Lecture 3: Fluctuations of the order parameter. Bogoliubov theory

Previous lecture: Mean field description of the condensates. Order parameter and its equation of motion.

$$\hat{\Psi}(\mathbf{x}) \longrightarrow \Psi_0$$

This lecture

- Fragmented condensate and its energy – role of interactions
- Quantum and thermal fluctuations. Bogoliubov theory of weakly interacting Bose gas
- Elementary excitations of the BEC. Particles and Quasiparticles
- Phonons and Quantum Hydrodynamics
- Beliaev decay of phonons

Order Parameter and Ground State

$$\hat{\Psi}(\mathbf{x}) = \Psi_0 + \delta\hat{\Psi}(\mathbf{x}) = \Psi_0 + \sum_{\mathbf{p}\neq 0} \psi_{\mathbf{p}}(\mathbf{x})\hat{a}_{\mathbf{p}}$$

$$\Psi_0 = \langle \hat{\Psi} \rangle = \langle N|\hat{\Psi}|N+1\rangle$$

$$|N\rangle = \frac{1}{\sqrt{N!}}\left(a_0^\dagger\right)^N |\mathrm{VAC}\rangle$$

Possible Fragmented BEC?

Suppose macroscopic occupation of states

$$\mathbf{p} = 0 \qquad \mathbf{p} = (2\pi\hbar/L, 0, 0)$$

$$N_0 \qquad N_1$$

$$N_0 + N_1 = N \qquad N_0, N_1 \sim N$$

Fragmented condensate

$$|\mathrm{Frag}\rangle = \frac{1}{\sqrt{N_0! N_1!}} \left(a_0^\dagger\right)^{N_0} \left(a_1^\dagger\right)^{N_1} |\mathrm{Vac}\rangle$$

Interaction Energy

$$H_{\mathrm{int}} = \frac{g}{2}\int \mathrm{d}\mathbf{r}\hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r})$$

Simplify field operator in the subspace of 1 and 2

$$\hat{\Psi}(\mathbf{r}) = \psi_0(\mathbf{r})\hat{a}_0 + \psi_1(\mathbf{r})\hat{a}_1$$

$$\hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}) =$$

$$= \left[(\psi_0^*)^2\left(\hat{a}_0^{\dagger}\right)^2 + (\psi_1^*)^2\left(\hat{a}_1^{\dagger}\right)^2 + 2\psi_0^*\psi_1^*\hat{a}_0^{\dagger}\hat{a}_1^{\dagger}\right] \times \left[(\psi_0)^2(\hat{a}_0)^2 + (\psi_1)^2(\hat{a}_1)^2 + 2\psi_0\psi_1\hat{a}_0\hat{a}_1\right]$$

Energy of fragmented state

$$\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}) =$$

$$= \left[(\psi_0^*)^2\left(\hat{a}_0^\dagger\right)^2 + (\psi_1^*)^2\left(\hat{a}_1^\dagger\right)^2 + 2\psi_0^*\psi_1^*\hat{a}_0^\dagger\hat{a}_1^\dagger\right] \times \left[(\psi_0)^2(\hat{a}_0)^2 + (\psi_1)^2(\hat{a}_1)^2 + 2\psi_0\psi_1\hat{a}_0\hat{a}_1\right]$$

$$E_{\mathrm{Frag}} = \frac{g}{2}\int \mathrm{d}\mathbf{r}\left(N_0^2|\psi_0|^4 + N_1^2|\psi_1|^4 + 4N_0N_1|\psi_0|^2|\psi_1|^2\right)$$

BEC state

$$N_0 + N_1 = N$$

$$E_{\mathrm{BEC}} = \frac{g}{2}\int \mathrm{d}\mathbf{r}(N_0 + N_1)^2|\psi_0|^4$$

