# Decision-theoretic design for a series of phase II trials with correlated treatment effects 

Siew Wan Hee
Statistics \& Epidemiology Unit, Warwick Medical School, University of Warwick, UK www.warwick.ac.uk/InSPiRe

## Setting

- Small population, $N$, is known
- > 2 treatments available for trial
- Only one of them can proceed to a phase III trial
- A series of single-arm phase II trials and one two-arm phase III trial
- Treatments for the same population may be related
- Extends Hee and Stallard (2012)


## Decision-theoretic design

- Start with experimental treatment, $E_{1}$
- Recruit $m$ patients
- Observe their responses and decide to:
- Action P: Stop and proceed to phase III
- Action A: Stop and abandon the programme
- Action T: Stop and start a new one with $E_{2}$
- Action R: Continue with another group of $m$ patients
- At each decision time point choose an optimal action based on utility


## Action P: Proceed to phase III

- At stage $i$ of trial $k$
- Remaining $N-\sum_{j=1}^{k-1} n_{j}$. $-n_{k i}$ patients are randomized to a 2 -arm phase III trial

$$
\theta_{k}=\log \left\{\frac{p_{k}\left(1-p_{C}\right)}{p_{C}\left(1-p_{k}\right)}\right\}, V_{k} \approx \frac{1}{4}\left(N-\sum n_{j}-n_{k i}\right) \bar{p}(1-\bar{p})
$$

- Test the null hypothesis: $H_{0}: \theta_{k}=0$

$$
1-\Phi\left(z_{1-\alpha / 2}-\theta_{k} \sqrt{V_{k}}\right)
$$

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$$
\begin{aligned}
& U\left(1-\Phi\left(z_{1-\alpha / 2}-\theta_{k} \sqrt{V_{k}}\right)\right)-c_{2 k} n_{k i} \\
& -c_{3 k}\left(N-\sum_{j=1}^{k-1} n_{k \cdot}-n_{k i}\right)-l_{3 k}
\end{aligned}
$$

## Action P: Proceed to phase III

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$$

- Test the null hypothesis: $H_{0}: \theta_{k}=0$

$$
\begin{aligned}
& \mathcal{G}_{P}\left(k, \boldsymbol{s}_{\boldsymbol{k} \boldsymbol{i}}, \boldsymbol{n}_{\boldsymbol{k} i}, N\right) \\
& =\int \ldots \int U\left(1-\Phi\left(z_{1-\alpha / 2}-\theta_{k} \sqrt{V_{k}}\right)\right) h\left(\boldsymbol{p} \mid \boldsymbol{S}_{\boldsymbol{k} \boldsymbol{i}}\right) d \boldsymbol{p}-c_{2 k} n_{k i} \\
& -c_{3 k}\left(N-\sum_{j=1}^{k-1} n_{k \cdot}-n_{k i}\right)-l_{3 k}
\end{aligned}
$$

## Action A: Abandon the programme

- Less the cost of patients recruited to the current trial so far,

$$
\mathcal{G}_{A}\left(k, \boldsymbol{s}_{\boldsymbol{k i}}, \boldsymbol{n}_{\boldsymbol{k} \boldsymbol{i}}, N\right)=-c_{2 k} n_{k i}
$$

## Action T: Start a new phase II trial

- The expected utility depends on the expected utility of the new trial and its resulting actions

$$
\begin{aligned}
& \mathcal{G}_{T}\left(k, \boldsymbol{s}_{\boldsymbol{k} \boldsymbol{i}}, \boldsymbol{n}_{\boldsymbol{k} \boldsymbol{i}}, N\right) \\
& =\mathcal{G}_{\text {Total }}\left(k+1, \boldsymbol{s}_{\boldsymbol{k}+\mathbf{1}, \mathbf{0}}, \boldsymbol{n}_{\boldsymbol{k}+\mathbf{1}, \mathbf{0}}, N\right)-c_{2 k} n_{k i}
\end{aligned}
$$

