

Funsilab

Magnetism and magnetic interactions

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◆ Introduction to magnetism:

single-particle magnetic hamiltonian; quantum origin; exchange interaction; atomic magnetism; band magnetism; DFT

◆ Magnetic interactions I

local moments; magnetic force theorem, Liechtenstein formula and Heisenberg model; magnetic groundstates and spin spirals; disordered local moments; energies of spin spirals; competing interactions; magnetic multilayers

◆ Magnetic interactions II

spin-orbit coupling; magnetic anisotropy, dipole-dipole interaction and magnetic stability; Dzyaloshinskii-Moriya interactions and chirality; chiral groundstates; symmetry; biquadratic and four-spin interactions

◆ Beyond the groundstate

statistical physics; spin waves; dynamical magnetic susceptibility

Introduction to magnetism

- ◆ Schrödinger hamiltonian in Coulomb gauge: $\vec{A} = \vec{r} \times \vec{B}$

$$\mathcal{H} = \frac{1}{2m} \left(\vec{p} - \frac{e}{c} \vec{A} \right)^2 + V(\vec{r}) = \frac{p^2}{2m} - \frac{e}{2mc} (\vec{r} \times \vec{p}) \cdot \vec{B} + \frac{e^2}{8mc^2} (\vec{r} \times \vec{B})^2 + V(\vec{r})$$

- ◆ Magnetic part is paramagnetic plus diamagnetic

- ◆ Orbital magnetic moment:

$$\hbar \vec{L} = \vec{r} \times \vec{p} \implies -\frac{e}{2mc} (\vec{r} \times \vec{p}) \cdot \vec{B} = -\mu_B \vec{L} \cdot \vec{B}, \quad \mu_B = \frac{e\hbar}{2mc}$$

- ◆ Dirac hamiltonian gives also spin moment and spin-orbit coupling:

$$2\hbar \vec{S} = \hbar \vec{\sigma} \implies -\mu_B \vec{\sigma} \cdot \vec{B} \quad V_{\text{soc}}(\vec{r}) \approx \frac{e\hbar^2}{4m^2c^2} \frac{1}{r} \frac{\partial V}{\partial r} \vec{L} \cdot \vec{\sigma} = \xi_{\text{soc}} \vec{L} \cdot \vec{\sigma}$$

Bohr – van Leeuwen theorem: the equilibrium magnetization of a system described by classical physics is zero

- ◆ The dependence on the magnetic field can be removed from the classical expression for the magnetization by a simple change of integration variable

Magnetism comes from the interaction among the electrons: the classical magnetic dipole-dipole interaction is too weak to explain it

- ◆ The Pauli principle and the Coulomb repulsion among the electrons combine to give different energies to parallel and antiparallel spins

Two electrons on one atom – I

- ◆ Schrödinger hamiltonian

for two electrons:

$$\mathcal{H} = \frac{p_1^2}{2m} - \frac{Ze^2}{r_1} + \frac{p_2^2}{2m} - \frac{Ze^2}{r_2} + \frac{e^2}{|\vec{r}_1 - \vec{r}_2|}$$

- ◆ No spin dependence: **why is energy spin dependent?**

- ◆ Pauli exclusion: $\Psi(\vec{r}_1, s_1; \vec{r}_2, s_2) = \phi(\vec{r}_1, \vec{r}_2)\chi(s_1, s_2) = -\Psi(\vec{r}_2, s_2; \vec{r}_1, s_1)$
 $\implies \phi(\vec{r}_1, \vec{r}_2) = \pm\phi(\vec{r}_2, \vec{r}_1) \text{ , } \chi(s_1, s_2) = \mp\chi(s_2, s_1)$

- ◆ Spinors: $\chi(s_1, s_2) = |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}, \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$

- ◆ Orbitals:

$$\phi(\vec{r}_1, \vec{r}_2) = |aa\rangle, |bb\rangle, \frac{|ab\rangle + |ba\rangle}{\sqrt{2}}, \frac{|ab\rangle - |ba\rangle}{\sqrt{2}}, |ab\rangle = \phi_a(\vec{r}_1)\phi_b(\vec{r}_2)$$

- ◆ Two electrons on the same orbital: spins antiparallel

R.M. White, *Quantum Theory of Magnetism*, Springer (2007)

Two electrons on one atom – II

◆ Kinetic plus nuclear part: $\langle ab | \frac{p_1^2}{2m} - \frac{Ze^2}{r_1} + \frac{p_2^2}{2m} - \frac{Ze^2}{r_2} | ab \rangle = E_a + E_b$

◆ Coulomb repulsion part: $\langle ab | \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} | ab \rangle = U_{abab}$

◆ Triplet (degenerate): $E_a + E_b + \frac{U_{abab} - U_{abba}}{2}$

◆ Singlet block:
$$\begin{pmatrix} 2E_a + U_{aaaa} & U_{aabb} & \frac{U_{aaab} + U_{aaba}}{\sqrt{2}} \\ U_{bbba} & 2E_b + U_{bbbb} & \frac{U_{bbab} + U_{bbba}}{\sqrt{2}} \\ \frac{U_{abaa} + U_{baaa}}{\sqrt{2}} & \frac{U_{abbb} + U_{babb}}{\sqrt{2}} & E_a + E_b + \frac{U_{abab} + U_{abba}}{2} \end{pmatrix}$$

◆ Coulomb repulsion lowers the energy of the triplet against the singlet

◆ Hamiltonian independent of spin + Pauli exclusion = spin-dependent energy

R.M. White, *Quantum Theory of Magnetism*, Springer (2007)

Hund's rules – groundstate of an atom

- ◆ Russel-Saunders coupling scheme: $\vec{L} = \sum_{i=1}^{N_e} \vec{L}_i$, $\vec{S} = \sum_{i=1}^{N_e} \vec{S}_i$
- ◆ **First rule:** maximize the total spin (think triplet vs singlet)
- ◆ **Second rule:** maximize the total orbital angular momentum (matrix elements of the Coulomb repulsion are smaller)
- ◆ **Third rule:** decide on the total angular momentum as to minimize the spin-orbit coupling energy, $E_{\text{soc}} \approx \xi \vec{L} \cdot \vec{S}$
- ◆ In molecules and solids the orbitals can spread across several atoms, and the rules hold approximately for those electrons localized on a single atom
- ◆ **How to describe the magnetism of delocalized electrons?**

R.M. White, *Quantum Theory of Magnetism*, Springer (2007)