Absence of fragmented state in uniform system. Nozières' argument

$$|\psi_0(\mathbf{r})|^2 = |\psi_1(\mathbf{r})|^2 = \mathrm{const}$$

$$\Delta E = E_{\mathrm{Frag}} - E_{\mathrm{BEC}} = gN_0N_1 \int \mathrm{d}\mathbf{r}|\psi_0|^4 > 0$$

and fragmentation is inhibited by repulsive interactions

NB. In a *non-uniform* system quantum fluctuations may favour fragmented state (see Josephson effect)

Bogoliubov theory

$$\hat{H} - \mu\hat{N} = \int \mathrm{d}\mathbf{x} \left[\frac{\hbar^2}{2m} \nabla\hat{\Psi}^\dagger \cdot \nabla\hat{\Psi} - \mu\hat{\Psi}^\dagger(\mathbf{x})\hat{\Psi}(\mathbf{x}) \right] + \frac{g}{2} \int \mathrm{d}\mathbf{x}\, \hat{\Psi}^\dagger(\mathbf{x})\hat{\Psi}^\dagger(\mathbf{x})\hat{\Psi}(\mathbf{x})\hat{\Psi}(\mathbf{x})$$

Substitute

$$\hat{\Psi}(\mathbf{x}) = \Psi_0 + \hat{\varphi}(\mathbf{x}) = \Psi_0 + \frac{1}{\sqrt{V}} \sum_{\mathbf{p}\neq 0} e^{i\mathbf{p}\cdot\mathbf{x}} \hat{a}_{\mathbf{p}}$$

Bogoliubov Hamiltonian

$$\hat{H} - \mu\hat{N} = \frac{g}{2}|\Psi_0|^4 - \mu|\Psi_0|^2$$

$$+ \int \mathrm{d}\mathbf{x} \left(g|\Psi_0|^2\Psi_0 - \mu\Psi_0\right)\hat{\varphi}^\dagger + \mathrm{h.c.}$$

$$+ \int \mathrm{d}\mathbf{x} \left[\frac{\hbar^2}{2m}\nabla\hat{\varphi}^\dagger \cdot \nabla\hat{\varphi} - \mu\hat{\varphi}^\dagger\hat{\varphi} + \frac{gn_0}{2}\left(\hat{\varphi}^\dagger\hat{\varphi}^\dagger + 2\hat{\varphi}^\dagger\hat{\varphi} + \hat{\varphi}\hat{\varphi}\right)\right]$$

$$+O(\hat{\varphi}^3)$$

Linear terms (GPE) $$|\Psi_0|^2 = n_0 = \frac{\mu}{g}$$

Quadratic terms

$$\hat{H} - \mu \hat{N} = -V \frac{\mu^2}{2g}$$

$$+ \sum_{\mathbf{p} \neq \mathbf{0}} \left(\frac{p^2}{2m} + g n_0 \right) a^\dagger_{\mathbf{p}} a_{\mathbf{p}} + \frac{g n_0}{2} \left(a^\dagger_{\mathbf{p}} a^\dagger_{-\mathbf{p}} + a_{-\mathbf{p}} a_{\mathbf{p}} \right)$$

$\left[\hat{N}, \hat{H} \right] \neq 0$ - particle number is not conserved

Quadratic, so can be diagonalised

Remark: to this order in the condensate density n_0 *can be replaced by* n

Linear equation of motion

To find the energy of elementary excitation we use Heisenberg equations of motion:

$$i\hbar\dot{a}_{\mathbf{p}} = [a_{\mathbf{p}}, \hat{H}] = \left(\frac{p^2}{2m} + gn_0\right) a_{\mathbf{p}} + gn_0 a^{\dagger}_{-\mathbf{p}}$$

and similarly for $a^{\dagger}_{-\mathbf{p}}$

Using $a_{\mathbf{p}}(t) = e^{-\frac{i\varepsilon t}{\hbar}} a_{\mathbf{p}}$ $\quad a^{\dagger}_{-\mathbf{p}}(t) = e^{+\frac{i\varepsilon t}{\hbar}} a^{\dagger}_{-\mathbf{p}}$

we obtain linear system

$$\begin{pmatrix} \lambda(p) - \varepsilon & gn_0 \\ gn_0 & \lambda(p) + \varepsilon \end{pmatrix} \begin{bmatrix} a_{\mathbf{p}} \\ a^{\dagger}_{-\mathbf{p}} \end{bmatrix} = 0 \quad \lambda(p) = \frac{p^2}{2m} + gn_0$$

