of emergence is the “distance” of a space-time phase from the set of products for independent units.
- Strong emergence means non-unique space-time phase (but not due to decomposability).

Examples

- “Climate” is an example of probability distribution over space-time compatible with the laws of weather.
- For equilibrium statistical mechanical systems the allowed probability distributions are the Gibbs phases for the energy βH (in units of temperature).

continued

- For Markov processes they are the Gibbs phases for -log p_{ij}.
- For deterministic dynamical systems with symbolic dynamics they are the Gibbs phases for log det Df^u (= SRB measures).
- For spatially extended deterministic dynamical systems with symbolic dynamics they are the space-time Gibbs phases for

  tr $(\log Df^u)_{ss}$.

Decomposability

- Non-uniqueness can arise trivially for topological reasons, e.g. more than one attractor, or a 2-piece attractor; more generally, because system is “decomposable”.
- A system with a space-time symbolic description is “indecomposable” if any allowable configurations on two sufficiently separated space-time patches can be joined into an allowable configuration (“specification property”).

Proved examples of (non-trivial) strong emergence

- Ferromagnetic phases of 2D Ising model
- Ferromagnetic phases of Toom's NEC majority voter PCA
- Period-2 phases of Toom's NEC voter PCA
- Endemic infection v disease-free phases of contact processes

Finite v infinite systems

- A catch with the definition of (non-trivial) strong emergence is that it can occur only for infinite systems.
- e.g. a finite piece of Ising model (with specified boundary conditions) has unique phase.
- Yet non-uniqueness for the infinite 2D Ising model is reflected in long-range correlations for finite versions.

3. Research Programme

- Make a zoo of possible phases
- Study their correlation structure
- Study robustness of phases/set of phases
- Universality classes/Aggregation
- Bifurcations/Scaling
- Control/Optimisation
- Fit to data
- Develop in contexts with dynamic network
- Develop for game-theoretic contexts
- What about systems that never settle down?

References

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