Critical Phenomena in a Heterogeneous Excitable System

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1. Aim of the Project

Modelling the changes in the muscular wall of the uterus in the days before labour. In particular, examining the excitation of a tightly coupled system of smooth muscle cells.

2. Background

- The muscular wall of the uterus only acquires the ability to expel a fœtus in the final days before labour
- Smooth muscle cells start at a stable equilibrium. Sufficient input current is needed to excite the cell causing a voltage surge (Fig.1)

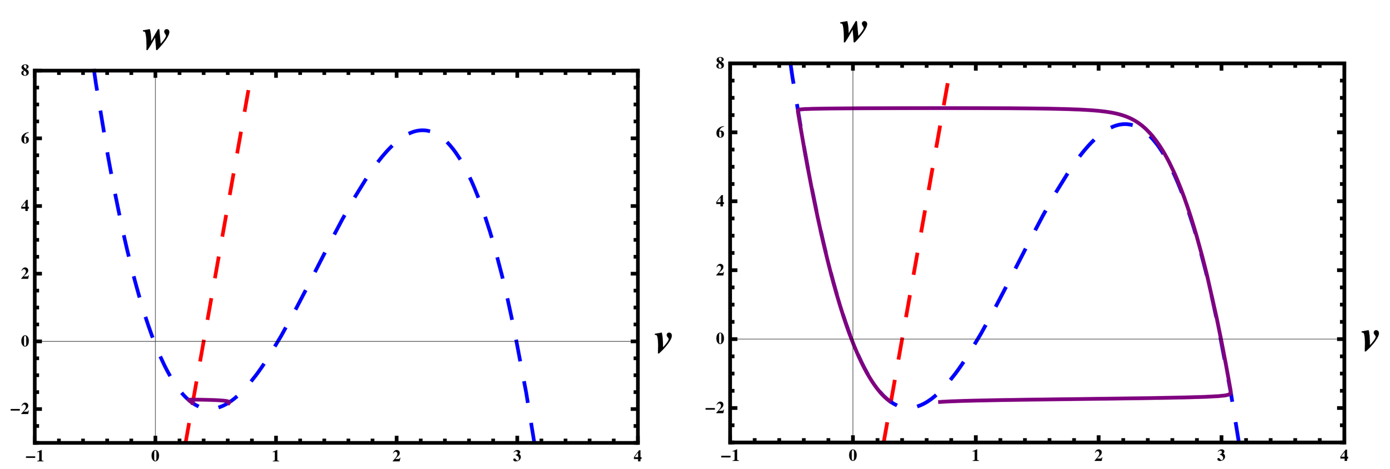


Fig.1 A sufficient perturbation in the initial voltage can excite a muscle cell a) Kick = 0.3; b) kick = 0.4.

- Potential returns rapidly to equilibrium after applied current is removed (Fig.2)
- Myometrial cell synchrony is achieved by electrical conduction through connecting microfibrils[1]
- The slow rise and fall of tension results in a contraction that lasts about a minute[2]

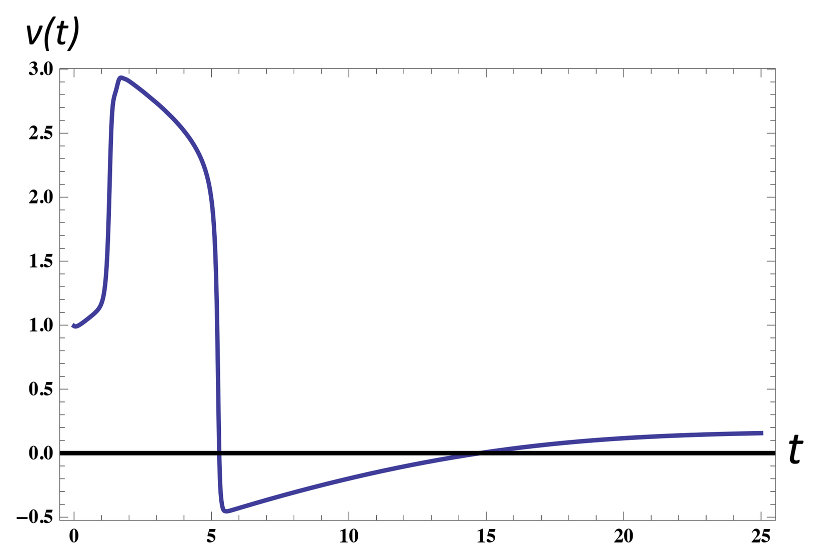


Fig.2 The rise and fall of voltage in an excited cell.

