Critical Phenomena in a Heterogeneous Excitable System

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Talk Outline

- Aim of the project
- Background to the problem
- Approach Taken
- *n* cell chain
- *(n x n)* cell lattice
- Further Work

Aim of the Project

To model the changes in the muscular wall of the uterus in the days before labour.

- Examine the excitation of a tightly coupled system of smooth muscle cells
- Investigate the spread of induced excitation through the system

Background

- Muscular wall of the uterus only acquires the ability to expel a foetus in the final days before labour

- The change is believed to be as a result of a phase transition from a locally excited to globally excited state

- Cell synchrony is achieved by electrical conduction through connecting microfibrils

- Activated myocytes produce prostaglandins which depolarise neighbouring myocytes

  - More cells recruited into contraction

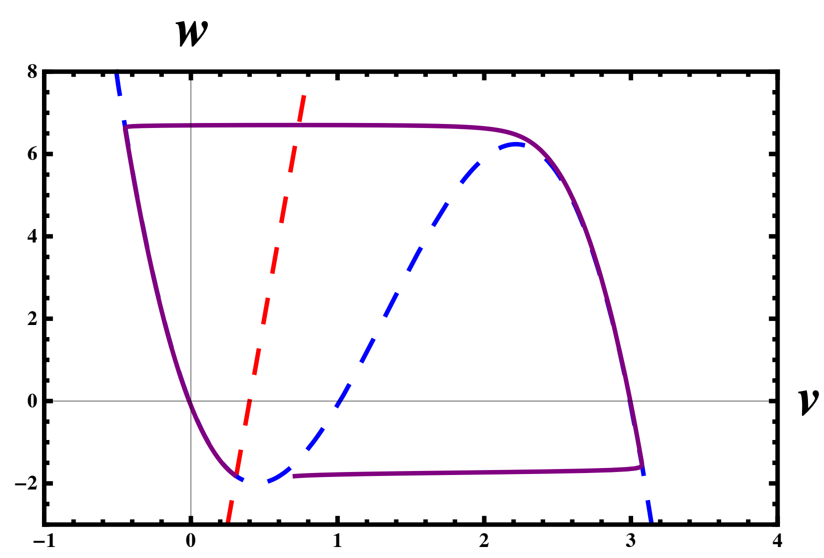
Background

- Smooth muscle cells of the uterus can be modelled using the Fitzhugh-Nagumo system of equations

$$\frac{d}{dt}v = \frac{1}{\varepsilon}(Av(1-v)(v-\alpha) - w - w_0)$$

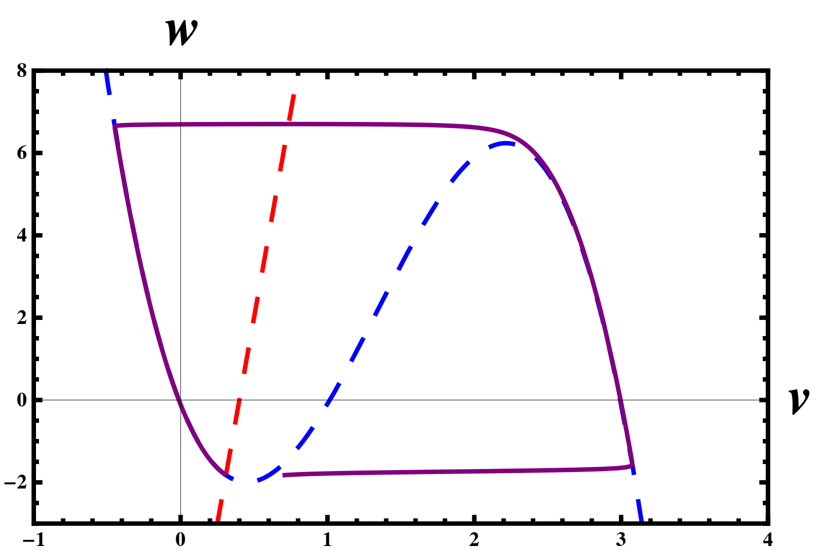
$$\frac{d}{dt}w = v - \gamma w - v_0$$

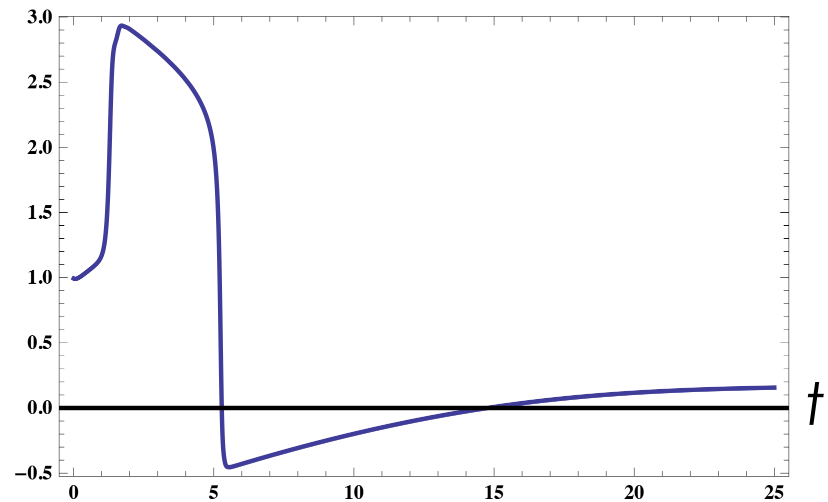
v – excitation variable
w – recovery variable