Excitation energies

Has solution only if $$\det \begin{pmatrix} \lambda(p) - \varepsilon & gn \\ gn & \lambda(p) + \varepsilon \end{pmatrix} = 0$$

or

$$\varepsilon^2 = \lambda^2(p) - (gn)^2 = \left(2gn + \frac{p^2}{2m}\right) \frac{p^2}{2m}$$

Bogoliubov dispersion $$\varepsilon(p) = \sqrt{\left(2gn + \frac{p^2}{2m}\right) \frac{p^2}{2m}}$$

Bogoliubov spectrum

$$\varepsilon(p) = \sqrt{\left(2gn + \frac{p^2}{2m}\right)\frac{p^2}{2m}}$$

describes 2 regimes:

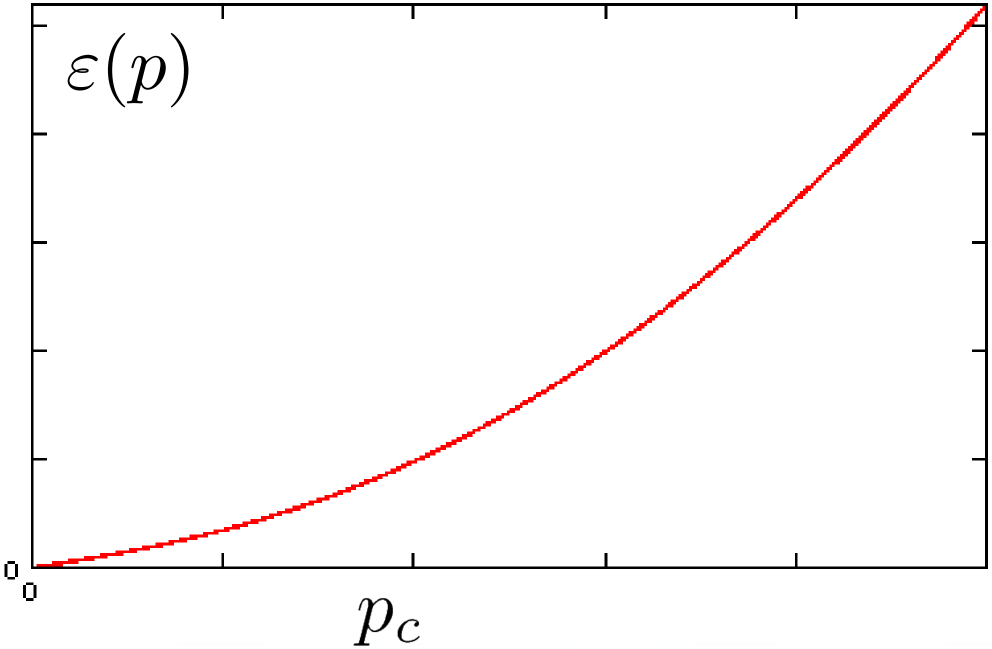
$$p < p_c = \sqrt{mgn} = \hbar/\xi$$

linear (phonons)

$$\varepsilon(p) = \sqrt{\frac{gnp^2}{m}} = c|p|$$

$p > p_c$ free particle

$$\varepsilon(p) = \frac{p^2}{2m} + gn$$



Bogoliubov transformation

Mix $a^\dagger_{\mathbf{p}}$ (particles) and $a_{-\mathbf{p}}$ (anti-particles)

$$a_{\mathbf{p}} = u_{\mathbf{p}} b_{\mathbf{p}} + v^*_{-\mathbf{p}} b^\dagger_{-\mathbf{p}} \qquad a^\dagger_{\mathbf{p}} = u^*_{\mathbf{p}} b^\dagger_{\mathbf{p}} + v_{-\mathbf{p}} b_{-\mathbf{p}}$$

