

Approaches to sample size calculation for clinical trials in small populations

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Joint work

- This talk is based on a joint work
 - together with Simon Day, Siew Wan Hee, Jason Madan, Martin Posch, Nigel Stallard, Märten Vågerö and Sarah Zohar
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Outline

- Example of a rare disease: Lyell's syndrome
- Methods for sample size calculation
 - Traditional
 - Assurance
 - Decision-theoretic
- Sample size for Lyell's case study
- Example 2: Adult-Onset Still's Disease
- Theoretical results

Lyell's syndrome

- Life-threatening syndrome
- Drug-induced severe adverse drug reaction:
Patient loses top layer skin, over whole body
- Mortality rate 22% in Europe
- In well controlled environment (specialized clinic)
lower mortality rate
- Rare: incidence $2/10^6$ inhabitants in Europe

Lyell's syndrome

- New cellular therapy:
- N=500 patients in EU could be reached by new therapy
- Hope that complete healing achieved after ~2 weeks
- Effect of new therapy measured by following "primary endpoint":
 - Is at least 90% of body surface area skin detachment completely healed at Day 10 of therapy?
- Anticipated based on experience:
 - Without new cellular therapy, positive primary endpoint for 50% of patients: $p_0=0.5$

Lyell's syndrome

- L'Assistance Publique-Hôpitaux de Paris sponsors a clinical study with n patients receiving new therapy
- Objective: Show that proportion of patients fulfilling primary endpoint is larger than 0.5
- How many patients (n) needed?

Methods for sample size calculation

- Traditional approach:
 - Probability to reject H_0 with a significance test of level α should be at least $1-\beta$ if the proportion of patients fulfilling the primary endpoint is p_1

Methods for sample size calculation

- Assurance approach (see O'Hagan et al., 2005):
 - Probability to reject H_0 with a significance test of level α should be at least $1-\beta$ if the proportion of patients fulfilling the primary endpoint p_1 **has a distribution F**
- E.g. p_1 assumed to have a beta-distribution $\text{Beta}(a,b)$;
 - expected value = $a/(a+b)$,
 - weight = $a+b$

Computation of sample size

- Traditional approach:

$$n = \frac{(z_{1-\alpha} + z_{1-\beta})^2 p_0(1-p_0)}{(p_1 - p_0)^2}$$

- Assurance approach:

$$\text{assurance} = \int \text{power}_{p_1} F(dp_1) = \int 1 - \Phi\left(z_{1-\alpha} - (p_1 - p_0) / \sqrt{\frac{p_0(1-p_0)}{n}}\right) F(dp_1)$$

Sample size for Lyell's syndrome

- Sample size for traditional and assurance approach (depending on prior)

Target / assumed mean response rate for new treatment (control response rate = 0.5)	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
Traditional approach	>500	197	88	50	32	22	17	13
Assurance, prior weight=20	*	>500	283	88	44	27	18	13
Assurance, prior weight=10	*	*	>500	158	59	31	20	14
Assurance, prior weight= 2	*	*	*	*	>500	79	27	18

Significance level $\alpha=0.05$, power= $1-\beta=0.8$, $p_0=0.5$, beta-distribution as prior for assurance

*for these cases, the assurance would be < 0.8 even for infinitively large sample size

Methods for sample size calculation

- Decision-theoretic approach:
 - Decision: n
 - Each result (depending on decision and on observed random variables) has specific utility $U = U(n, \mathbf{X})$
 - Expected utility is (here) called gain: $G = EU$
 - Gain depends on unknown parameters (here p_1)
 - If we have a prior for unknown parameters, we can calculate the expected gain $G = EG$
- Hee et al. (2015) review decision-theoretic designs

Decisions for Lyell's syndrome

- Decisions in the Lyell's syndrome case:
 - Sample size n
 - After study:
Decide about treatment for future patients