Background

- Input current needed to move muscle cells out of equilibrium
- Rapid increase in voltage
- Maintains a pseudo-equilibrium on the right branch
- Rapid decrease in voltage to below equilibrium value
- Maintains a pseudo-equilibrium on the left branch
- Return to equilibrium





Approach Taken

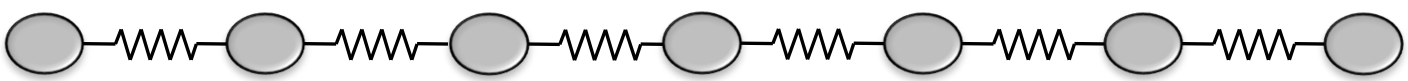
1. Consider a chain of *n* excitable elements, each obeying Fitzhugh-Nagumo dynamics and connected by a resistor

2. Perturb one cell from its equilibrium state so it becomes excited

Approach Taken

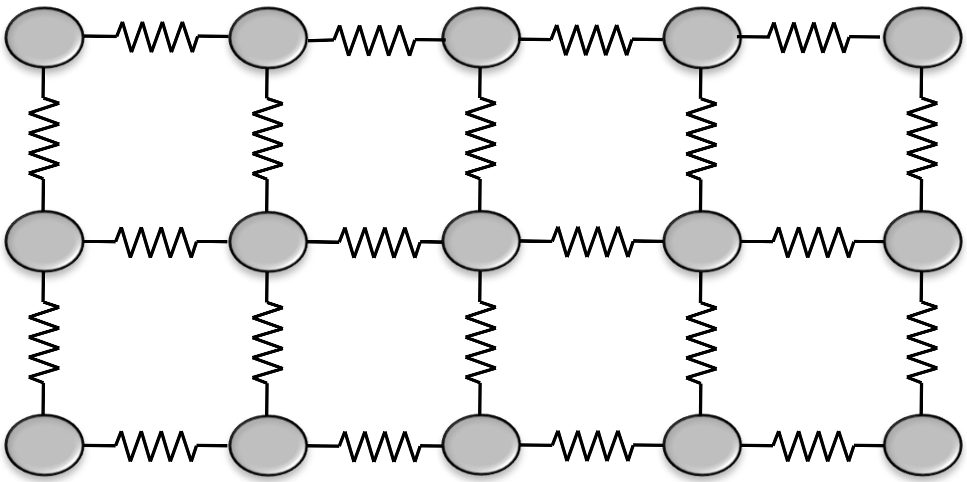
3. Investigate the spread of the excitation wave down the cell chain for different values of coupling

