PX391 Nonlinearity, chaos and complexity.

Prof. S. C. Chapman

In previous courses you will have met generic equations, such as the wave equation which describes propagation of water waves, electromagnetic waves and waves on a string. Almost all of the techniques that you will have seen so far for solving these analytically apply to *linear* systems. In this course we will deal with ***nonlinear*** systems. Just as the wave equation describes a class of behaviour (waves) our approach is to look at simple models for classes of nonlinear behaviour- for example, the predator-prey equations that model both fish populations in the Adriatic, and certain chemical interactions. These nonlinear systems cannot be solved analytically, and some can only be expressed as a set of rules for computer, so we will look at techniques that reveal the dynamics without obtaining a full analytical solution.

Syllabus:

1. General introduction to non-linear phenomena and universality.
2. Landau theory of phase transitions, order parameters, Bifurcation diagrams. First and second order phase transitions.
3. First order non-linear differential equations. Fixed points and linear stability analysis. Global stability (1D phase plane).
4. Second order non-linear differential equations. Phase plane analysis and classification of fixed points. Limit cycles (Attractor).
5. Difference Equations and maps. The tent map and global chaos, Lyapunov exponents. The logistic map, fixed points and bifurcation sequence to chaos. Feigenbaum universality.
6. Self organisation, and emergent behaviour many degree of freedom systems. Examples by computer: avalanche and forest fire models, preferential attachment, flocking, segregation. Concept of few order parameters, critical behaviour, phase transitions and scaling.

Recommended Texts:

G. Rowlands, *Non linear models in Science and Engineering*, Ellis Horwood, ISBN 0-13-62487-4

E. Ott, *Chaos in Dynamical Systems*, CUP, ISBN 0-521-43799-7

P. G. Drazin, Nonlinear Systems, CUP, ISBN 0-521-40668-4

Further Reading:

J. P Sethna, Entropy, order parameters and complexity, OUP, ISBN 978-0-19-856677-9

Netlogo: Many of the systems that we will study have been implemented in the Netlogo suite of models. You can interactively play with these using a web interface on http://ccl.northwestern.edu/netlogo/models/ .You can download the software for free (just click *download*) which allows you to modify the codes or write your own.