PX391 Nonlinearity, Chaos and Complexity: Problem Sheet 1

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Question 1

By suitable normalization identify the characteristic length and timescales for the following:

i) a single charged particle moving in a magnetic field with force given by

$$\mathbf{F} = q\mathbf{v} \wedge \mathbf{B}$$

ii) a wave in the absence of dispersion

iii) conservation equation

What do these scales tell you about the solutions to these equations?

Question 2

Assume that the free energy of a Ferromagnet is a simple function of order parameter (magnetization) M:

$$F = F_0 + F_1 M + F_2 M^2 + F_3 M^3 + F_4 M^4 \tag{1}$$

Assuming that a good approximation is $F_2 = \alpha(T - T_c), F_3 = \gamma, F_4 = \beta$ describe the behaviour of the minimum of free energy:

i) for symmetric F

ii) for asymmetric F

What is the role of fluctuations in both cases?

iii) The Van der Vaal model for liquid-vapour transition is

$$F = \frac{T}{b}(1 - bM)ln(1 - bM) + MT - \frac{aM^2}{2} \tag{2}$$

where T is the temperature and $1/M$ the volume.

Show that this reduces to the form of (1) in the limit of small M and hence identify the critical temperature in terms of constants a, b.

Question 3

Use stability analysis to study the following and obtain the global behaviour in both cases:

i)

$$\frac{dq}{dt} = sinq$$

ii)

$$\frac{dq}{dt} = \alpha q - \beta q^2$$