4. Move the kicked cell to the middle of the chain

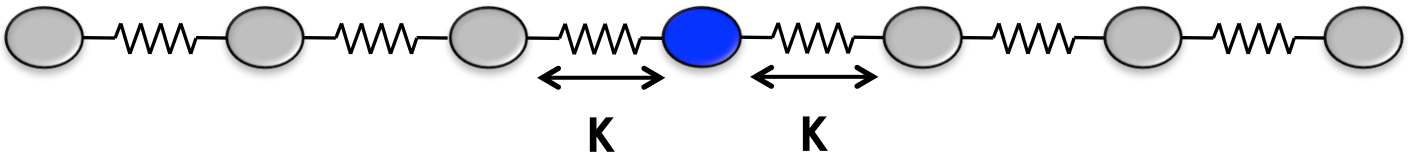


Approach Taken

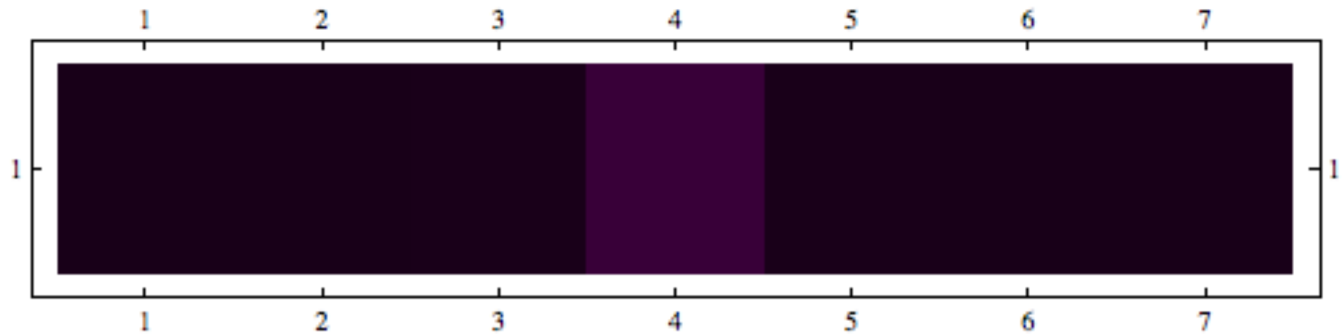
5. Extend to an *(m x n)* grid



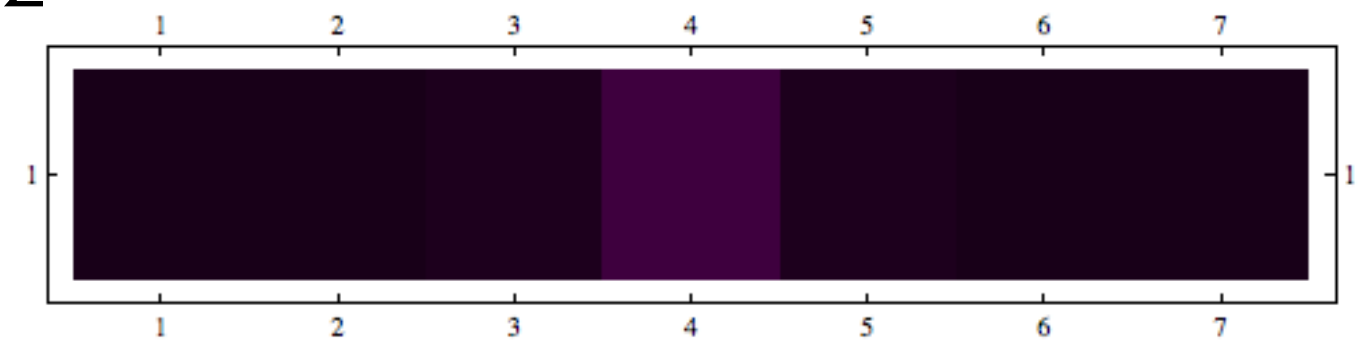