- Cells are considered to obey Fitzhugh-Nagumo dynamics[3]:

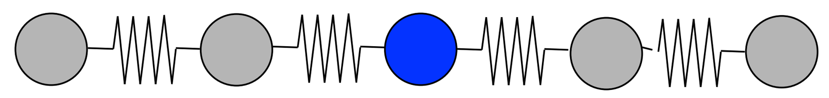
Excitation Variable: $$\frac{d}{dt}v = \frac{1}{\varepsilon}(Av(1-v)(v-\alpha) - w - w_0)$$

Recovery Variable: $$\frac{d}{dt}w = v - \gamma w - v_0$$

- Simulations were run for cells in chains and lattices of an arbitrary size

3. Cell Chains

- Excite a cell with a 'kick'
- Minimum kick needed to excite highlighted cell increases with coupling (Fig. 3)
- Maximum amplitude is consistent for all coupled cells
- There is a threshold coupling value for exciting coupled cells (Fig.4)



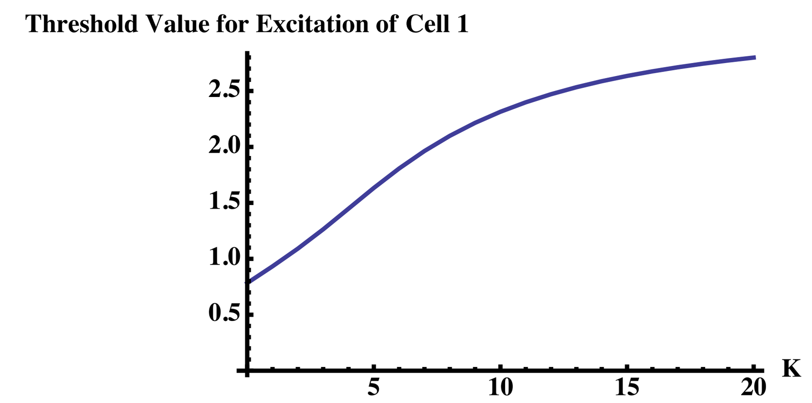


Fig.3 Threshold value for exciting the kicked cell.

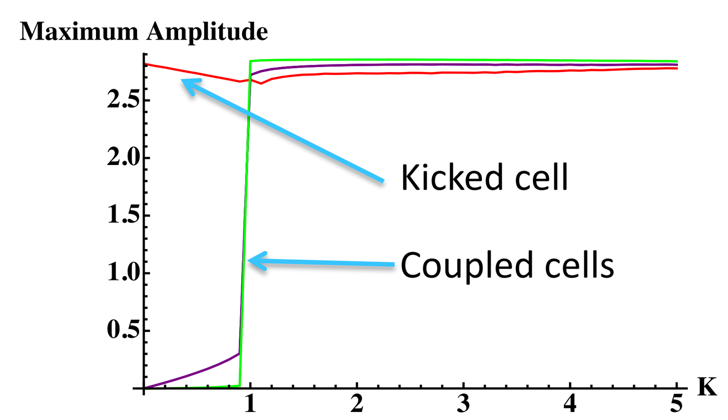
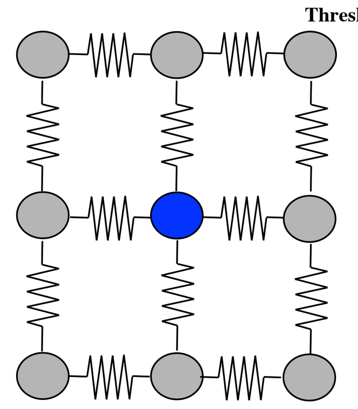


Fig.4 Maximum amplitude of cells with increasing coupling.

4. Cell Lattices

- Excite middle cell
- Minimum kick needed to excite highlighted cell increases with coupling (Fig.5) faster than for a cell chain
- Maximum amplitude is consistent for all coupled cells and comparable to a cell chain
- There is a threshold for exciting coupled cells (Fig.6) which is lower than for a cell chain



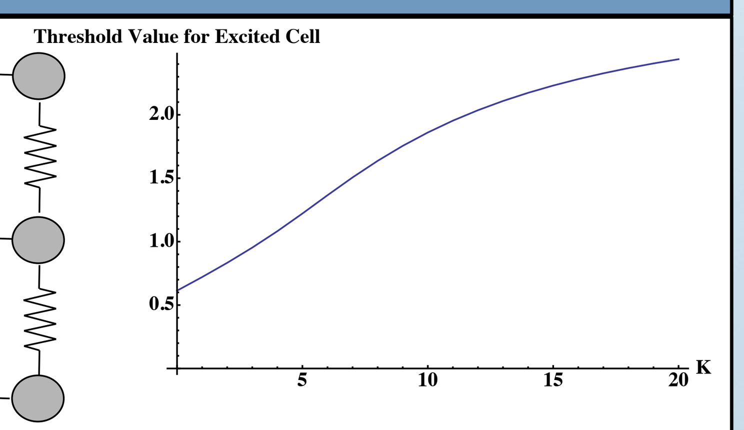


Fig.5 Threshold value for exciting kicked cell.