- ◆ All the tricks in the book (spin up and spin down with same energy):

$$E_{\text{band}} = 2 \sum_{\vec{k}}^{k \leq k_F} \frac{\hbar^2 k^2}{2m} = 2 \int^{\varepsilon_F} d\varepsilon \varepsilon \frac{V}{(2\pi)^3} \int d\vec{k} \delta(\varepsilon - \varepsilon_{\vec{k}}) = \int^{\varepsilon_F} d\varepsilon \varepsilon (n_{\uparrow}(\varepsilon) + n_{\downarrow}(\varepsilon))$$

- ◆ Stoner exchange energy: $E_{\text{total}} = E_{\text{band}} - \frac{I}{2}(N_{\uparrow} - N_{\downarrow})^2 = E_{\text{band}} - \frac{I}{2}M^2$

- ◆ Splitting the non-magnetic state: $\delta M = \int^{\varepsilon_F} d\varepsilon (n_{\uparrow}(\varepsilon - \Delta) - n_{\downarrow}(\varepsilon + \Delta)) \approx 2n(\varepsilon_F) \Delta$

- ◆ Magnetic instability: $\delta E_{\text{total}} \approx \left(\frac{1}{n(\varepsilon_F)} - I \right) \frac{(\delta M)^2}{2} < 0$ if $n(\varepsilon_F) I > 1$

- ◆ **Contrast:** localized electrons favour magnetism, itinerant ones not really

- ◆ **How to describe the magnetism of real materials?**

J. Kübler, *Theory of Itinerant Electron Magnetism*, OUP (2000)

- ◆ Kohn-Sham hamiltonian and Green function (cell-centered coordinates):

$$\begin{aligned}\mathcal{H}_i^{\text{KS}}(\vec{r}) &= -\nabla_{\vec{r}}^2 + 2 \sum_j \int d\vec{r}' \frac{n_j(\vec{r}') - Z_j \delta(\vec{r}')}{|\vec{r} - \vec{r}' + \vec{R}_{ij}|} + V_i^{\text{xc}}(\vec{r}) + \left[\vec{B}_i^{\text{xc}}(\vec{r}) + \vec{B}_i^{\text{soc}}(\vec{r}) \right] \cdot \vec{\sigma} \\ &= -\nabla_{\vec{r}}^2 + V_i^{\text{KS}}(\vec{r}) + \vec{B}_i^{\text{KS}}(\vec{r}) \cdot \vec{\sigma} \quad , \quad [\varepsilon - \mathcal{H}_i^{\text{KS}}(\vec{r})] G_{ij}(\vec{r}, \vec{r}'; \varepsilon) = \delta_{ij} \delta(\vec{r} - \vec{r}')\end{aligned}$$

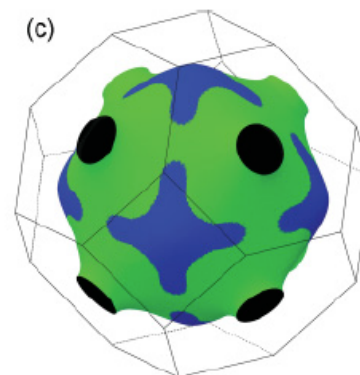
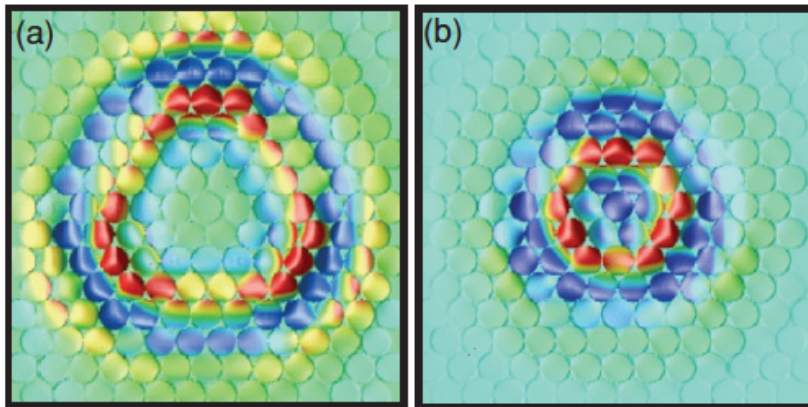
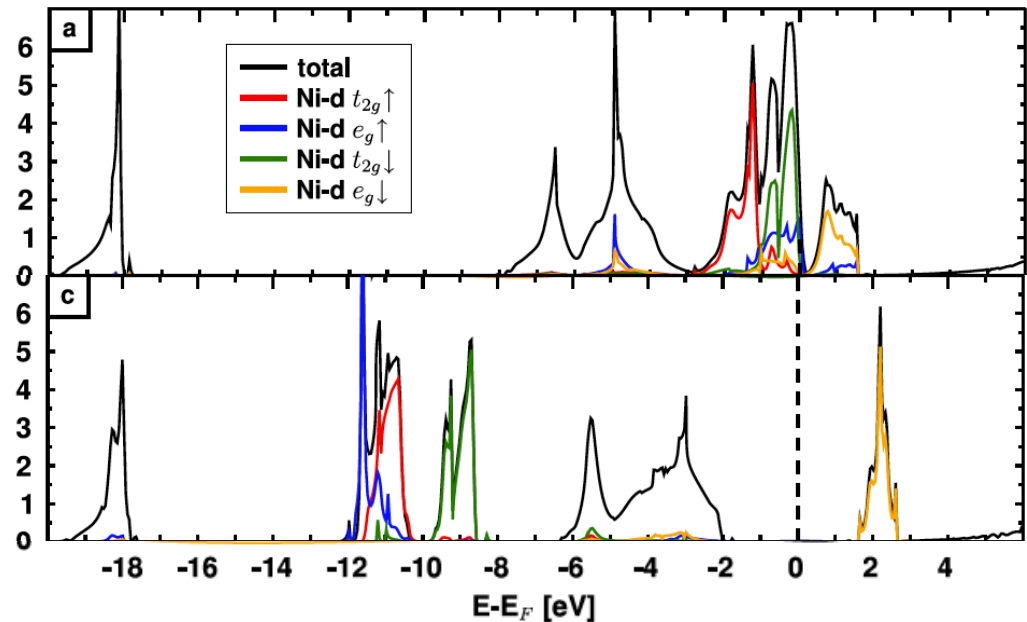
- ◆ Total energy: $E_{\text{tot}} = \underbrace{E_{\text{band}} - E_{\text{dc}}}_{T_s} + E_{\text{Coulomb}} + E_{\text{xc}} + E_{\text{ext}}$
- ◆ Each electron moves in mean-field created by all other electrons
- ◆ Complicated electron-electron interactions approximated by xc functional
- ◆ Structure similar to Stoner model – **when should DFT perform well?**