Cell Chains



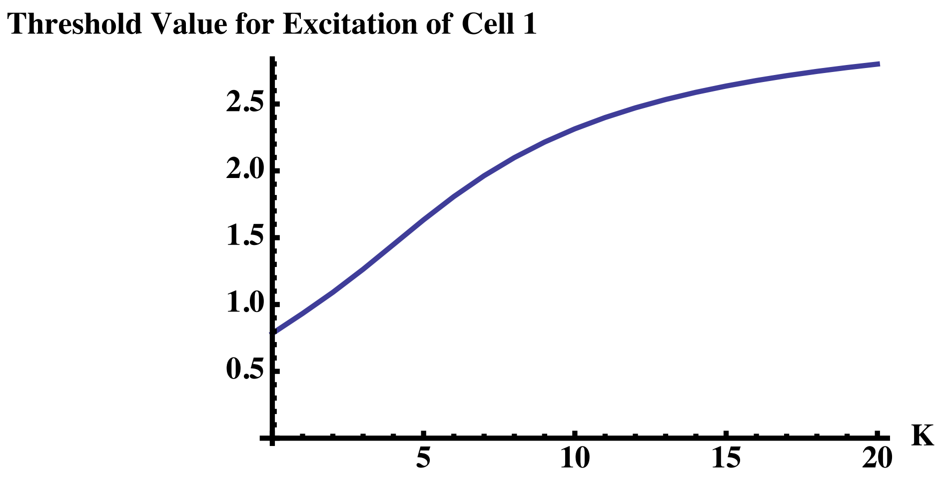
K = 0.5

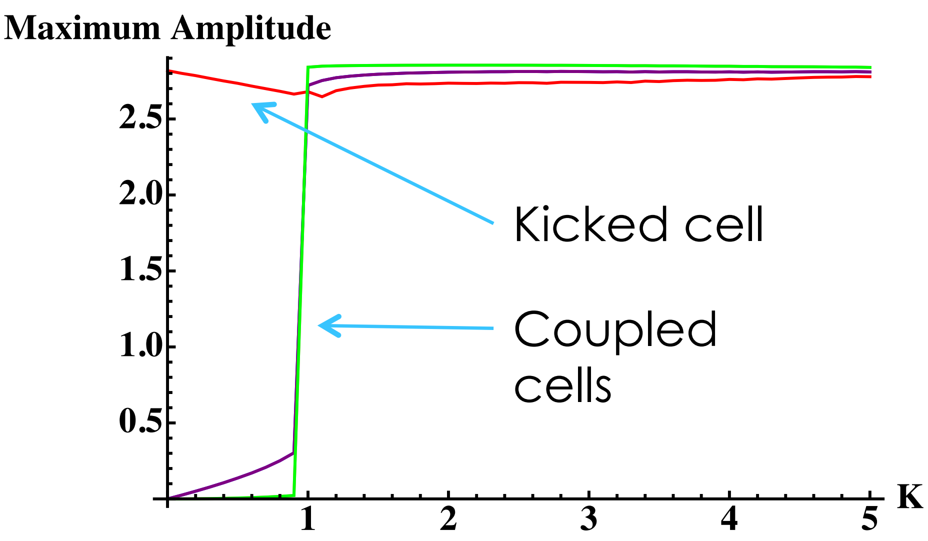


K = 2

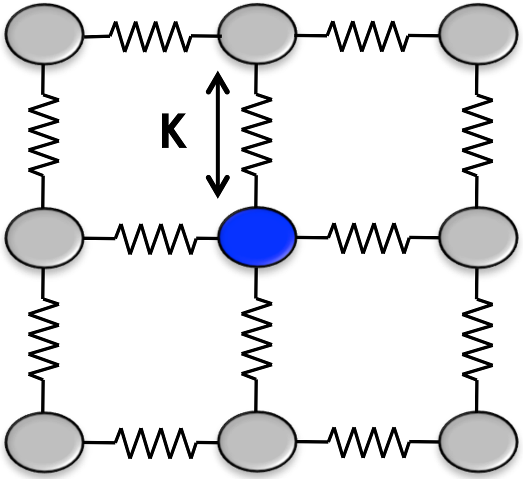


Cell Chains

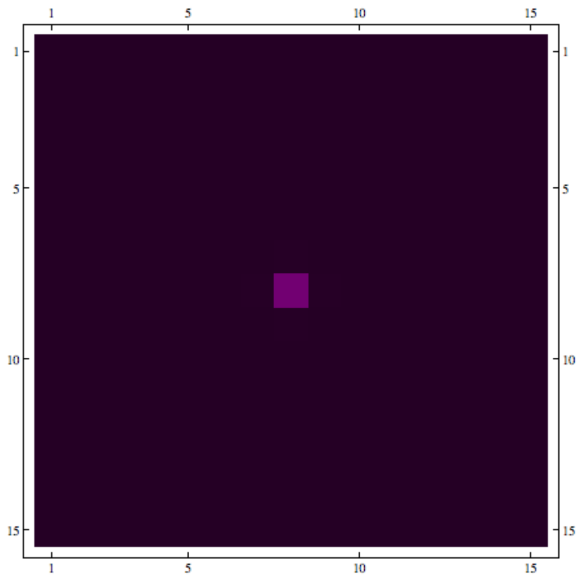

