Landau Theory II

This is the summary given in the lecture – see handout for full details.

Now relax assumption (2) of symmetric $F(M)$. Now $F_3 \neq 0$.

Simplest case: let $F_3 = \gamma \quad \gamma > 0$ constant, (eg: ferromagnet with applied *B* field).

Now we have

$$F = \alpha(T - T_c)M^2 + \gamma M^3 + \beta M^4$$
$$\frac{\partial F}{\partial M} = 2\alpha\left(T - T_c\right)M + 3\gamma M^2 + 4\beta M^3$$
$$= M\left[2\alpha(T - T_c) + 3\gamma M + 4\beta M^2\right]$$

extrema are $M = 0$ - again, and

$$M = \frac{-3\gamma \pm \sqrt{9\gamma^2 - 4.2.\alpha(T - T_c)4\beta}}{2.4\beta}$$

We will use notation as follows: write this as

$$M = \frac{-3\gamma \pm 3\sqrt{\gamma^2 - \gamma_c^2}}{8\beta}$$
$$\gamma_c^2 = \frac{32}{9}\alpha\beta\left(T - T_c\right)$$

The real extrema are:

$M = 0$, and $M \neq 0$ when $\gamma^2 > \gamma_c^2$ – 2 roots
when $\gamma^2 = \gamma_c^2$ – 1 root

Importantly - $M \neq 0$ extrema are at $T > T_c$ (appear at)
different to $F_3 = 0$ case

- now look for max/min.

$$\frac{\partial^2 F}{\partial M^2} = 2\alpha\left(T - T_c\right) + 6\gamma M + 12\beta M^2$$

For $M = 0$: min $T > T_c$, max $T < T_c$ — as in $F_3 = 0$ case

For $M \neq 0$

We have $2\alpha(T - T_c) + 3\gamma M + 4\beta M^2 = 0$

from $\rightarrow$ $\left(\frac{\partial F}{\partial M} = 0,\ M \neq 0\right)$

So, $$\frac{\partial^2 F}{\partial M^2} = 3\gamma M + 8\beta M^2 \quad \text{for the case where } M \neq 0$$
$$= 8\beta M\left(\frac{3\gamma}{8\beta} + M\right)$$

passes through zero at $M = \frac{-3\gamma}{8\beta}$

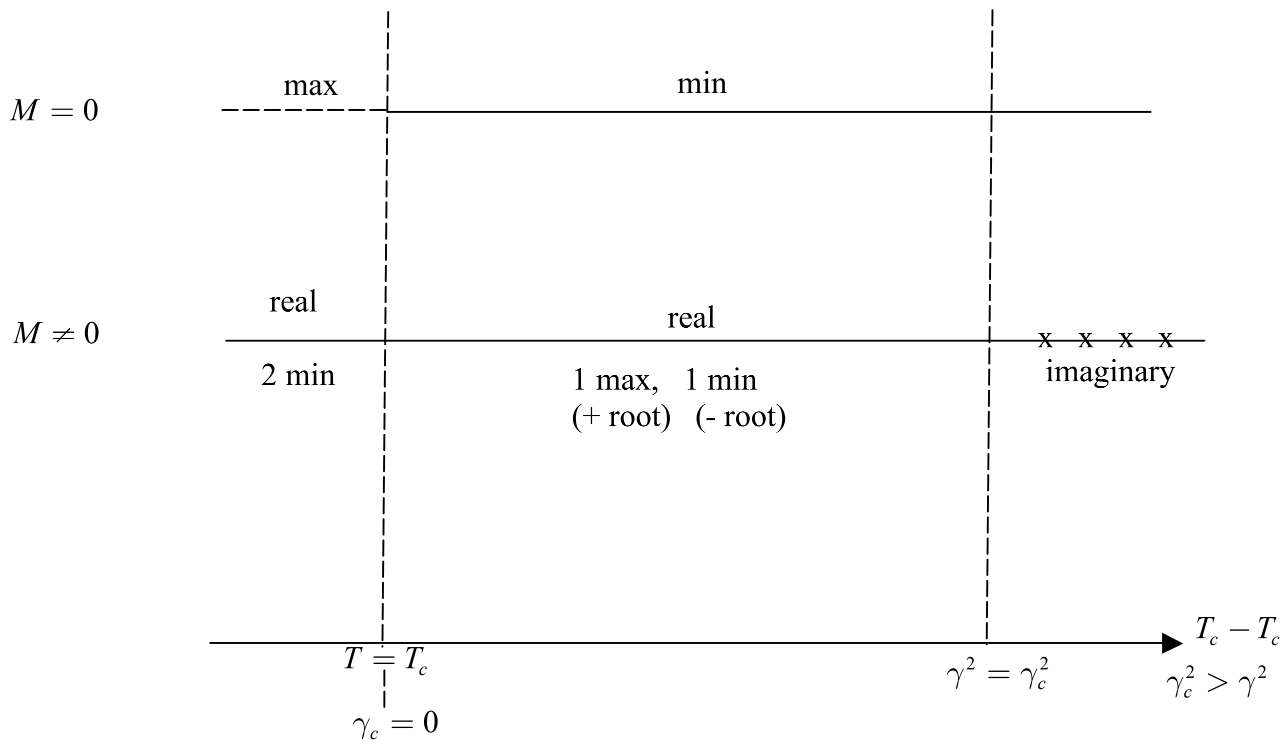
this is when $\gamma^2 = \gamma_c^2$ - so one real $M \neq 0$ root.