R.M. Martin, *Electronic Structure – Basic Theory and Practical Methods*, CUP (2004)

◆ Transition metal oxides:

Compound	SIC configuration
MnO	$3t_{2g}^{\uparrow} + 2e_g^{\uparrow}$
FeO	$3t_{2g}^{\uparrow} + 2e_g^{\uparrow} + 1t_{2g}^{\downarrow}$
CoO	$3t_{2g}^{\uparrow} + 2e_g^{\uparrow} + 2t_{2g}^{\downarrow}$
NiO	$3t_{2g}^{\uparrow} + 2e_g^{\uparrow} + 3t_{2g}^{\downarrow}$
CuO	$3t_{2g}^{\uparrow} + 2e_g^{\uparrow} + 3t_{2g}^{\downarrow} + 1e_g^{\downarrow}$

M Däne et al, J. Phys. Condens. Matter **21**, 045604 (2009)



◆ Fermi surface sonar

S. Lounis et al, Phys. Rev. B **83**, 035427 (2011)

Magnetic interactions I

Localized magnetic moments (?)

- ◆ **Gedankenexperiment:** freeze the system in some magnetic configuration.

If we rotate the moment on site i only: $\vec{M}_i(\hat{e}_i) \equiv \int_{V_i} d\vec{r} \vec{m}_i(\vec{r}) = M_i^{\parallel}(\hat{e}_i) \hat{e}_i + \vec{M}_i^{\perp}$

- ◆ **Local moment** – approximate Hund's rule: $M_i^{\perp} \ll M_i^{\parallel}(\hat{e}_i)$, $M_i^{\parallel}(\hat{e}_i) \approx M_i^{\parallel}$

- ◆ Good candidates: narrow band systems (3d, 4f...); band gap systems

- ◆ Other electrons in the system may be polarized by the local moments

- ◆ **Rigid spin approximation:** the xc field depends on the orientation only trivially

$$\vec{B}_i^{\text{xc}}(\vec{r}) \approx B_i^{\text{xc}}(\vec{r}) \hat{e}_i \implies \underline{t}_i(\varepsilon; \hat{e}_i) \approx \underline{\mathcal{R}}_i(\hat{e}_i) \underline{t}_i(\varepsilon; \hat{z}) \underline{\mathcal{R}}_i(\hat{e}_i)^{-1}$$

- ◆ Dependence of the total energy on the magnetic configuration:

$$E(\{\hat{e}\}) = E_0 + \sum_i E_i^{(1)}(\hat{e}_i) + \sum_{ij} E_{ij}^{(2)}(\hat{e}_i, \hat{e}_j) + \sum_{ijk} E_{ijk}^{(3)}(\hat{e}_i, \hat{e}_j, \hat{e}_k) + \dots$$

- ◆ **Magnetic force theorem:** $E(\{\hat{e} + \delta\hat{e}\}) - E(\{\hat{e}\}) \approx E_{\text{band}}(\{\hat{e} + \delta\hat{e}\}) - E_{\text{band}}(\{\hat{e}\})$

- ◆ **Lichtenstein formula:** follows directly from Lloyd's formula in KKR

$$\delta E_{\text{band}}(\delta\hat{e}_i, \delta\hat{e}_j) \approx \frac{1}{\pi} \text{Im Tr} \int^{\epsilon_F} d\epsilon \delta \underline{t}_i^{-1}(\epsilon) \underline{\tau}_{ij}(\epsilon) \delta \underline{t}_j^{-1}(\epsilon) \underline{\tau}_{ji}(\epsilon) = J_{ij} \delta\hat{e}_i \cdot \delta\hat{e}_j$$

- ◆ Heisenberg model (great leap of faith): $E_{\text{iso}}(\{\hat{e}\}) = -\frac{1}{2} \sum_{ij} J_{ij} \hat{e}_i \cdot \hat{e}_j$

- ◆ From a single calculation the energy of a whole class of magnetic states can be estimated – **spin spirals**

Magnetic force theorem: A. I. Liechtenstein et al, J. Mag. Magn. Mater. **67**, 65 (1987)

Magnetic groundstates and spin spirals

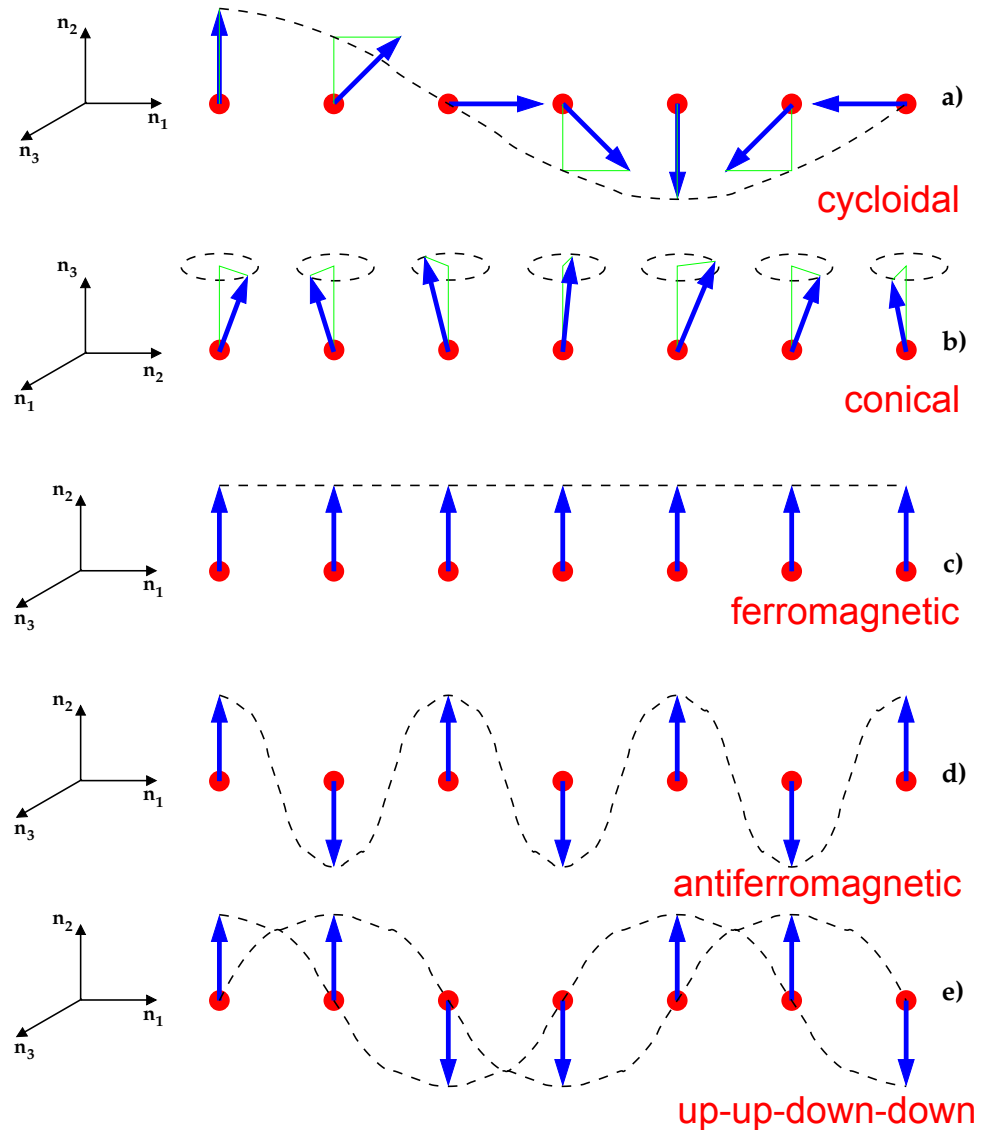
◆ Spin spirals:

$$\vec{M}_i(\vec{q}) = M(\vec{q}) \left(\cos(\vec{q} \cdot \vec{R}_i + \phi) \sin \theta \hat{n}_1 + \sin(\vec{q} \cdot \vec{R}_i + \phi) \sin \theta \hat{n}_2 + \cos \theta \hat{n}_3 \right)$$

◆ Generalized Bloch theorem

◆ May need constraining fields

◆ Comparison of total energies of spin spirals with force theorem calculations checks assumptions

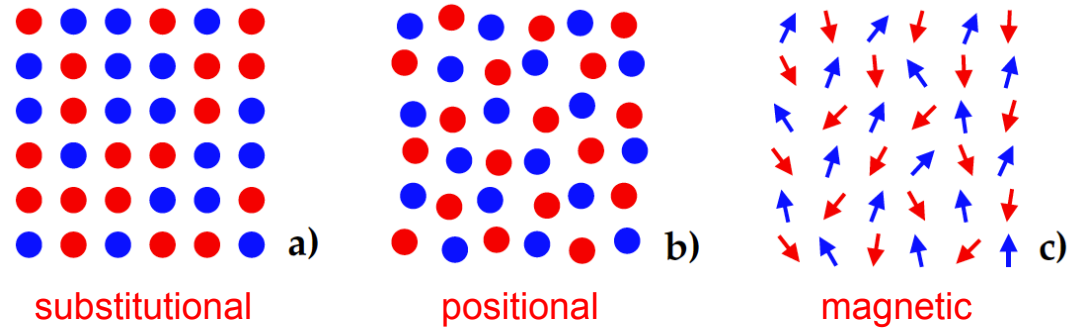


Noncollinear magnetism: L.M. Sandratskii, Adv. Phys. **47**, 91 (1998)

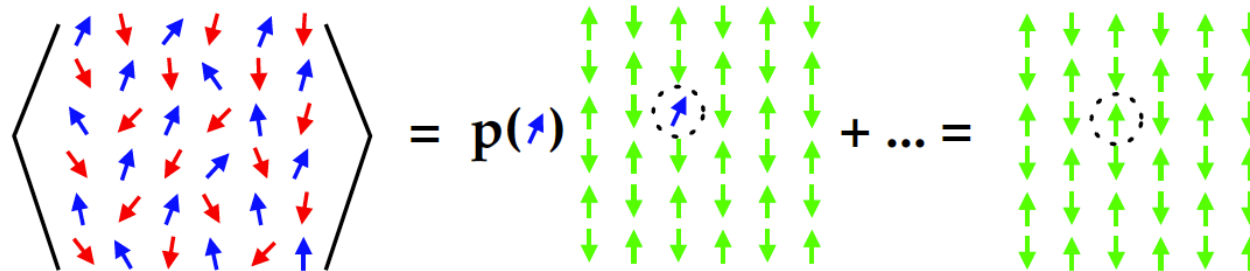
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Disordered Local Moment (DLM) state

◆ Disorder types:



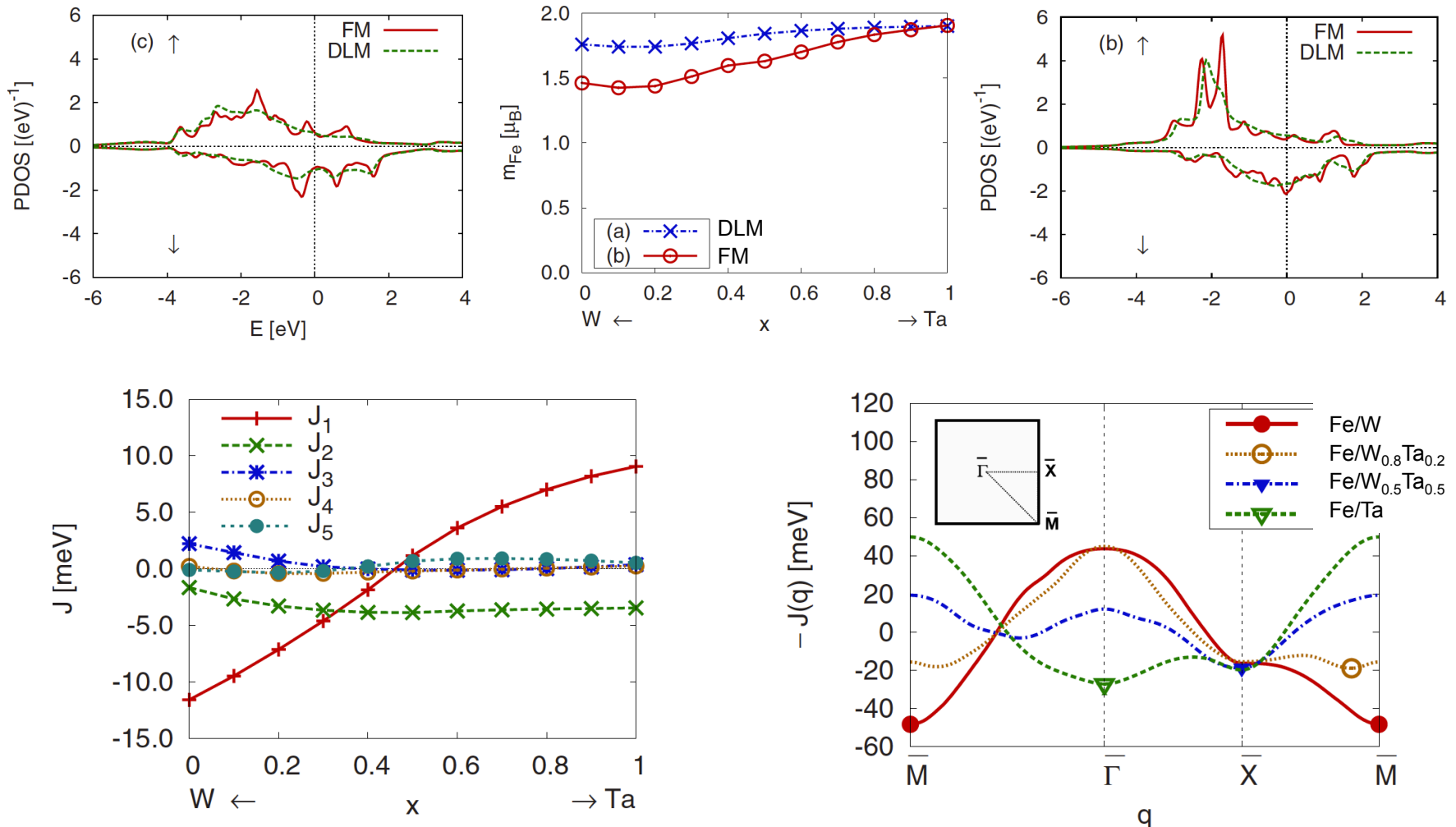
◆ Effective medium (CPA): $\langle G_{ii} \rangle = \int d\hat{e}_i P_i(\hat{e}_i) G_{ii}(\hat{e}_i; \{\vec{m}\}) \quad \vec{m}_i = \int d\hat{e}_i P_i(\hat{e}_i) \hat{e}_i$



- ◆ Each local moment sees average magnetic medium set up by all other moments
- ◆ Paramagnetic instabilities estimate stable magnetic states, transition temperature

Fe monolayer on (W,Ta) substrate

◆ Electronic structure for ferromagnetic and disordered local moment states:



LMT0 CPA+DLM: M. Ondracek et al, Phys. Rev. B **81**, 064410 (2010)

Magnetic interactions II

- ◆ **Goldstone theorem:** it costs no energy to rotate all the magnetic moments by the same angle

$$E_{\text{iso}}(\{\hat{e}\}) = -\frac{1}{2} \sum_{ij} J_{ij} \hat{e}_i \cdot \hat{e}_j$$

- ◆ **Spin-orbit coupling** – coupling of spin and orbital angular momentum; energy now depends on how the moments point with respect to the lattice
- ◆ **Magnetic anisotropy energy** – can be magnetocrystalline or magnetostatic

$$E_{\text{dd}}(\{\hat{e}\}) = \frac{1}{2} \sum_{ij} \left[\frac{3}{R_{ij}^3} \vec{M}_i \cdot \vec{M}_j - \frac{1}{R_{ij}^5} (\vec{M}_i \cdot \vec{R}_{ij})(\vec{M}_j \cdot \vec{R}_{ij}) \right]$$

$$E_{\text{a}}^{(1)}(\{\hat{e}\}) = \sum_i K_i(\hat{e}_i) = \sum_i [K_{1i} \sin^2 \theta_i + (K_{2i} + K'_{2i} \cos 4\phi_i) \sin^4 \theta_i + \dots] \quad (C_{4v})$$

- ◆ The last expression is for the onsite anisotropy, tetragonal symmetry.

The pair interaction will also become anisotropic...

R. Skomski, *Simple Models of Magnetism*, OUP (2008)

◆ Bilinear pair interaction tensor:
$$E^{(2)}(\{\hat{e}\}) = -\frac{1}{2} \sum_{ij} \hat{e}_i \cdot \underline{J}_{ij} \cdot \hat{e}_j$$

◆ Isotropic part:

$$J_{ij}^{\text{iso}} = \frac{1}{3} \text{Tr} \underline{J}_{ij} \longrightarrow J_{ij}^{\text{iso}} \hat{e}_i \cdot \hat{e}_j$$

◆ Dzyaloshinskii-Moriya (DM) part:

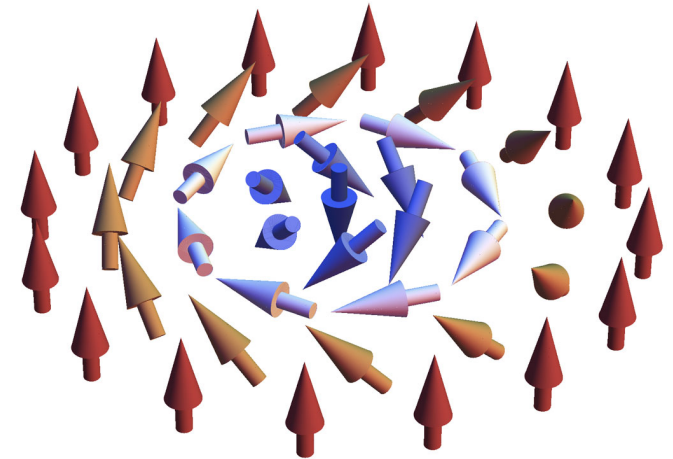
$$\underline{J}_{ij}^{\text{anti}} = \frac{1}{2} (\underline{J}_{ij} - \underline{J}_{ij}^{\text{T}}) \longrightarrow \vec{D}_{ij} \cdot (\hat{e}_i \times \hat{e}_j)$$

◆ Symmetric traceless part similar to dipole-dipole term

◆ DM term needs broken inversion symmetry

◆ This leads to **chirality** – meet the skyrmion:

◆ Anisotropic terms on same energy scale as higher order isotropic terms...



