

PX391 Nonlinearity, Chaos and Complexity: Problem Sheet 4

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You can find implementations of both these models in Netlogo. see <http://ccl.nortwestern.edu/netlogo/models/>

Question 1

Recall for the Landau theory of ferromagnets we found that in the absence of an applied field, the system's behaviour was given by the extrema:

$$M = 0, M = \pm \sqrt{\frac{\alpha(T_c - T)}{2\beta}}$$

and with applied field B_0 the extrema:

$$M = 0, M = \frac{-3\gamma \pm \sqrt{9\gamma^2 - 32\alpha\beta(T - T_c)}}{8\beta}$$

where M is the nett magnetization, T is the temperature, T_c the critical temperature and α, β, γ are constants that specify details of the system.

(a) Identify the dimensionless groups of parameters needed specify M at a given fixed temperature T for both cases.

(b) Consider a microscopic dynamical model of the ferromagnet composed of many individual spins on a 2D grid, each with magnetization m , separation η , in a box of size L_0 . At each timestep Δt the spins change their orientation (and hence their vector magnetization) by an interaction with their neighbours, and by a random fluctuation which on average has amplitude ϵ . Use dimensional analysis (Buckingham II theorem) to identify the dimensionless groups of parameters that order behaviour in the system for both cases and identify the corresponding macroscopic dimensionless parameters in the Landau theory.

Question 2

A model for fireflies has the following rules:

- each firefly flashes cyclically every $t = \tau_c$ with duration τ_d
- the start time of the cycle is initially set at random
- a firefly resets to the beginning of its cycle if N_f flashes are seen within radius R
- there are N fireflies in a box of size L_0 with periodic boundary conditions and they move at random every dt with steps of size $dx = vdt$ where v is a characteristic speed.

By means of Buckingham II theorem or otherwise, write down the dimensionless groups of parameters that order behaviour in the system and discuss their meaning. Use these to identify a single nontrivial criterion for synchronization of flashes.