# Exam topics MA933 - 2019

**Basic probability, DTMCs**

- Definition of probability space, events, probability distribution, independence, conditional probability, law of total probability
- Random variables, distribution function (CDF), mass function, PDF, expectation, variance, joint distribution, marginal (definitions and compute simple examples)
- Simple random walk Y_n, compute expectation and variance of Y_n
- State weak law of large numbers and central limit theorem
- Definition of discrete-time Markov process, sample path, simple examples
- Transition functions and Chapman Kolmogorov equations (including proof (premium)), matrix formulation, computation of probabilities for simple examples
- Stationary/reversible distributions, detailed balance, eigenvectors of transition matrix
- absorbing states, linear recursion for absorption probabilities, solution of those
- Random walks with different boundary conditions (Q1.1): transition matrices, stationary distributions
- Polya urn models and generalization (Q1.3)

**Continuous-time Markov chains (CTMCs)**

- Definition and path space of CTMCs, sample paths
- Transition functions and Chapman Kolmogorov equations, definition of the generator matrix, forward and backward equations
- Solution of forward equation by matrix exponential, Gershgorin theorem, Perron Frobenius, computations as in Q1.2
- State and explain Master equation
- Sketch sample paths of a CTMC and explain holding times, jump times, jump chain and their properties
- Stationary/reversible distributions, detailed balance, eigenvectors of generator matrix
- Result on existence of stat. distributions and uniqueness, irreducibility
- birth-death chain (Q2.4), compute stationary distributions
- Definition of ergodicity and sufficient condition, state ergodic theorem
- Time-reversal of stationary Markov chains, reversibility, simple examples (RWs)
- Return time, transience, null/positive recurrence, explosion, simple examples

**Stochastic particle systems**

- definition of contact process, voter model, exclusion process
- mean field scaling limit of contact process
- mean field approximation and rate equation
- Adding iid Poisson processes gives Poisson processes, thinning, simulation of particle systems and continuous-time Markov chains (hand-out)

**Processes with continuous state space**

- Transition kernel, Chapman Kolmogorov equations, finite dimensional distributions
- Jump processes, ansatz for transition function, Kolmogorov Feller equation
- Gaussian processes, definition, multivariate Gaussians
- Stationary independent increments, equivalent characterization of Brownian motion, existence of a version with continuous paths
- Properties of BM: Heat kernel, diffusion equation, self-similarity, white noise
- Generators as operators for Markov processes, Brownian motion, jump processes
- BM as a scaling limit of jump processes using Taylor expansion of the generator
- Definition of diffusion processes, and Fokker-Planck equation

- Ornstein-Uhlenbeck, Brownian bridge, Geometric Brownian motion (Q2.2, Q3.1)
- scaling limits (Kingman Q2.1, Moran Q2.3), mean-field contact process
- Stochastic differential equations and Ito's formula
- Generalized Ito formula and martingales, conservation laws, exponential martingale
- Levy processes, fractional Brownian motion, fractional noise, anomalous diffusion

**Graph properties**

- Definition, directed, undirected, adjacency matrix
- Definition of paths, distance, characteristic path length, diameter, connected components, compute for simple example
- In/out degree sequence, degree distribution, average, variance, compute for simple example
- Simple examples of graphs, complete, regular lattices, trees (characterizations)
- Degree correlations, edged biased sampling, definition of (dis)assortative, uncorrelated
- Definition of subgraph, cliques, spanning trees, simple examples
- Definition of clustering coefficients, compute for simple examples
- Graph spectra, graph Laplacian, Wigner semi-circle law

**Random graph models**

- Define E-R graphs, distribution of number of edges, expected degree distribution (binomial -> Poisson), explain local tree-like topology, compute clustering coefficient
- Explain percolation on a graph, resulting connected components
- State result on giant components for E-R random graphs, sketch relevant plots (Q3.3)
- Explain preferential attachment mechanism, define Barabasi-Albert model, state basic properties of degree distribution (Q3.2), sketch relevant plots, non-linear generalizations
- Define small-world property and Watts-Strogatz random graphs, explain small-world regime in terms of average path length and clustering coefficient
- Define configuration model, simple examples including non-graphical sequences

Extreme value statistics stuff on hand-out 6

**Examinable material.**

includes the lecture notes, all problem sheets and hand-outs (except hand-out 5)