Threshold Value for Excitation of Cell 1
2.5
2.0
1.5
1.0
0.5
5
10
15
20
K


Maximum Amplitude
2.5
2.0
1.5
1.0
0.5
1
2
3
4
5
K
Kicked cell
Coupled cells

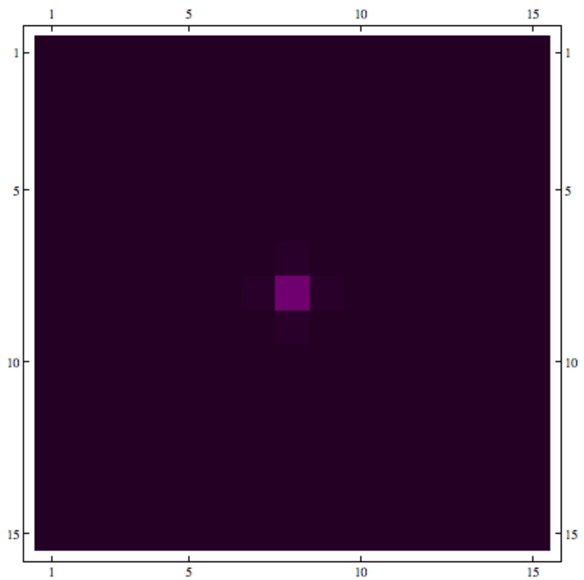
Cell Lattices



Cell Lattices

