Landau Theory – I

of phase transitions of systems in thermodynamic equilibrium

Many component system (ferromagnet) has few parameter behaviour.

Postulate

I) Free energy $F(M)$ $\quad$ M = nett magnetisation
captures state of system.

II) System has time/means to always relax to equilibrium given by minimum in $F(M)$

$$\frac{\partial F}{\partial M} = 0 \qquad \frac{\partial^2 F}{\partial M^2} > 0$$

- thus we only need behaviour close to the min (universal behaviour).

Landau Theory

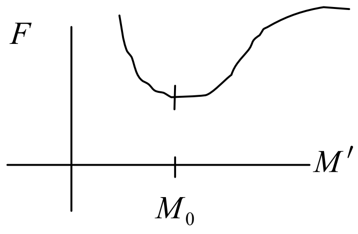
Assume $F(M)$ is a simple power law expansion (we only need behaviour close to the minima – details are irrelevant).

Let

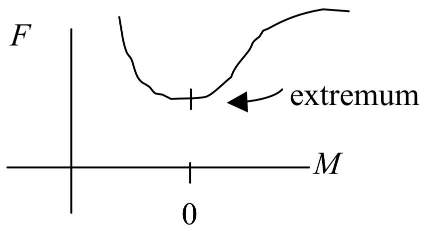
$$F(M) = F_0 + F_1 M + F_2 M^2 + F_3 M^3 + F_4 M^4 + ...$$

with the F_k to be specified.

1) Insist $M = 0$ is an extremum:



Can always transform this to system where an extremum is at M_0 with transformation $M' = M + M_0$



$$\text{thus } \left.\frac{\partial F}{\partial M}\right|_{M=0} = 0$$

$$and \quad F_1 = 0$$

2) Insist on symmetry with M – ie: M equally likely to be +ve or -ve c.f. bar magnet. Then $F_3 = 0$ - no terms in odd powers of M.

3) Since we are only interested in $\frac{\partial F}{\partial M}$, F_0 is irrelevant
$F_0 = 0$.

<u>Our model is then:</u>

$$F(M) = F_2 M^2 + F_4 M^4$$
$$\frac{\partial F}{\partial M} = 2F_2 M + 4F_4 M^3 = 2M\left(F_2 + 2F_4 M^2\right).$$

extrema: $M = 0, \quad M^2 = -\frac{F_2}{2F_4}$

Are these max or min? Depends on F_2, F_4...

Now assume $F_N = F_N(T)$ only.

i.e. T is the only relevant parameter, and that close to some (constant) T_c

$$F_2 = \alpha(T - T_c) \quad F_4 = \beta \quad \alpha, \beta > 0$$

this is the simplest model that we can think of – now explore behaviour.

$$\left.\begin{array}{l} F_2 \text{ changes sign as } T \text{ passes through } T_c \\ F_4 \text{ doesn't} \end{array}\right\} \text{relevant property}$$

extrema are : $M = 0, \qquad M = \pm\sqrt{\frac{\alpha(T_c - T)}{2\beta}}$

are these max or min?

$$\frac{\partial^2 F}{\partial M^2} = 2F_2 + 12F_4 M^2$$

For $M = 0 \qquad \frac{\partial^2 F}{\partial M^2} = 2F_2 = 2\alpha\left(T - T_c\right)$

so $\quad M = 0 \qquad \begin{array}{l} \min T > T_c \\ \max T < T_c \end{array}$

passes smoothly through inflexion at $T = T_c$.

For $M = \pm\sqrt{\dfrac{\alpha(T_c - T)}{2\beta}}$ now recall $M^2 = \dfrac{-F_2}{2F_4}$

$$\frac{\partial^2 F}{\partial M^2} = 2F_2 + 12F_4\left(-\frac{F_2}{2F_4}\right) = 2F_2 - 6F_2$$
$$= -4F_2$$

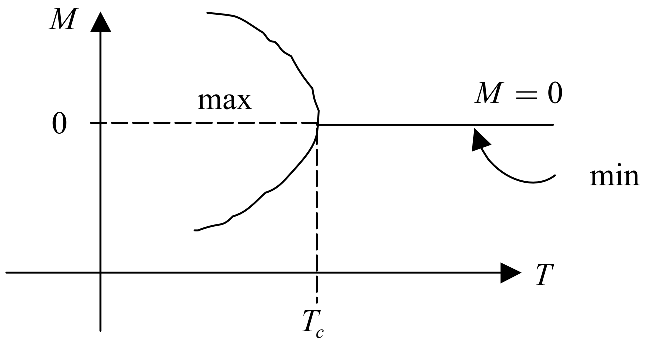
ie: $$\frac{\partial^2 F}{\partial M^2} = 4\alpha\left(T_c - T\right)$$

min for $T_c > T$
imaginary when $T > T_c$

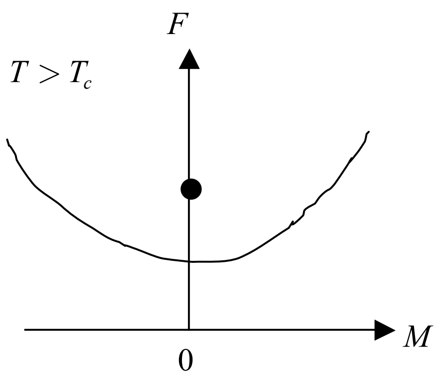
passes through inflexion smoothly at $T = T_c$.

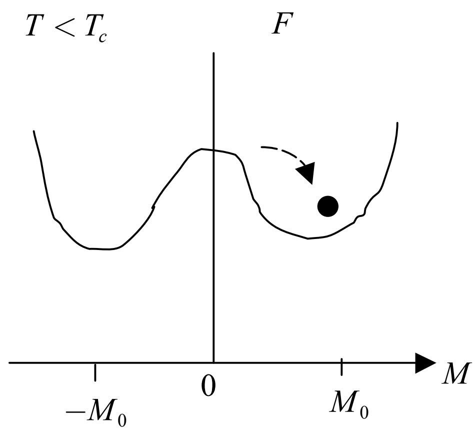
Sketch of behaviour (locations of minima):

$$M = \pm\sqrt{\frac{\alpha(T_c - T)}{2\beta}}$$



- pitchfork bifurcation at $T = T_c$





<u>2nd order</u> – phase transition
- continuous evolution of $M(T)$

[discontinuous = 1st order – comes next]

As we move from $T > T_c$ to $T < T_c$ (reversible path)
- starts at $M = 0$ - unique
- ends at one of $\pm M_0$ - symmetry breaking
- small fluctuations at T_c determine which one $\rightarrow$ NONLINEAR.