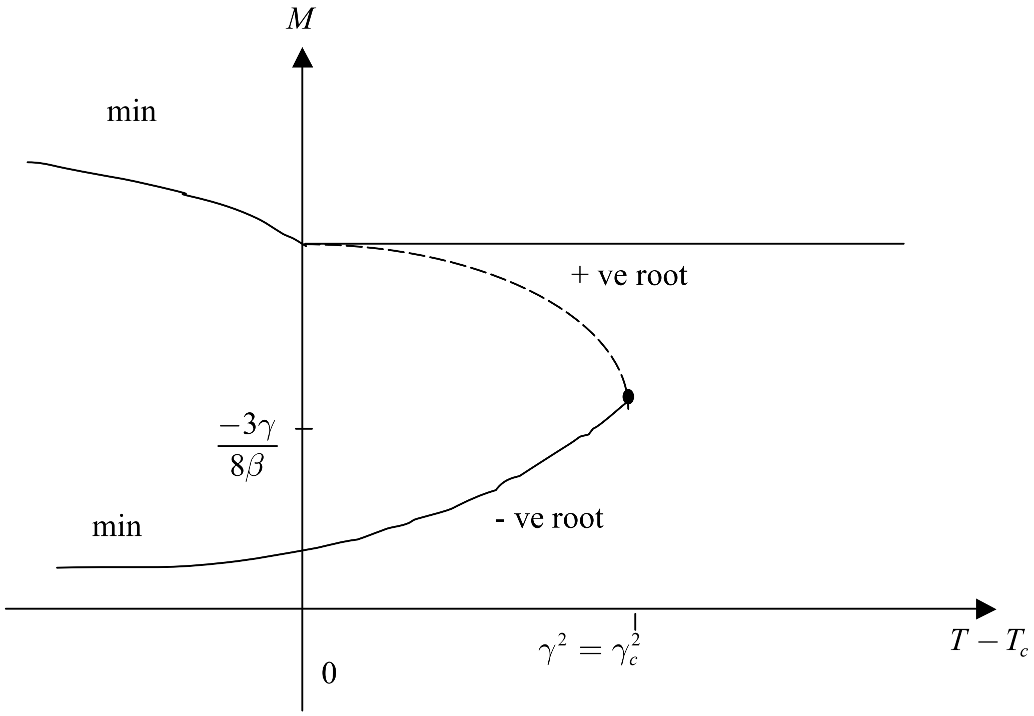
otherwise $\gamma \neq \gamma_c^2$:

$$\frac{\partial^2 F}{\partial M^2} = M\left[\pm 3\sqrt{\gamma^2 - \gamma_c^2}\,\right]$$

- need to look at signs (handout) to classify max/min
- but see that as we go from large $T \to$ small T the root goes *Im*→ *Re*