Study:
 n patients
treated with
new cellular
therapy

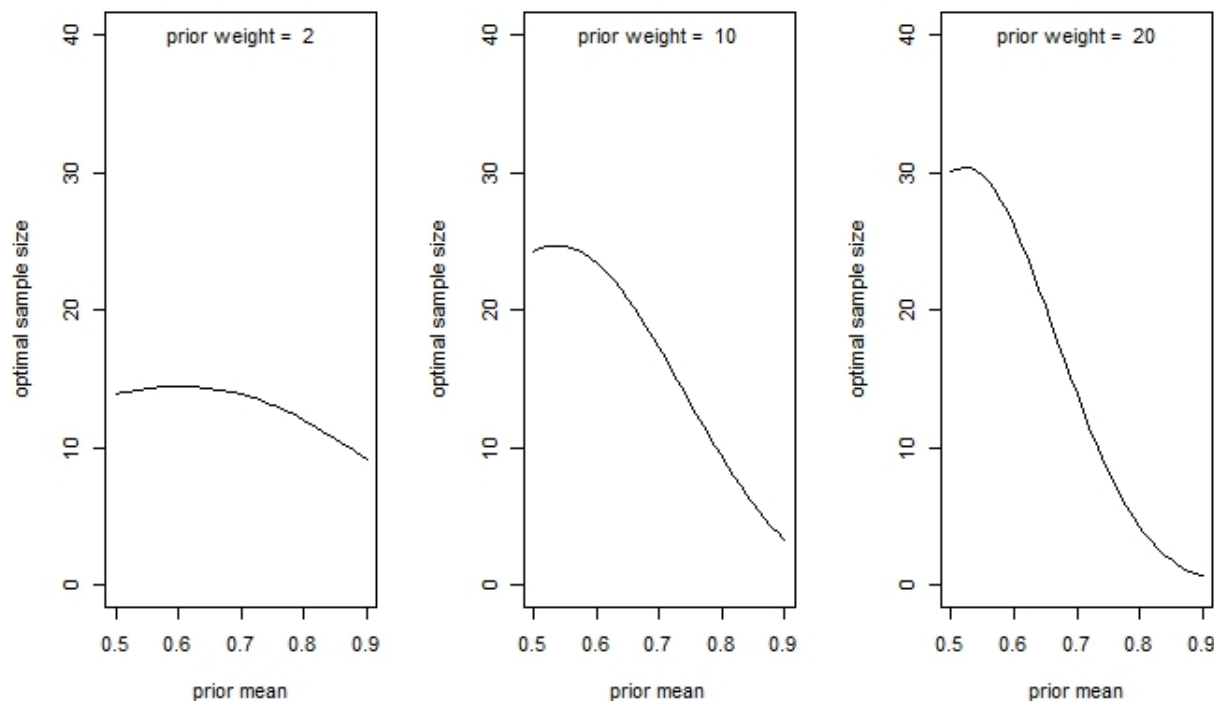
After study:
 $N-n$ patients
(treatment
depends on
study result)

Utility for Lyell's syndrome

- Utility for the Lyell's syndrome case:
 - Patients treated successfully has utility which is valued as 100 000 €
 - Costs of a patient being in the study and for new therapy: 25 000 €
 - Costs of patient being treated with new therapy after study: 5 000 €
- $X_i=1$ if patient i treated successfully; $X_i=0$ if not
- Utility for a patient in study: $U = 100 X_i - 25$
- Utility for a patient after study: $U = 100 X_i - 5$