## Action R: Recruit more to the current trial

- Action R requires us to recruit an additional $m$ patients
- Subsequently, take an optimal action on these future observations
- The gain depends on the action taken based on the observations from subsequent stages and trials

$$
\begin{aligned}
& \mathcal{G}_{R}\left(k, \boldsymbol{s}_{\boldsymbol{k} \boldsymbol{i}}, \boldsymbol{n}_{\boldsymbol{k} \boldsymbol{i}}, N\right) \\
& =\sum_{y=0}^{m} \max _{a \in\{P, A, T, R\}}\left\{G_{a}\left(k, \boldsymbol{s}_{\boldsymbol{k} \boldsymbol{i}}+y, \boldsymbol{n}_{\boldsymbol{k} \boldsymbol{i}}+m, N\right)\right\} \times g\left(y \mid \boldsymbol{s}_{\boldsymbol{k} \boldsymbol{i}}, \boldsymbol{n}_{\boldsymbol{k} \boldsymbol{i}}\right)
\end{aligned}
$$

## Case study

- Total hip arthroplasty (standard) vs. resurfacing arthroplasty (experimental) trial for patients with arthritis of the hip joint (Costa et al., BMJ, 2012;344)
- For our illustration, assume 2 newer resurfacing arthroplasty procedures that differ in the technical aspects
- Only one of them can proceed to a phase III trial


## Assumptions

- Binary outcome

$$
\begin{gathered}
Y_{k i} \sim \operatorname{Bin}\left(m, p_{k}\right), S_{k i}=\sum_{j=1}^{i} Y_{k j} \sim \operatorname{Bin}\left(n_{k i}, p_{k}\right) \\
p_{k} \sim \operatorname{Beta}\left(a_{k}, b_{k}\right), k=1,2
\end{gathered}
$$

- The Sarmanov bivariate beta distribution is

$$
h\left(p_{1}, p_{2}\right)=f\left(p_{1}\right) f\left(p_{2}\right)\left(1+\omega \phi\left(p_{1}\right) \phi\left(p_{2}\right)\right)
$$

where $\phi\left(p_{k}\right)=p_{k}-\mu_{k}$ and $\rho=\omega \sigma_{1} \sigma_{2}$

## Illustration

- $p_{C}=0.5$
- $\operatorname{Beta}(1,1), \operatorname{Beta}(3,2)$ and $\operatorname{Beta}(2,3)$
- $U=£ 3$ million; $l_{2 k}=£ 30,000 ; l_{3 k}=£ 300,000 ; c_{2 k}=c_{3 k}=£ 750$
- Projected size, $N=350$
- Patients are recruited in groups of $m=5$
- Minimum phase III size, $n_{\text {min }}=300$
- Mixing parameter, $\omega=0,4$


## Optimal action for the first phase II trial, Beta $(1,1)$




## Optimal action for the second phase II trial, $\rho=0.33$

$$
s_{1 .}=1, n_{1 .}=10
$$

$$
s_{1 .}=2, n_{1 .}=10
$$

$$
s_{1 .}=3, n_{1 .}=10
$$





## Trivariate case

$\operatorname{Beta}(2,3)$
$\operatorname{Beta}(1,1)$
$\operatorname{Beta}(3,2)$




## Conclusion

- Ordering of treatments matters
- Different priors:
- $p_{1} \sim \operatorname{Beta}(1,1), p_{2} \sim \operatorname{Beta}(3,2)$
- $p_{1} \sim \operatorname{Beta}(3,2), p_{2} \sim \operatorname{Beta}(1,1)$
- $p_{1} \sim \operatorname{Beta}(12,8), p_{2} \sim \operatorname{Beta}(3,2)$
- Start with less informative prior


## Discussion

- The Sarmanov family of distribution is slightly more flexible than those of the Farlie-Gumbel-Morgenstern (FGM) distribution
- For $a, b \geq 1$, the correlation is limited to $[-1 / 3,1 / 3]$
- Olkin and Trikalinos (2015) bivariate beta distribution allows $\rho$ in $[-1,1]$ but has no closed form


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