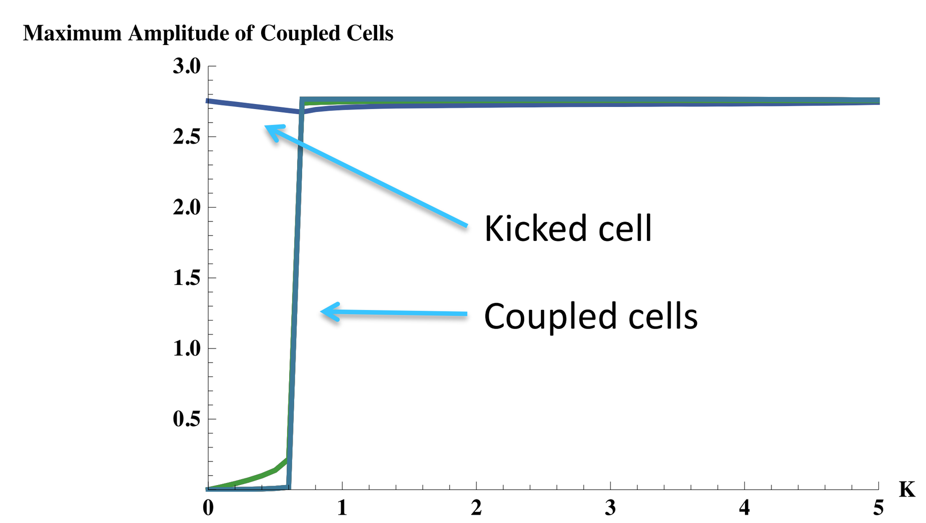


Fig.6 Maximum amplitude of cells with increasing coupling.

5. Conclusions

- The system has two states:
  - Local excitation which does not spread across the entire tissue
  - Global excitation where all cells are excited
- The phase transition from a locally excited state to a globally excited state has a coupling threshold and is instantaneous at this point
- Excited cells follow the same voltage pathway over time

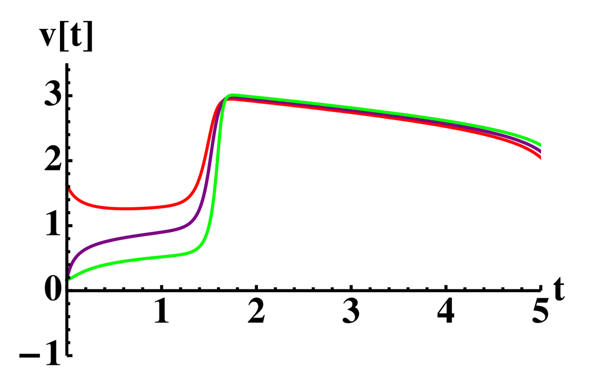
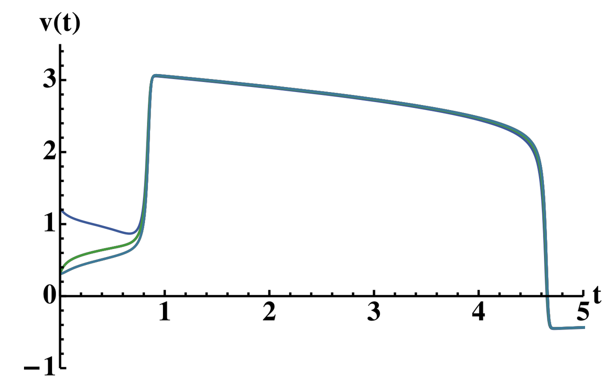
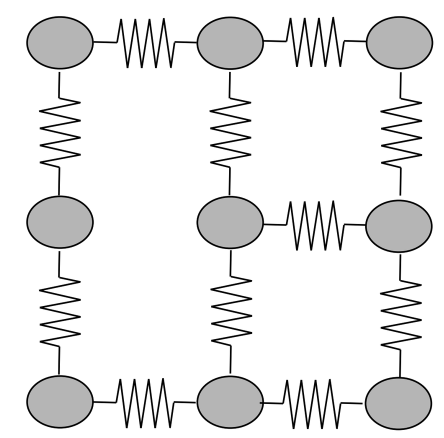


Fig.6 Excitation variable v over time for a) *n* length chain b) *(n x n)* lattice.



6. Further Work

- Introduce variable coupling values between cells
- Remove resistor couplings at random and investigate the spread of excitation using percolation theory



7. References

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2 Blanks, A.M. *et.al.* Myometrial function in prematurity. *Best Pract Res Clin Obstet Gynaecol.* **2007**, 21(5): 807-819.

3 Keener, J.; Sneyd, J. *Mathematical Physiology*; Springer-Verlag: New York, 1998.

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