Sample size for Lyell's syndrome

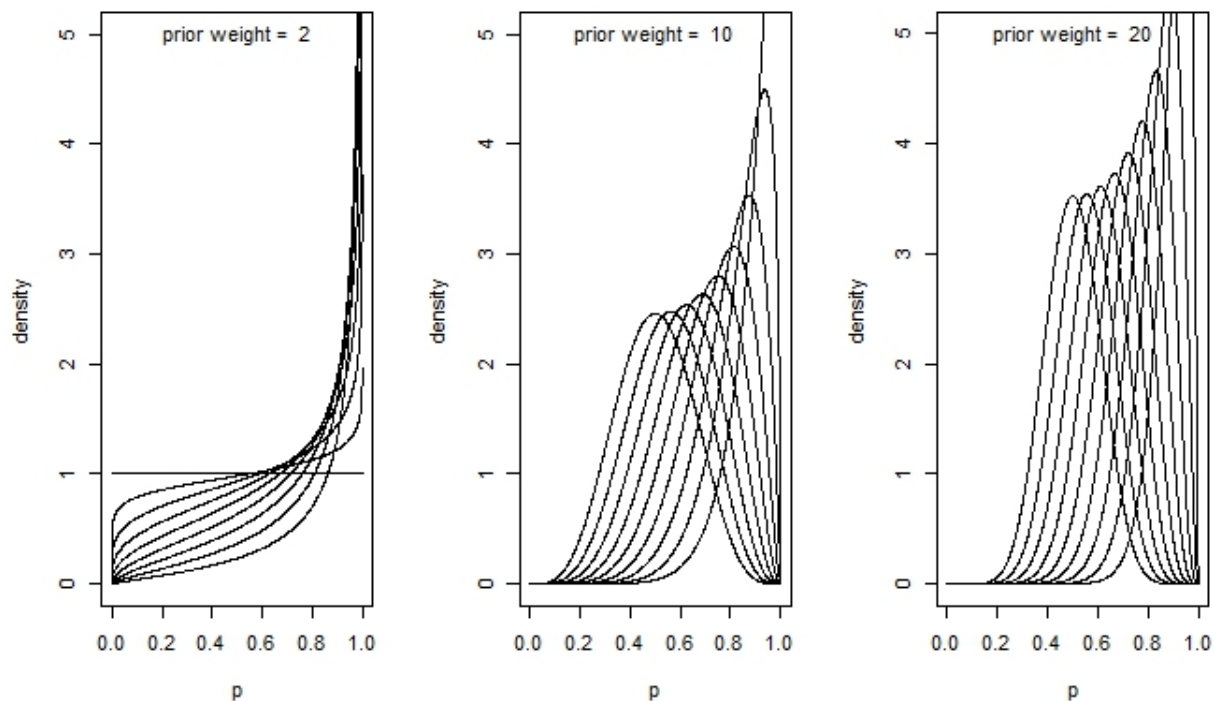
- Optimal sample size (depending on prior)



- Chosen sample size for study: $n=15$

Sample size for Lyell's syndrome

- Different priors



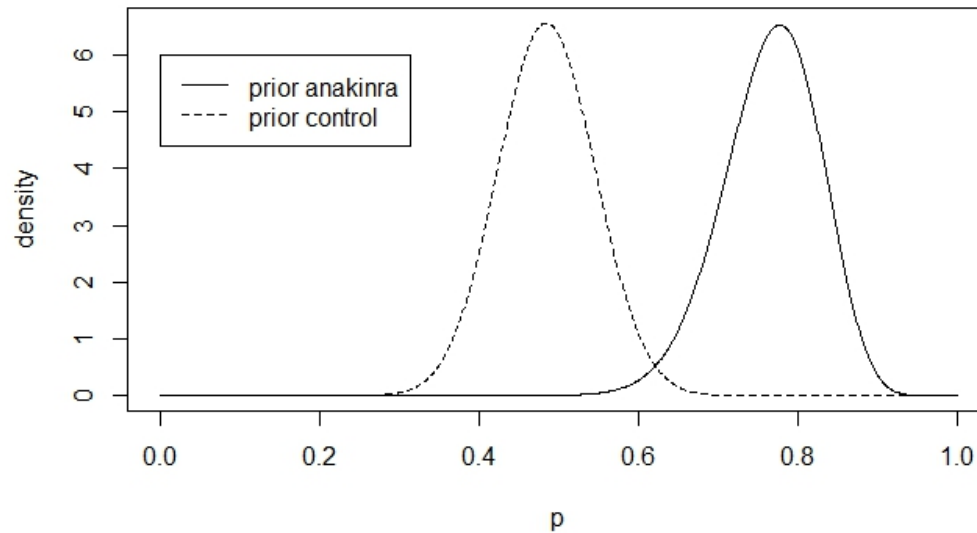
Adult-Onset Still's Disease

- Adult-Onset Still's Disease (AoSD) is a chronic symptomatic disease affecting around N=1000 patients in the EU
- A randomized clinical trial comparing the treatment anakinra (n patients) with control (n patients) is planned
- Measurement of primary interest: remission (one binary variable for each patient)

