

# Applied decision theory for clinical trials in small populations

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# Joint work



- This talk is based on joint work together with **Simon Day, Siew Wan Hee, Jason Madan, Martin Posch, Nigel Stallard, Mårten Vågerö** and **Sarah Zohar** for the **InSPiRe** project.
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Integrated D<sup>E</sup>sign and AnaLysis  
of small population group trials



# Case study: Lyell's syndrome

- Life-threatening syndrome
- Drug-induced severe adverse drug reaction:  
Patient loses top layer skin, over whole body
- Mortality rate 22% in Europe
- In well controlled environment (specialized clinic)  
lower mortality rate
- Rare: incidence  $2/10^6$  inhabitants in Europe

# Lyell's syndrome

- New cellular therapy:
- **N=500** patients in EU could be reached by new therapy
- Hope that complete healing achieved after  $\sim 2$  weeks
- Primary endpoint: Is at least 90% of body surface area skin detachment completely healed at Day 10 of therapy?
- Without new cellular therapy: anticipated that positive primary endpoint for 50% of patients:  **$p_0=0.5$**
- With new therapy:  **$p_1=?$**

# Lyell's syndrome

- L'Assistance Publique-Hôpitaux de Paris sponsors a **clinical study with  $n$  patients receiving new therapy**

$n$  ?

- Objective: Show that proportion of patients fulfilling primary endpoint is larger than  $p_0=0.5$

# Computation of sample size

- Traditional approach:

$$\text{power}_{p_1} = 1 - \Phi \left( z_{1-\alpha} - (p_1 - p_0) / \sqrt{\frac{p_0(1-p_0)}{n}} \right) \longrightarrow n = \frac{(z_{1-\alpha} + z_{1-\beta})^2 p_0(1-p_0)}{(p_1 - p_0)^2}$$

- Assurance approach (see O'Hagan *et al.*, 2005):

$$\text{assurance} = \int \text{power}_{p_1} \pi(dp_1) = \int 1 - \Phi \left( z_{1-\alpha} - (p_1 - p_0) / \sqrt{\frac{p_0(1-p_0)}{n}} \right) \pi(dp_1)$$

# Sample size for Lyell's syndrome

- Sample size for traditional and assurance approach (depending on prior)

Target / assumed mean response rate for new treatment (control response rate = 0.5)	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
<b>Traditional approach</b>	>500	197	88	50	32	22	17	13
<b>Assurance, prior weight=20</b>	*	>500	283	88	44	27	18	13
<b>Assurance, prior weight=10</b>	*	*	>500	158	59	31	20	14
<b>Assurance, prior weight= 2</b>	*	*	*	*	>500	79	27	18

Significance level  $\alpha=0.05$ , power= $1-\beta=0.8$ ,  $p_0=0.5$ , beta-distribution as prior for assurance

\*for these cases, the assurance would be  $< 0.8$  even for infinitively large sample size

In our case study,  $p_1$  assumed to have a Beta(a,b)-distribution:

expected value =  $a/(a+b)$ , weight =  $a+b$

# Lyell's syndrome

- L'Assistance Publique-Hôpitaux de Paris sponsors a **clinical study with  $n$  patients receiving new therapy**

$n$  ?

- Objective: Show that proportion of patients fulfilling primary endpoint is larger than  $p_0=0.5$  ??
- Objective:  
Make good treatment **decisions** for the patients



# Decisions for Lyell's syndrome

- Decisions in the Lyell's syndrome case:
  - Sample size **n**
  - After study:  
Decide about **treatment for future patients**

Study:  
**n** patients  
treated with  
new cellular  
therapy

After study:  
N-n patients  
(**treatment**  
depends on  
study result)

- We have a certain utility depending on our decision

# Utility for Lyell's syndrome

- Utility for the Lyell's syndrome case:
  - Patients treated successfully has utility which is valued as 100 000 €
  - Costs of a patient being in the study and for new therapy: 25 000 €
  - Costs of patient being treated with new therapy after study: 5 000 €

