PX391 Nonlinearity, Chaos and Complexity : Problem Sheet 2

Sandra Chapman

Question 1

The equation of motion for a damped oscillator is:

$$\frac{d^2x}{dt^2} + \lambda\frac{dx}{dt} + \omega^2 sinx = 0$$

where λ is the frictional coefficient.

i) For $\lambda = 0$ the pendulum is undamped. Show that the energy is conserved, and discuss the dynamics in this case.

Give the value of the energy that separates bounded from unbounded motion.

ii) For both positive and negative λ discuss the dynamics. Compare the phase plane for both cases.

Question 2

The Lotka Volterra (predator-prey) equations are:

$$\frac{dR}{dt} = (\lambda - \alpha F)R$$

$$\frac{dF}{dt} = -(\eta - \beta R)F$$

Discuss the dynamics of this system and sketch the phase plane.

Show that a constant of the motion is:

$$C = (\eta lnR - \beta R) - (\alpha F - \lambda lnF)$$

Question 3

Show that the system:

$$\frac{dx}{dt} = x - y - x(x^2 + 2y^2)$$

$$\frac{dy}{dt} = x + y - y(x^2 + y^2)$$

must possess a limit cycle (hint: transform to polar coordinates).