This is a linear canonical transformation. Canonical means that it preserves bosonic commutation relations:

$$[a_{\mathbf{p}}, a^\dagger_{\mathbf{p}}] = u^*_{\mathbf{p}} u^*_{\mathbf{p}} [b_{\mathbf{p}}, b^\dagger_{\mathbf{p}}] + u^*_{\mathbf{p}} v^*_{-\mathbf{p}} [b^\dagger_{-\mathbf{p}}, b^\dagger_{\mathbf{p}}] +$$

$$+ u_{\mathbf{p}} v_{-\mathbf{p}} [b_{\mathbf{p}}, b_{-\mathbf{p}}] + v^*_{-\mathbf{p}} v_{-\mathbf{p}} [b^\dagger_{-\mathbf{p}}, b_{-\mathbf{p}}]$$

$$= |u_{\mathbf{p}}|^2 - |v_{-\mathbf{p}}|^2 = 1$$

Bogoliubov transformation

Parametrisation $u_{\mathbf{p}} = \cosh \alpha_{\mathbf{p}} \qquad v_{-\mathbf{p}} = \sinh \alpha_{\mathbf{p}}$

$$|u_{\mathbf{p}}|^2 - |v_{-\mathbf{p}}|^2 = 1$$

* Choose $\alpha_{\mathbf{p}}$ to eliminate nondiagonal terms $a^\dagger_{\mathbf{p}} a^\dagger_{-\mathbf{p}} + a_{-\mathbf{p}} a_{\mathbf{p}}$ in the Hamiltonian

Result

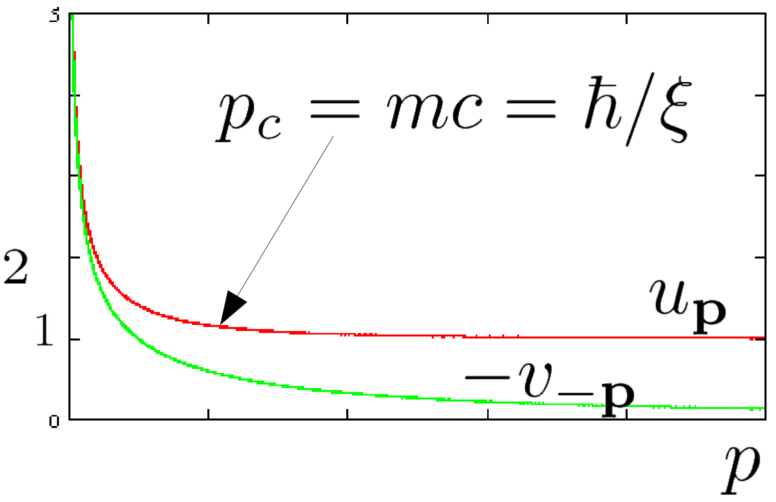
$$\frac{gn}{2}\left(|u_{\mathbf{p}}|^2 + |v_{-\mathbf{p}}|^2\right) + \left(\frac{p^2}{2m} + gn\right) u_{\mathbf{p}} v_{-\mathbf{p}} = 0$$

$\cosh 2\alpha = \cosh^2 \alpha + \sinh^2 \alpha$ $\qquad$ $\sinh 2\alpha = 2 \cosh \alpha \sinh \alpha$

Bogoliubov coefficients

$$\coth 2\alpha_{\mathbf{p}} = 1 - \frac{p^2}{2mgn}$$

$$u_{\mathbf{p}}, v_{-\mathbf{p}} = \pm \left(\frac{p^2/2m + gn_0}{2\epsilon(p)} \pm \frac{1}{2} \right)^{1/2}$$