- Total utility:

$$U(n, p_1) = n (100 p_1 - 25) + (N-n) (100 p_0 - 5)$$

if old treatment chosen after study;

$$U(n, p_1) = n (100 p_1 - 25) + (N-n) (100 p_1 - 5)$$

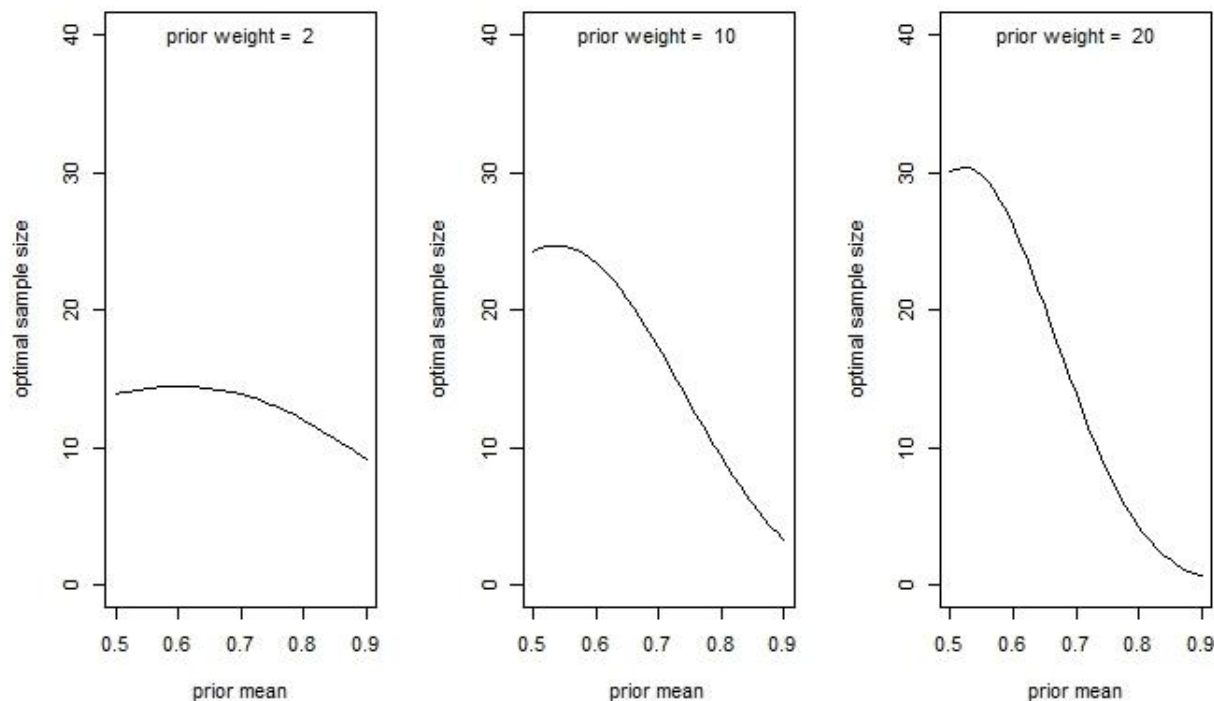
if new treatment chosen after study

# Decision-theoretic sample size

- Expected utility (gain):  $G(n, p_1) = EU(n, p_1)$   
(*expectation over possible study outcomes*)
- Prior distribution assumed for unknown parameters  
(here Beta-distribution for  $p_1$ ) and expected gain  
 $\mathcal{G}(n) = EG(n, p_1)$  can be calculated  
(*expectation over  $p_1$* )
- $\mathcal{G}(n)$  can be optimized over  $n$

# Sample size for Lyell's syndrome

- Optimal sample size (depending on prior)



- Chosen sample size for study:  $n=15$

# Theoretical results for optimal decision-theoretic sample size

- General context (Stallard *et al.*, 2016):
  - Clinical study: one- or two-sample case
  - Observed variable has distribution from one-parameter exponential family
  - Unknown parameter has prior distribution of conjugate form
  - Gain function in study  $h_i$ , after study  $g_i$  for treatment  $i=1,2$
  - Size of population:  $N$
- The optimal sample size(s) is/are of order  $N^{1/2}$

# Theoretical results for optimal decision-theoretic sample size

- Approximations for optimal sample sizes in the general situation (Stallard *et al.*, 2016):

$$n_1^* = \sqrt{N \frac{\int v_1(g_1^{-1}(g_2(\xi_2)))g_1'(g_1^{-1}(g_2(\xi_2)))\pi(g_1^{-1}(g_2(\xi_2)), \xi_2)d\xi_2}{2(E_0(\max_{i=1,2} g_i(\xi_i)) - E_0(h_1(\xi_1)))}}$$

$$n_2^* = \sqrt{N \frac{\int v_2(g_2^{-1}(g_1(\xi_1)))g_2'(g_2^{-1}(g_1(\xi_1)))\pi(\xi_1, g_2^{-1}(g_1(\xi_1)))d\xi_1}{2(E_0(\max_{i=1,2} g_i(\xi_i)) - E_0(h_2(\xi_2)))}}$$