R. Skomski, *Simple Models of Magnetism*, OUP (2008)

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Mn₁/W(001) as detailed case study – I

- ◆ Mn monolayer on W(001) has a chiral spin spiral as groundstate –

P. Ferriani et al, Phys. Rev. Lett. **101**, 027201 (2008) – **why?**

- ◆ Spin spiral energy:

$$E_{\text{iso}}(\vec{q}; \theta) = -\frac{1}{2N} \sum_{ij} J_{ij} \vec{m}_i(\vec{q}) \cdot \vec{m}_j(\vec{q}) = \frac{J(0) - J(\vec{q})}{2} \sin^2 \theta - \frac{J(0)}{2}$$

$$J(\vec{q}) = \sum_j J_{0j} \cos(\vec{q} \cdot \vec{R}_{0j}) = 2J_1 [\cos(\pi q_x) + \cos(\pi q_y)] + 4J_2 \cos(\pi q_x) \cos(\pi q_y)$$

- ◆ Extremal values of the energy:

➤ Ferromagnetic: (0,0)

$$\sin(\pi q_x) \left(1 + \frac{2J_2}{J_1} \cos(\pi q_y) \right) = 0$$

➤ Row-wise antiferromagnetic: (1,0), etc.

$$\sin(\pi q_y) \left(1 + \frac{2J_2}{J_1} \cos(\pi q_x) \right) = 0$$

➤ Checkerboard antiferromagnetic: (1,1), etc.

$$(J(0) - J(\vec{q})) \sin 2\theta = 0$$

➤ Spiral if J_2 sufficiently large w.r.t. J_1 ...

◆ ... which it isn't.

- ◆ DM energy depends on the orientation of the spin spiral:

$$E_{\text{DM}}(\vec{q}; \theta; \hat{n}_3) = -\frac{1}{2N} \sum_{ij} \vec{D}_{ij} \cdot (\vec{m}_i \times \vec{m}_j) = -\frac{1}{2N} \sum_{ij} \vec{D}_{ij} \cdot \hat{n}_3 \sin(\vec{q} \cdot \vec{R}_{ij}) \sin^2 \theta$$

- ◆ **Chirality:** DM energy is not the same for q and $-q$ – rotational sense matters

$$\begin{aligned} E_{\text{DM}}(\vec{q}; \theta; \hat{n}_3) &= D [(\hat{e}_x \cdot \hat{n}_3) \sin(\pi q_y) - (\hat{e}_y \cdot \hat{n}_3) \sin(\pi q_x)] \sin^2 \theta \\ &= D [\cos \alpha \sin(\pi q_y) - \sin \alpha \sin(\pi q_x)] \sin^2 \theta \end{aligned}$$

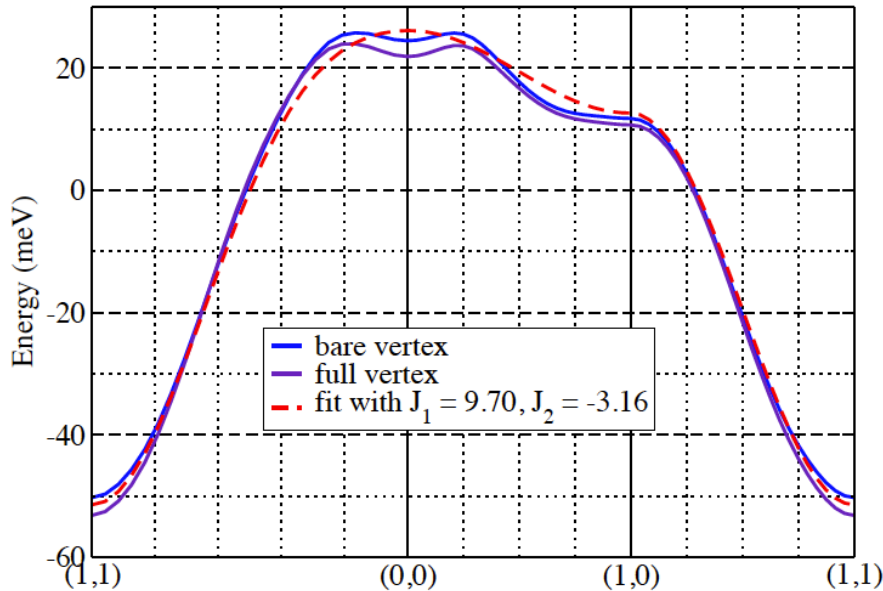
- ◆ Extremal values of the energy:

$$\begin{aligned} \sin(\pi q_x) \left[1 + \frac{2J_2}{J_1} \cos(\pi q_y) \pm \frac{D}{J_1} \frac{\cos(\pi q_x)}{\sqrt{\sin^2(\pi q_x) + \sin^2(\pi q_y)}} \right] &= 0 \\ \sin(\pi q_y) \left[1 + \frac{2J_2}{J_1} \cos(\pi q_x) \pm \frac{D}{J_1} \frac{\cos(\pi q_y)}{\sqrt{\sin^2(\pi q_x) + \sin^2(\pi q_y)}} \right] &= 0 \end{aligned}$$

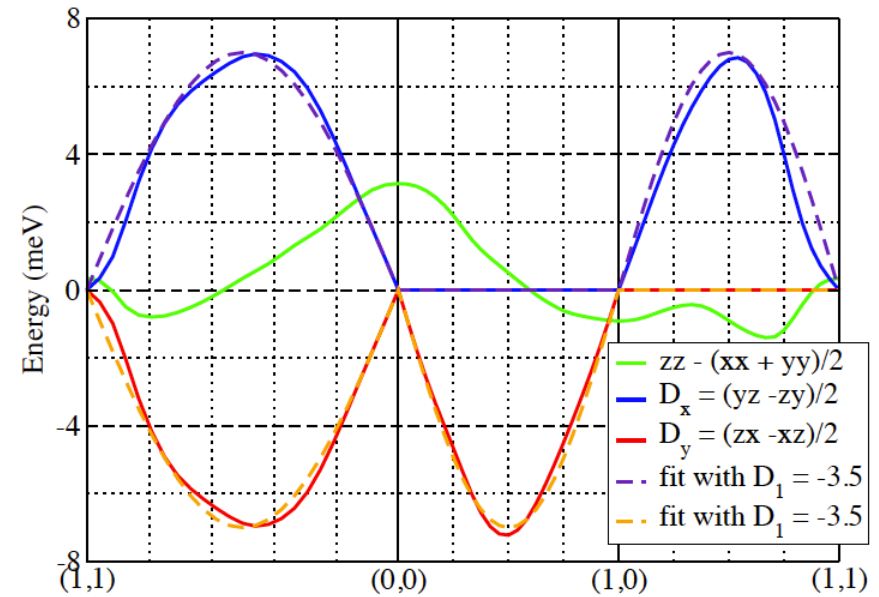
- Spiral also possible if J_2 zero; cycloidal spirals: $\hat{n}_3 \cdot \vec{q} = 0$