Phonons

$$u_{\mathbf{p}} = -v_{-\mathbf{p}} = \sqrt{mc/2p}$$

Free particles

$$u_{\mathbf{p}} \simeq 1 \qquad v_{-\mathbf{p}} \simeq 0$$

Ground State Energy

* $$\hat{H} - \mu\hat{N} = E_0 - \mu N + \sum_{\mathbf{p}\neq\mathbf{0}} \varepsilon(p) b^\dagger_{\mathbf{p}} b_{\mathbf{p}}$$

(difference of) zero point oscillations

$$E_0 = -\frac{gnN}{2} + \mu N$$
$$+\frac{V}{2}\int \frac{\mathrm{d}\mathbf{p}}{(2\pi\hbar)^d}\left(\varepsilon(p) - \frac{p^2}{2m} - gn\right)$$

A problem...

The integral

$$\int \frac{\mathrm{d}\mathbf{p}}{(2\pi\hbar)^d} \left(\varepsilon(p) - \frac{p^2}{2m} - gn \right)$$

has ultra violet (large momenta) divergence in $d \geq 2$
since

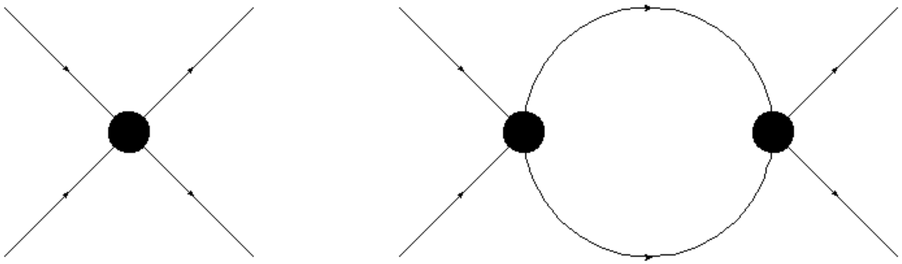
$$\varepsilon(p) \simeq \frac{p^2}{2m} + gn - \frac{m(gn)^2}{p^2} + O(p^{-4})$$

second order in coupling

Renormalisation of coupling

Exactly the same divergence occurs when trying to use effective interaction potential

$$V_{\text{eff}}(\mathbf{x}) = g\delta^{(3)}(\mathbf{x})$$



to calculate second order mean-field energy

$$* \quad g\frac{N^2}{2V} \to g\frac{N^2}{2V} + \frac{m(gn)^2}{2}\sum_{\mathbf{p}\neq 0}\frac{1}{p^2}$$

Putting all together

$$E_0 = \frac{gnN}{2} + \frac{V}{2}\int \frac{d\mathbf{p}}{(2\pi\hbar)^d}\left(\varepsilon(p) - \frac{p^2}{2m} - gn + \frac{m(gn)^2}{p^2}\right)$$

$$\frac{E_0}{V} = \frac{gn^2}{2}\left[1 + \frac{128}{15\sqrt{\pi}}\left(na^3\right)^{1/2}\right]$$ *

$$\mu = \frac{\partial E_0}{\partial N} = gn\left[1 + \frac{32}{3\sqrt{\pi}}\left(na^3\right)^{1/2}\right]$$

Corrections are proportional to $g^{5/2}$ - <u>not a perturbation theory</u>

Ground state properties

Ground state is a vacuum of Bogoliubov quasiparticles

$$b_{\mathbf{p}}|0\rangle = \left(u_{\mathbf{p}}a_{\mathbf{p}} - v^*_{-\mathbf{p}}a^\dagger_{-\mathbf{p}}\right)|0\rangle = 0$$

and NOT of real particles!

$$a_{\mathbf{p}}|0\rangle = \left(u_{\mathbf{p}}b_{\mathbf{p}} + v^*_{-\mathbf{p}}b^\dagger_{-\mathbf{p}}\right)|0\rangle \neq 0$$

