Predator – prey problem

Volterra (1926) – fish in the adriatic
Lotka (1920) – chemical reactions

Lotka-Volterra equations

$$\frac{dR}{dt} = \left(\lambda - \alpha F\right) R$$

$$\frac{dF}{dt} = -\left(\eta - \beta R\right) F$$

F = foxes

R = rabbits

hence rabbit growth rate $\left(\frac{dR}{dt}\right)$ is constant $\times R$ λR

in absence of foxes (birth). $\left(\alpha \text{ grass assumption}\right)$

rabbits are eaten at rate α foxes and rabbits, ie: αFR.

Similarly, growth of foxes depends on number of foxes (birth) and rabbits (food) βRF
and is adversely affected by other foxes (competition) $-\eta F$

so $\lambda,\ \alpha,\ \eta,\ \beta > 0$.

As usual find fixed points

$$\overline{R} = 0,\ \overline{F} = 0$$

$$\overline{R} = \frac{\eta}{\beta}\ \overline{F} = \frac{\lambda}{\alpha}$$

look at stability: (done on problem sheet 2)

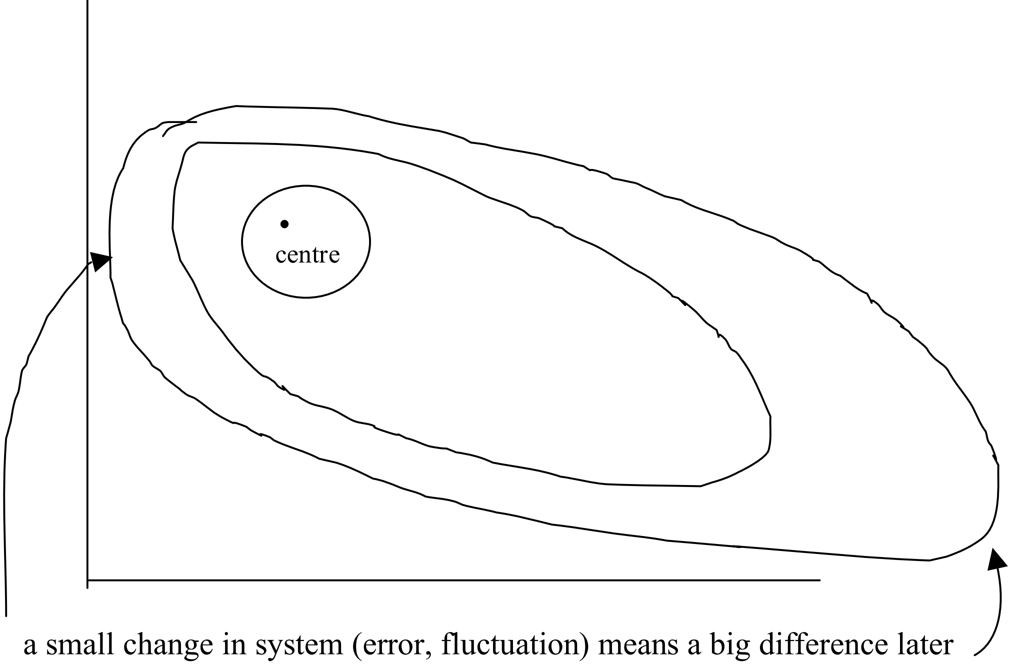
Find:

$\overline{R} = 0,\ \overline{F} = 0$ saddle

$\overline{R} = \frac{\eta}{\beta}\ \overline{F} = \frac{\lambda}{\alpha}$ centre

Note $R > 0$
$F > 0$
only physically meaningful solutions!

NB: Centre ⇒ structural instability;
eg: consider the topology:



Therefore, centre based mathematical models <u>NOT</u> a good idea for physical, biological systems.

Instead, consider models with limit cycle dynamics– won't be affected by fluctuations

<u>Lotka-Volterra – motion about centre implies a constant of the motion</u>

As with nonlinear pendulum – expect in principle that system behaves like particle in a potential well – means should be able to integrate to obtain a constant of the motion.

We have $\frac{dR}{dt} = (\lambda - \alpha F)R \qquad \frac{dF}{dt} = -(\eta - \beta R)F$

then $\frac{dR}{dF} = \frac{-(\lambda - \alpha F)R}{(\eta - \beta R)F}$

$$\frac{dR}{R}(\eta - \beta R) = -(\lambda - \alpha F)\frac{dF}{F}$$

integrates to give $\eta \ln R - \beta R = -\lambda \ln F + \alpha F + C$

ie: C <u>is a constant of the motion</u>. Each value of C specifies an orbit around the centre. Therefore, we know the equation for each orbit C = constant.
NB – this is <u>global</u> information – we did not linearise to obtain it