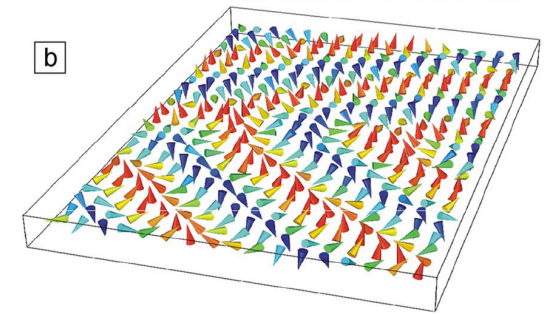
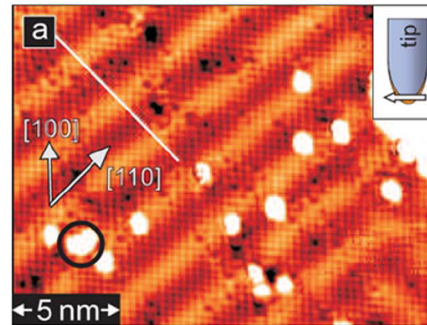
Isotropic part of J(q):



Anisotropic part of J(q):



- Maximum in isotropic part of J(q) is a Fermi surface effect
- Min energy for $q = (0.17, 0.17)$ good agreement with experiment

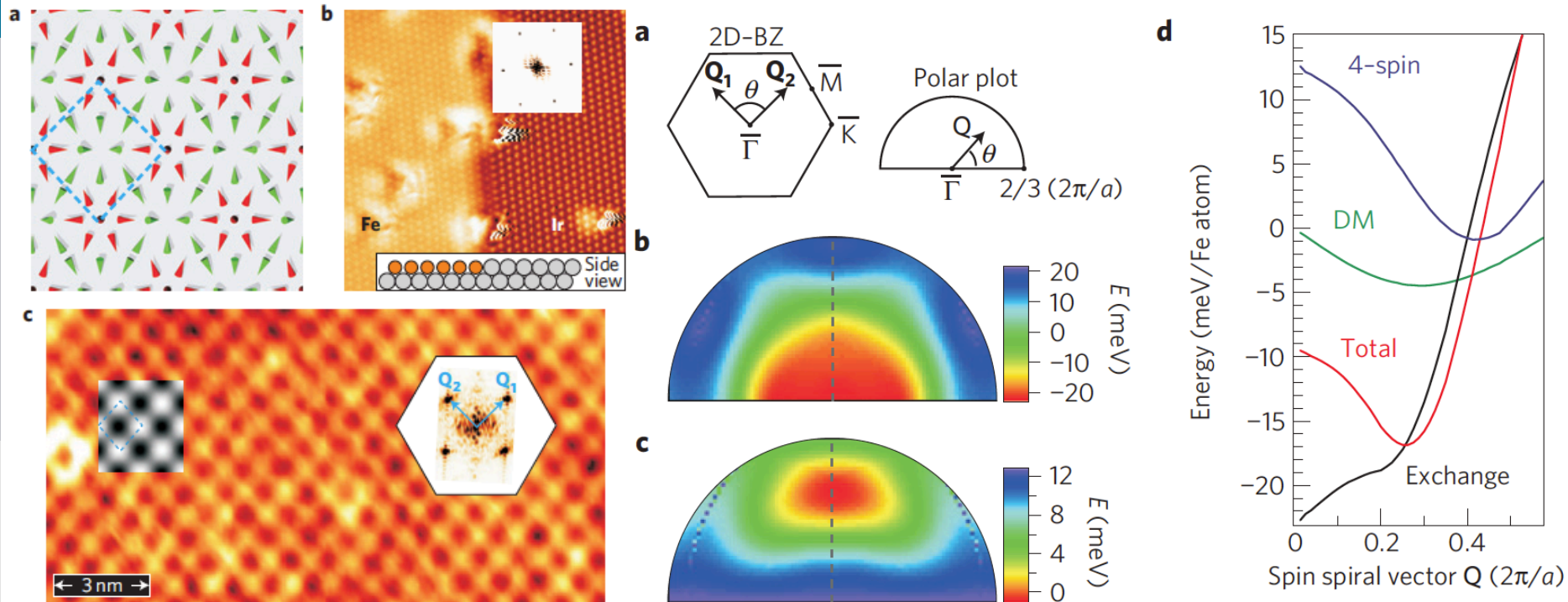


- For relativistic DLM formalism, see M. dos Santos Dias et al, Phys. Rev. B 83, 054435 (2011)

Higher order interactions – Fe₁/Ir(111)

◆ Biquadratic and four-spin interactions:

$$E_{\text{more}}(\{\hat{e}\}) = \sum_{ij} B_{ij}(\hat{e}_i \cdot \hat{e}_j)^2 + \sum_{ijkl} Q_{ijkl} [(\hat{e}_i \cdot \hat{e}_j)(\hat{e}_k \cdot \hat{e}_l) - (\hat{e}_i \cdot \hat{e}_k)(\hat{e}_j \cdot \hat{e}_l) + (\hat{e}_i \cdot \hat{e}_l)(\hat{e}_j \cdot \hat{e}_k)]$$



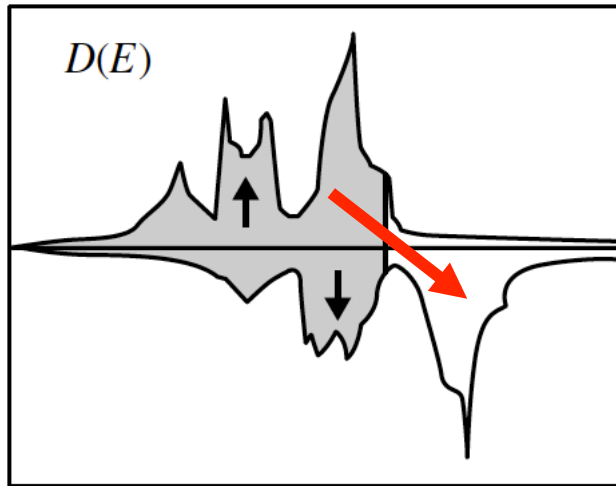
◆ Squarish skyrmion lattice out of an hexagonal magnetic monolayer