In fact it is a coherent superposition of pairs with momenta p and $-p$

Occupation numbers and depletion

Using Bogoliubov transformation and commutation relations

$$n_{\mathbf{p}} = \langle a^{\dagger}_{\mathbf{p}} a_{\mathbf{p}} \rangle = |v_{-\mathbf{p}}|^2 + |u_{\mathbf{p}}|^2 \langle b^{\dagger}_{\mathbf{p}} b_{\mathbf{p}} \rangle + |v_{-\mathbf{p}}|^2 \langle b^{\dagger}_{-\mathbf{p}} b_{-\mathbf{p}} \rangle$$

At zero temperature the number of particles "depleted" from the condensate is

$$n - n_0 = \frac{1}{V} \sum_{\mathbf{p}\neq 0} |v_{\mathbf{p}}|^2 = \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} \left(\frac{p^2/2m + gn}{2\varepsilon(p)} - \frac{1}{2} \right)$$

$$= \frac{8n}{3\sqrt{\pi}} (na^3)^{1/2}$$

Phonons and quantum hydrodynamics

Low energy excitations of Bogoliubov theory are phonons, quantum analogue of sound

$$\epsilon(p) = cp$$

This remains true in strongly interacting superfluids (see next lecture).

Indeed, at low energy quantum pressure term is small and one can use hydrodynamic equations

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\mathbf{v}_s \rho) = 0$$ - continuity

$$m\frac{\partial \mathbf{v}_s}{\partial t} + \nabla\left(\frac{m\mathbf{v}_s^2}{2} + \mu(\rho)\right) = 0$$ - Euler equation

Hydrodynamic hamiltonian

The density ρ and superfluid velocity $\mathbf{v}_s = \frac{\hbar}{m}\nabla\phi$ configuration has energy

$$H = \int \mathrm{d}\mathbf{r} \left[\frac{1}{2} m\rho\mathbf{v}_s^2 + e(\rho)\right]$$

$$\mu(\rho) = \frac{\mathrm{d}e}{\mathrm{d}\rho}$$

In 1941 Landau suggested a way to quantise this Hamiltonian by imposing canonical commutation relations

$$\left[\hat{\rho}(\mathbf{r}), \hat{\phi}(\mathbf{r}')\right] = i\hbar\delta(\mathbf{r} - \mathbf{r}')$$

Hydrodynamic hamiltonian

and properly symmetrising the Hamiltonian:

$$\hat{H} = \int \mathrm{d}\mathbf{r} \left[\frac{1}{2m} \nabla \hat{\phi}\, \hat{\rho}\, \nabla \hat{\phi} + e(\hat{\rho}) \right]$$

This quantum Hamiltonian yields the same hydrodynamic equations of motion if one uses the Heisenberg time evolution

$$i\hbar \partial_t \hat{\rho} = \left[\hat{\rho}, \hat{H} \right] \qquad i\hbar \partial_t \hat{\phi} = \left[\hat{\phi}, \hat{H} \right]$$

Density – Phase representation

Density – phase representation of bosonic operators $\hat{\Psi}(\mathbf{x}) = \sqrt{\hat{\rho}(\mathbf{x})}e^{i\hat{\phi}(\mathbf{x})}$

$$\frac{\hbar^2}{2m}\nabla\hat{\Psi}^\dagger \cdot \nabla\hat{\Psi} \longrightarrow \frac{\hbar^2}{2m}\left[\left(\nabla\sqrt{\hat{\rho}}\right)^2 + \hat{\rho}\left(\nabla\hat{\phi}\right)^2\right]$$

$$\frac{g}{2}\hat{\Psi}^\dagger(\mathbf{x})\hat{\Psi}^\dagger(\mathbf{x})\hat{\Psi}(\mathbf{x})\hat{\Psi}(\mathbf{x}) \longrightarrow e(\hat{\rho}) = \frac{g\hat{\rho}^2}{2}$$