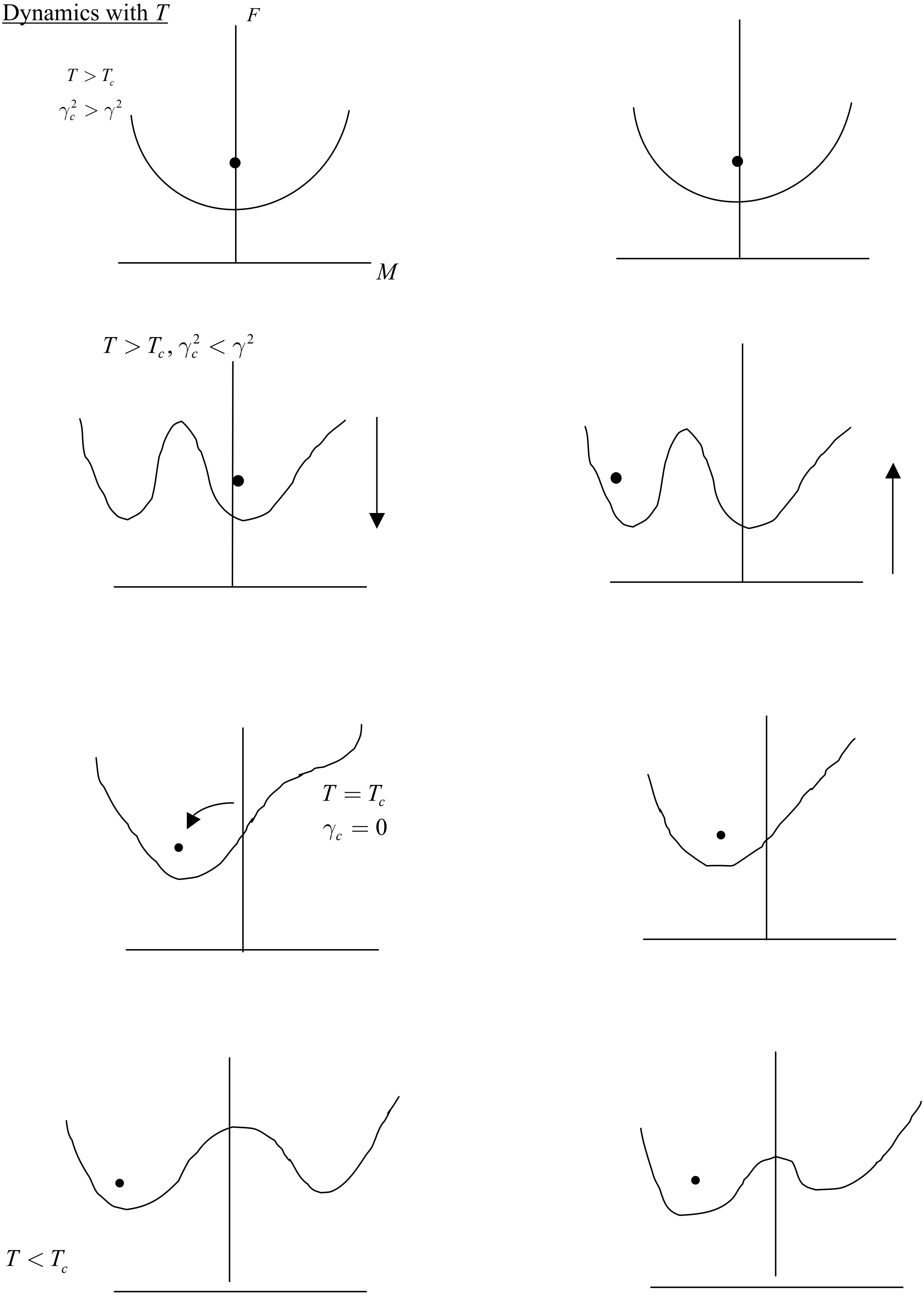
<u>To summarise</u> - max/min behaviour

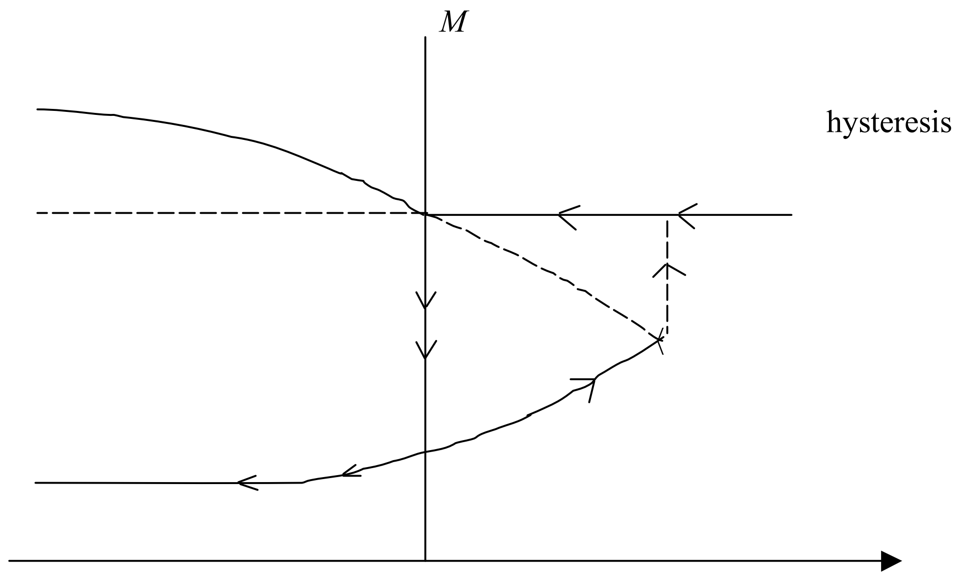




M
min
+ ve root
$\frac{-3\gamma}{8\beta}$
min
- ve root
$\gamma^2 = \gamma_c^2$
$T - T_c$
0

<u>Dynamics with T</u>





- Hysteresis

- Fluctuations don't matter

- 1st order phase transition

- discussed ferromagnets but anything can do this if satisfies $F(M)$, etc. – universality.