Adult-Onset Still's Disease: Prior information

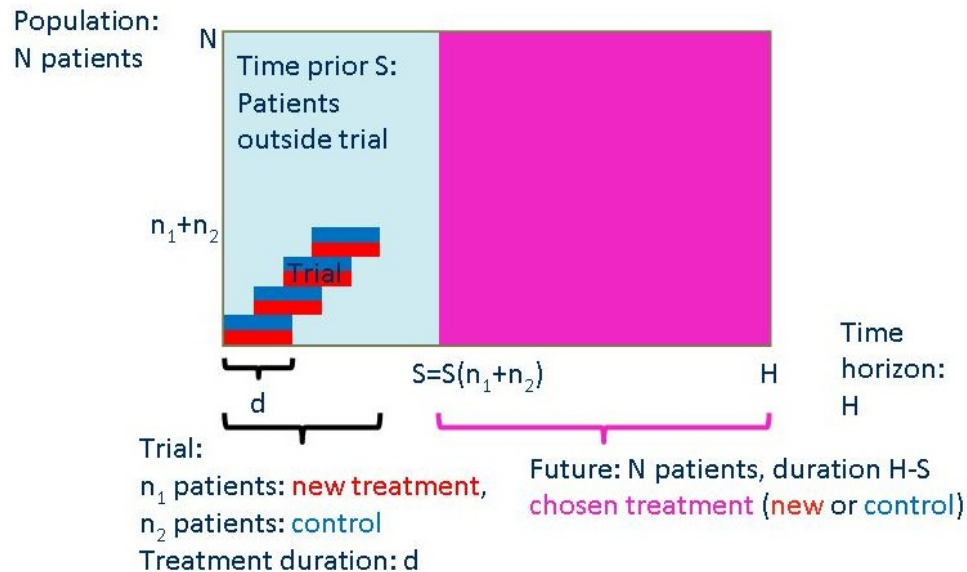
- Meta analysis based on observational remission data for anakinra-treated patients and controls available (Hong et al., 2014):
 - **36 of 47 anakinra**-treated patients and
 - **33 of 68 controls** experienced remission

Adult-Onset Still's Disease: Prior information



Adult-Onset Still's Disease: Gain function

- Gain for patients in the trial: $p - 0.05$,
- gain for patients outside-trial: $p - 0.01$,
- $p =$ remission rate ($p=p_1$ anakinra, $p=p_2$ control)



Adult-Onset Still's Disease: Sample sizes

- Traditional approach: $n=46$
- Assurance approach: $n=56$
(where $\alpha=0.05$, $1-\beta=0.8$)
- Decision-theoretic approach: $n=0$ (!)

Theoretical results for optimal decision-theoretic sample size

- General context (Stallard et al., 2016):
 - Clinical study: one- or two-sample case
 - Observed variable has distribution from one-parameter exponential family (includes binomial and normal case)
 - Unknown parameter has prior distribution of conjugate form
 - Gain function in study h_i , after study g_i for treatment $i=1,2$
 - Size of population: N
- The optimal sample size(s) is/are of order $N^{1/2}$

Theoretical results for optimal decision-theoretic sample size

- Expressions for optimal sample sizes in the general situation (Stallard et al., 2016):

$$n_1^* = \sqrt{\frac{N \int v_1(g_1^{-1}(g_2(\xi_2)))g_1'(g_1^{-1}(g_2(\xi_2)))\pi(g_1^{-1}(g_2(\xi_2)), \xi_2)d\xi_2}{2(E_0(\max_{i=1,2} g_i(\xi_i)) - E_0(h_1(\xi_1)))}}$$

$$n_2^* = \sqrt{\frac{N \int v_2(g_2^{-1}(g_1(\xi_1)))g_2'(g_2^{-1}(g_1(\xi_1)))\pi(\xi_1, g_2^{-1}(g_1(\xi_1)))d\xi_1}{2(E_0(\max_{i=1,2} g_i(\xi_i)) - E_0(h_2(\xi_2)))}}$$

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