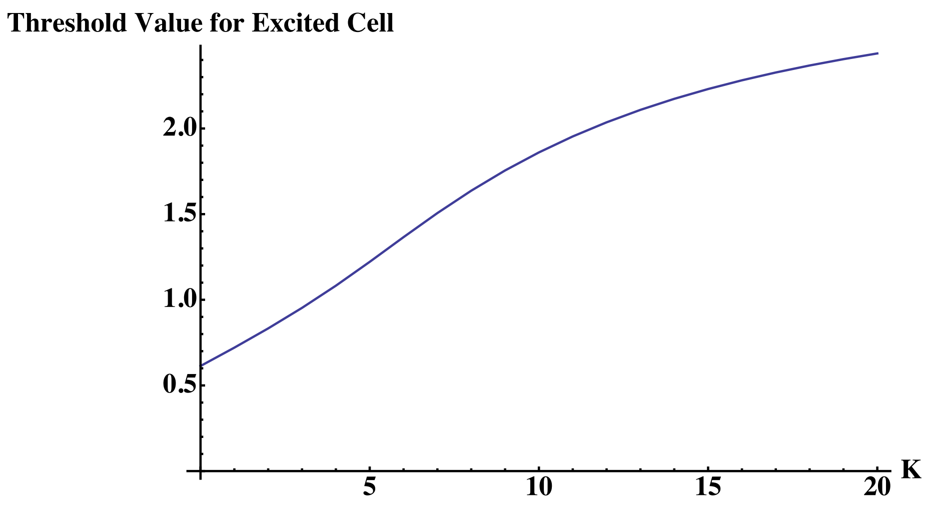


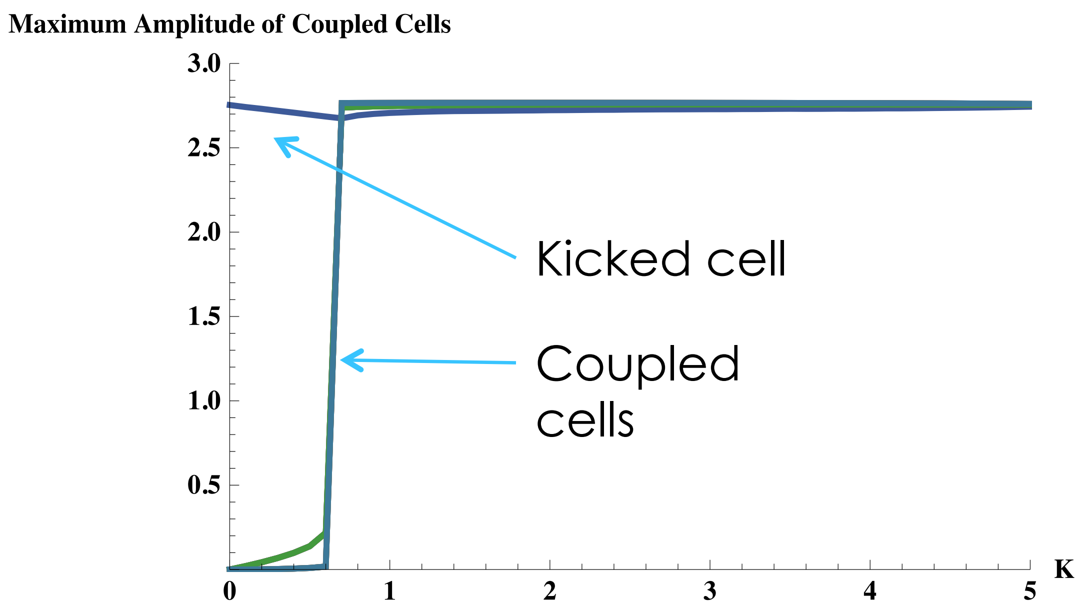
K = 0.5



K = 0.7

Cell Lattices

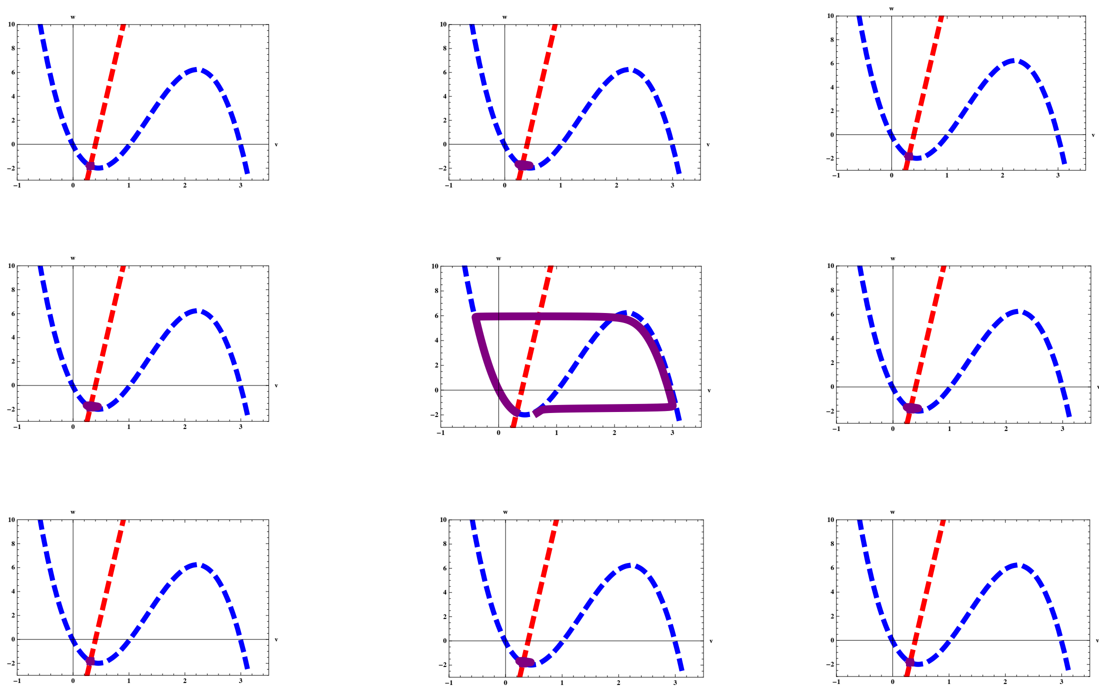




Conclusions

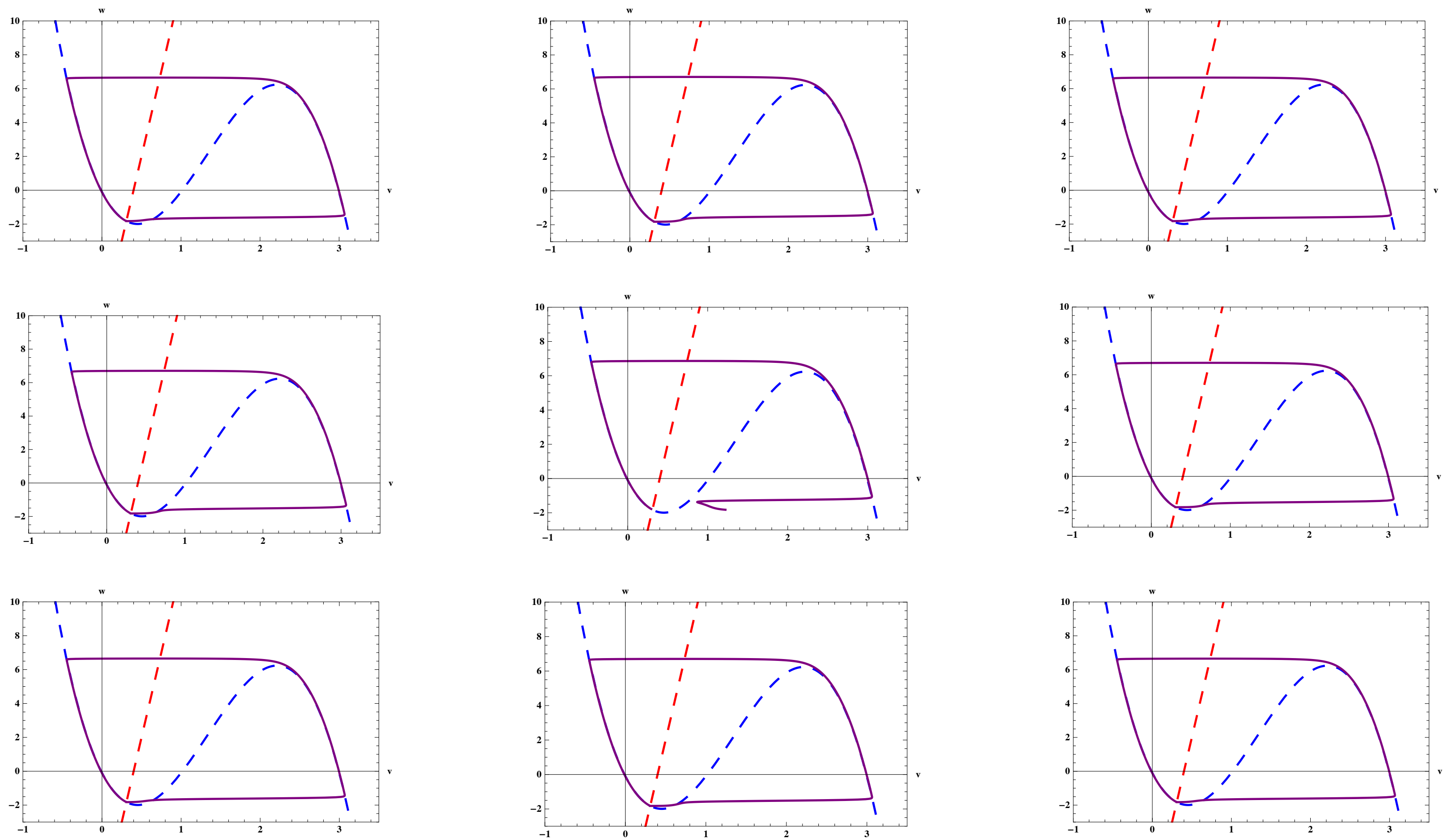
The system has two states:

- **Local excitation** which does not spread across the entire tissue



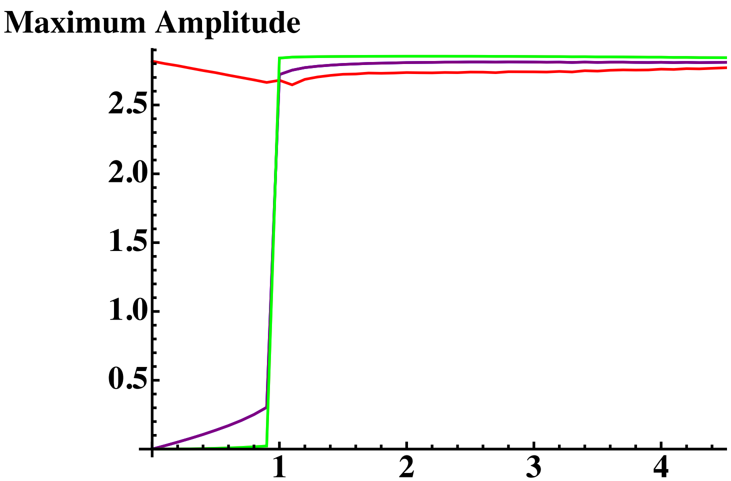
Conclusions

- **Global excitation** where all the cells in the tissue are excited

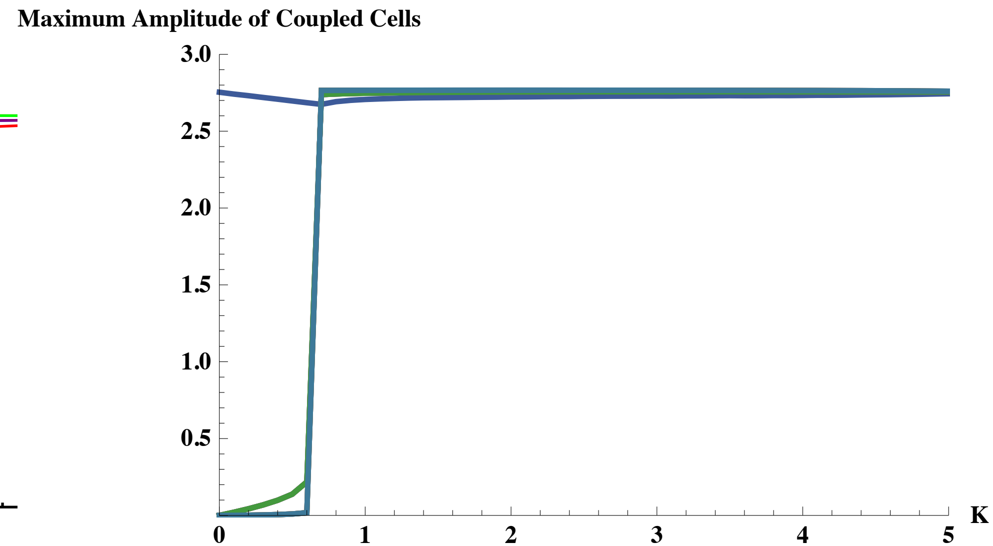


Conclusions

The transition between the two states has a coupling threshold and is instantaneous