The last substitution is valid for weak interactions only. As interactions become stronger higher terms "dress" the value of $e(\rho)$

Normal mode expansion

Consider small deviations around uniform density $\rho = \rho_0 + \rho'$

$$\rho' = \frac{1}{\sqrt{2V}} \sum_{\mathbf{k} \neq 0} \left(\frac{\rho_0 k}{mc} \right)^{1/2} (b_{\mathbf{k}} + b^{\dagger}_{-\mathbf{k}}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

and from the commutation relations

$$\phi = \frac{i}{\sqrt{2V}} \sum_{\mathbf{k} \neq 0} \left(\frac{mc}{\rho_0 k} \right)^{1/2} (b_{\mathbf{k}} - b^{\dagger}_{-\mathbf{k}}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

Hamiltonian of phonons

The coefficients in this expansion are chosen to diagonalise the quadratic Hamiltonian

$$\hat{H}_0 = \int \mathrm{d}\mathbf{r} \left[\frac{\rho_0}{2m} (\nabla \hat{\phi})^2 + \frac{mc^2}{2\rho_0} (\rho')^2 \right] = \sum_{\mathbf{k} \neq 0} ck b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}}$$

which describes free phonons (sound waves). Sound velocity is obtained from compressibility

$$\frac{mc^2}{\rho_0} = \frac{\mathrm{d}^2 e}{\mathrm{d}\rho_0^2} = \frac{\mathrm{d}\mu}{\mathrm{d}\rho_0}$$

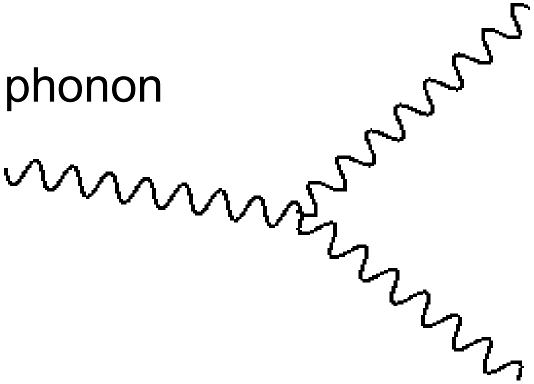
Anharmonic terms (phonon interactions)

Next term in the expansion gives 3-phonon interactions

$$\hat{H}_3 = \int \mathrm{d}\mathbf{r} \left[\frac{1}{2m} \nabla\hat{\phi}\,\hat{\rho}'\,\nabla\hat{\phi} + \frac{1}{6}\left(\frac{\mathrm{d}}{\mathrm{d}\rho_0} \frac{mc^2}{\rho_0} \right) (\rho')^3 \right]$$

absent in Bogoliubov theory

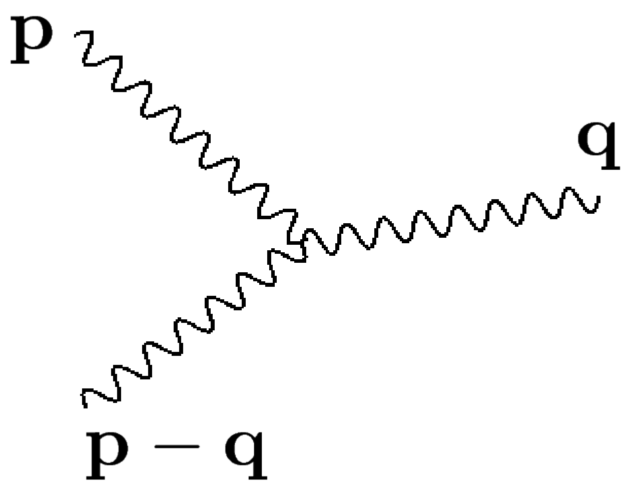
which may lead to decay of a phonon into two phonons