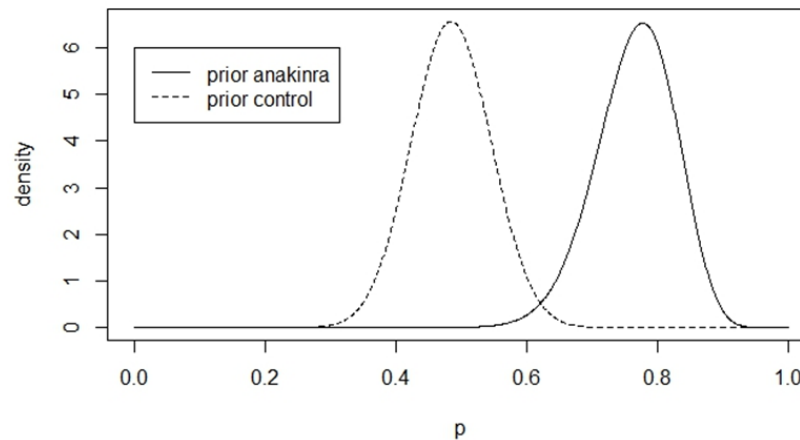
where  $\xi_i$  prior mean and  $v_i$  variance.

## Case study 2: Adult-Onset Still's Disease

- Adult-Onset Still's Disease (AoSD) is a **chronic symptomatic disease** affecting around **N=1000** patients in the EU
- A randomized clinical trial comparing the treatment anakinra ( $n_1=n$  patients) with control ( $n_2=n$  patients) is planned
- Measurement of primary interest: remission (one binary variable for each patient)

# Adult-Onset Still's Disease: Prior information

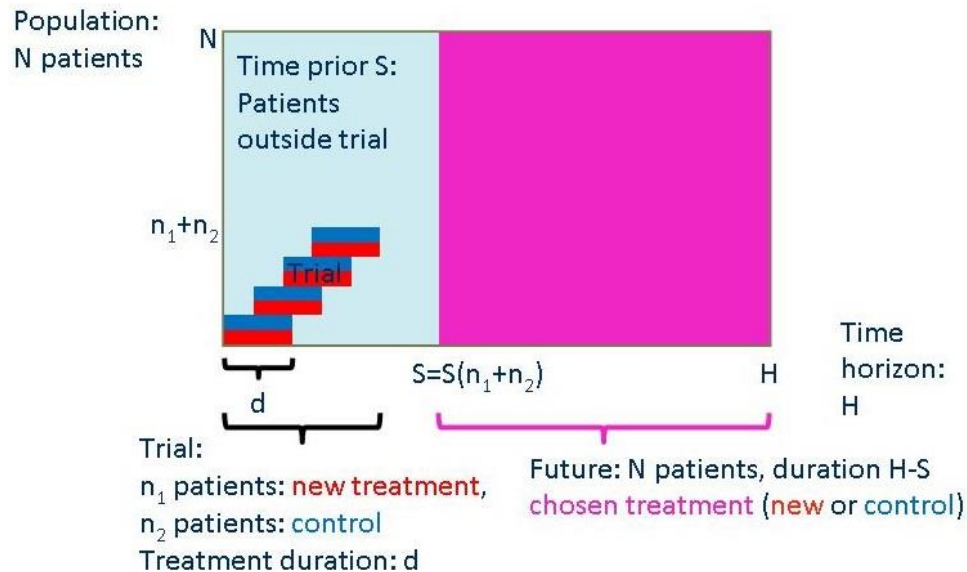
- Meta analysis based on observational remission data for anakinra-treated patients and controls available (Hong *et al.*, 2014):
  - **36 of 47 anakinra**-treated patients and
  - **33 of 68 controls** experienced remission





# Adult-Onset Still's Disease: Utility and gain

- For a chronic treatment, **duration of treatment in study** and **time of introduction of post-study treatment recommendation** is relevant



## Adult-Onset Still's Disease: Sample sizes

- Traditional approach:  $n=46$
- Assurance approach:  $n=56$   
(where  $\alpha=0.05$ ,  $1-\beta=0.8$ )
- Decision-theoretic approach:  $n=0$  (!)

# Discussion

- Sample size justified by decision-theoretic arguments can be considerably different from traditional sample size
- Reasonable that decision-theoretic sample size  $n$  depends on population size  $N$
- Hee *et al.* (2016) review decision-theoretic designs and distinguish “simple” and “more realistic” utility

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