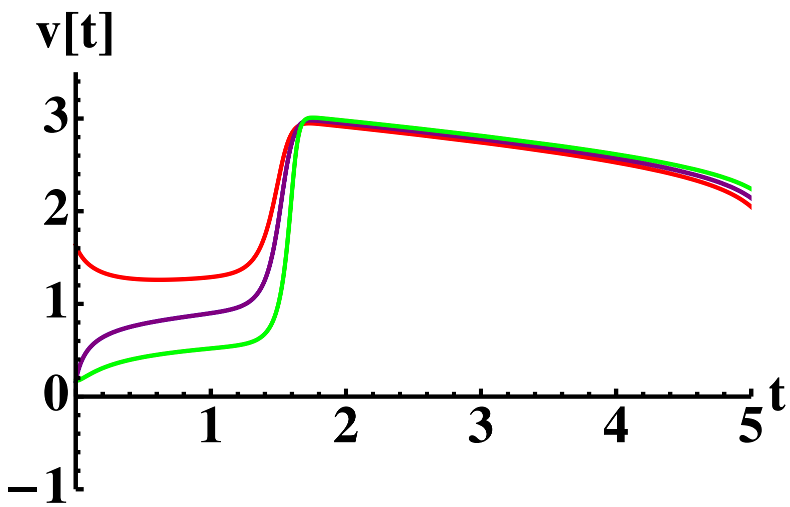
Linearly coupled cells



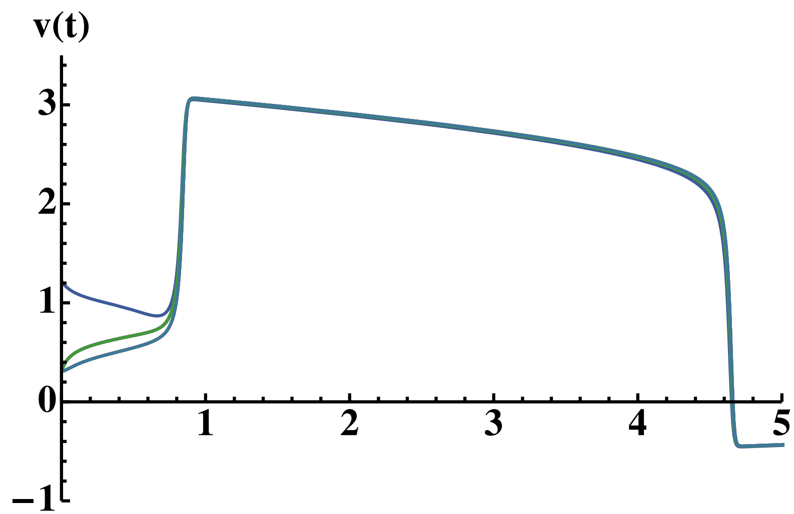
Cell lattice

Conclusions

Excited cells follow the same voltage pathway over time



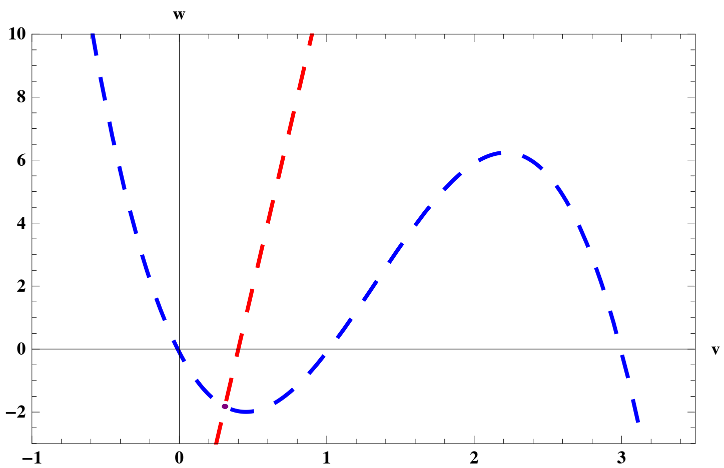
K = 5

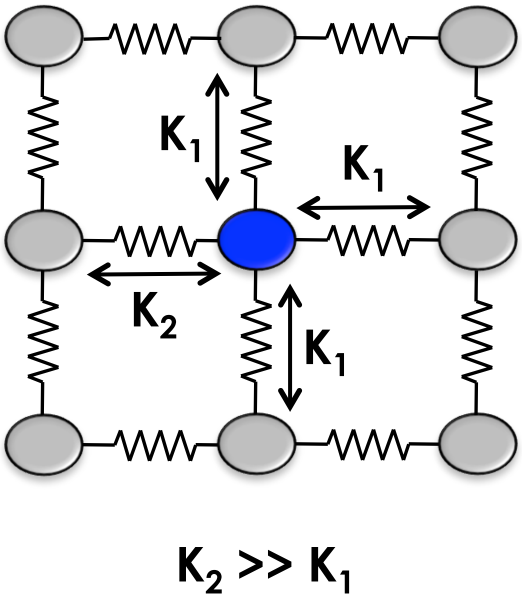


K = 5

Further Work

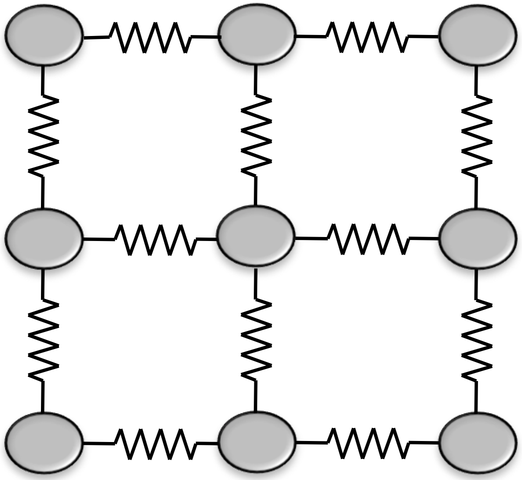
1. Adjust starting parameters





Further Work

2. Remove resistor couplings at random



Thanks!

Thank you to:

- Hugo van den Berg
- MOAC
- EPSRC

- Smith, R. Parturition. *N Engl J Med*. **1997**, 356: 271-283.
- Keener, J.; Sneyd, J. *Mathematical Physiology*; Springer-Verlag: New York, 1998.
- Blanks,A.M. *et.al.* Myometrial function in prematurity. *Best Pract Res Clin Obstet Gynaecol*. **2007**, 21(5): 807-819.

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