Beliaev decay

Probability of decay in unit time is given by Fermi Golden Rule

$$\Gamma = \frac{2\pi}{\hbar} \sum_f \left| \langle f | \hat{H}_3 | i \rangle \right|^2 \delta(\epsilon(q) + \epsilon(|\mathbf{p} - \mathbf{q}|) - \epsilon(p))$$



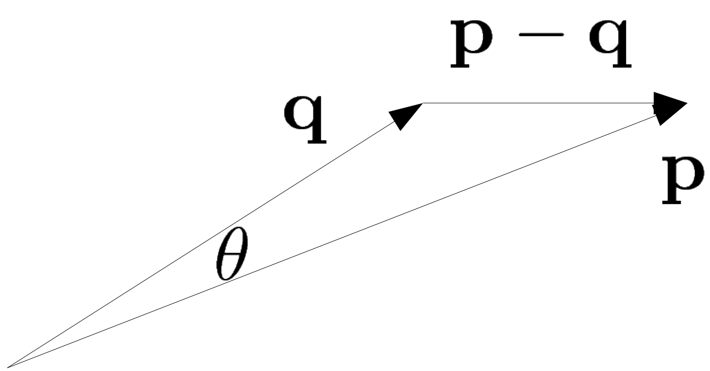
initial state: 1 phonon with momentum $\mathbf{p}$

final state: 2 phonons with momenta

$\mathbf{q}$ and $\mathbf{p} - \mathbf{q}$

Energy conservation

$$\epsilon(p) = \epsilon(q) + \epsilon(|\mathbf{p} - \mathbf{q}|)$$



Linear dispersion leads to *divergent result*.
Include nonlinearity

$$\epsilon(p) = cp + \alpha p^3$$

$$c(p - q - |\mathbf{p} - \mathbf{q}|) \simeq -\frac{cpq}{p-q}(1 - \cos\theta)$$

$$= -\alpha(p^3 - q^3 - |\mathbf{p} - \mathbf{q}|^3) \simeq -3\alpha pq(p - q)$$

$$1 - \cos\theta = \frac{3\alpha}{c}(p - q)^2 \geq 0$$

Matrix element

$$\rho' \sim \frac{1}{\sqrt{2V}}\left[\left(\frac{\rho_0 p}{mc}\right)^{1/2} b_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}} + \left(\frac{\rho_0 q}{mc}\right)^{1/2} b^\dagger_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}} + \left(\frac{\rho_0|\mathbf{p}-\mathbf{q}|}{mc}\right)^{1/2} b^\dagger_{\mathbf{p}-\mathbf{q}} e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}}\right]$$

and similarly for $\nabla\hat{\phi}$

$$\langle\mathbf{q},\mathbf{p}-\mathbf{q}|\hat{H}_3|\mathbf{p}\rangle = \langle\mathbf{q},\mathbf{p}-\mathbf{q}|\frac{\nabla\hat{\phi}\rho'\nabla\hat{\phi}}{2m}|\mathbf{p}\rangle = = \frac{-i}{\sqrt{2V}}\left[\frac{mc}{\rho_0}pq(p-q)\right]^{1/2}$$

Decay rate

$$\Gamma = p^5 \frac{3}{320\pi\rho_0\hbar^4} \sim \int_0^p \frac{q^2 \mathrm{d}q}{2\pi}(p-q)^2$$

Note that there is no dependence on α and on the coupling constant

The only small parameter is the smallness of low energy phase space:

$$\frac{p}{mc} \ll 1$$

Conclusions of Lecture 3

- Repulsive interactions favour a non-fragmented condensate in uniform systems. Not the case in inhomogeneous situation
- Fluctuations of order parameter can be described by elementary excitations: quasiparticles. At low energies their properties are drastically different from those of particles
- Quasiparticles are phonons with linear dispersion law and survive for strong interactions. This is the base of superfluidity
- Quasiparticles are weakly interacting. The weak interactions lead to a 2-phonon decay (Beliaev decay of phonons)
- The uniform approach of this lecture can be generalised to inhomogeneous (trapped) systems by expanding around Thomas Fermi background