PX391 Nonlinearity, Chaos and Complexity: Problem Sheet 3

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Question 1

For the general map

$$x_{n+1} = f(x_n)$$

obtain an expression for the Lyapunov exponent λ which characterizes the exponential divergence of neighbouring trajectories x_n and $\bar{x}_n = x_n + \epsilon_n$ such that:

$$\bar{x}_{n+1} - x_{n+1} = \epsilon_0 e^{n\lambda}$$

and where subscript 0 denotes the initial separation of the trajectories.

What is the nature of the dynamics for $\lambda > 0$ and $\lambda < 0$?

Question 2

Consider the map $M(x)$ given by:

$$x_{n+1} = \frac{x_n}{a} \quad 0 < x < a$$
$$x_{n+1} = \frac{1 - x_n}{1 - a} \quad a < x < 1$$

where $0 < a < 1$.

Sketch the map and find the fixed points. Show that the fixed point of $M(x)$ is unstable for all $0 < a < 1$.

By finding the 'folding points' or otherwise sketch the twice iterated map $M^2(x)$, for $a \neq \frac{1}{2}$.

Find the Lyapunov exponent for $M(x)$.

Finally consider the special cases $a = 1$ and $a = 0$. Discuss the fixed points and their stability, find the Lyapunov exponent and obtain the behaviour for all $0 < x < 1$.

Question 3

The growth of grass g in the presence of rabbits R is given by

$$\frac{dg}{dt} = \lambda_g g - eR$$

where e is the constant rate at which the rabbits consume the grass.

If the growth rate of rabbits in the presence of foxes F is given by assuming that the rabbit birth rate λ_b just depends on availability of grass such that

$$\frac{dR}{dt} = \lambda_b g - \alpha F R$$

find the dynamics of grass, rabbits and foxes for fast growing grass, that is, $\lambda_g >> \lambda_b$.