S. Heinze et al, Nature Phys. 7, 713 (2011)

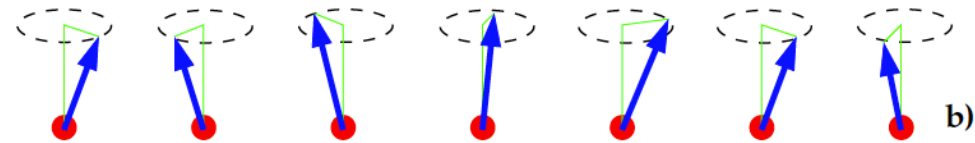
Beyond the groundstate

Spin waves and spin flips

◆ Stoner excitations:

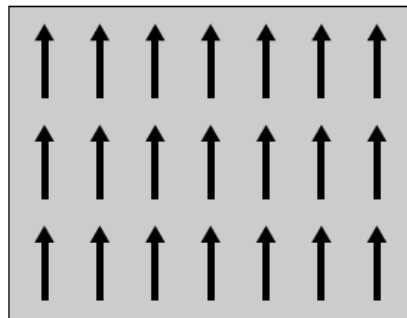


◆ Spin waves:

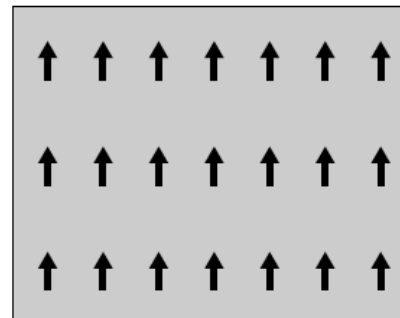


- Spin spirals also called frozen magnons
- Spin spiral energies approximate spin wave energies?
- **Adiabatic approximation**

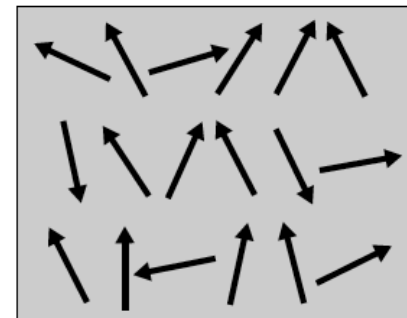
◆ What does the high temperature magnetic state look like?



$T = 0$: ferromagnet (DFT)



$T > 0$: Stoner

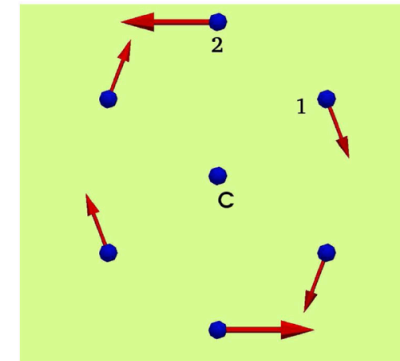
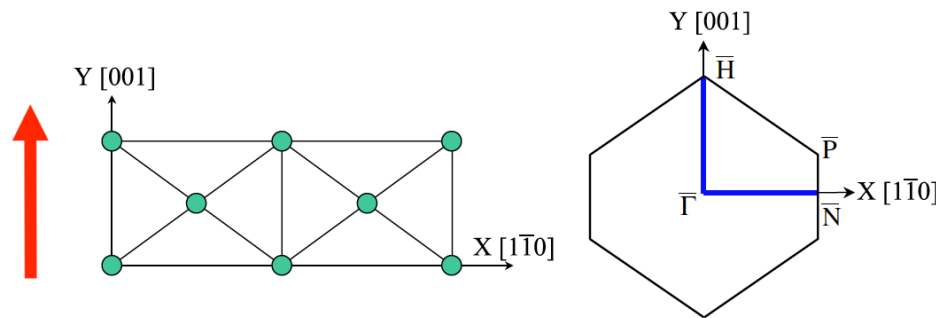


$T > 0$: Heisenberg

Spin wave spectra in adiabatic approx

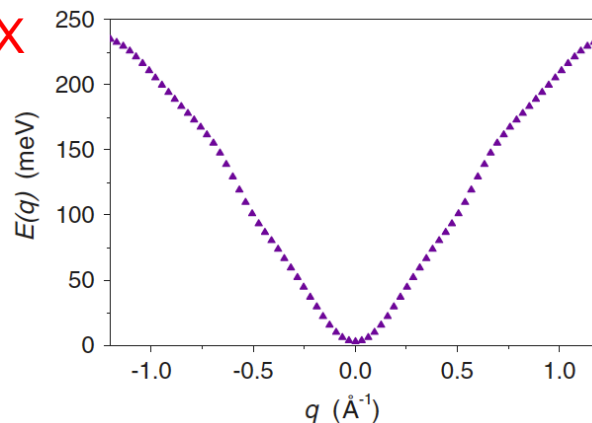
◆ Landau-Lifshitz equation:
$$\frac{d}{dt} \vec{M}_i \approx M_i \frac{d\hat{e}_i}{dt} = -\gamma \vec{M}_i \times \frac{\delta \mathcal{F}}{\delta \hat{e}_i} = -\gamma \vec{M}_i \times \vec{B}_i$$

◆ Fe₁/W(110): magnetization in-plane, along Y axis

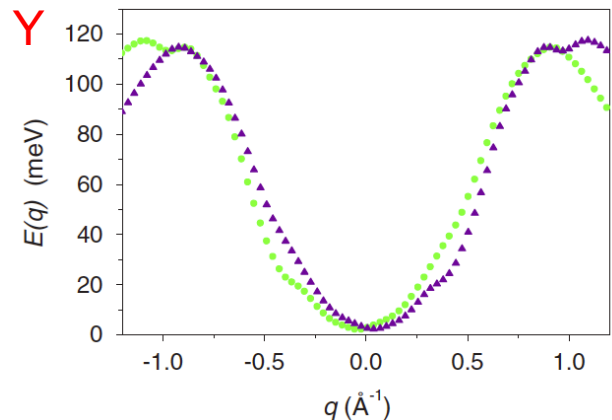


➤ DM vectors, nn and nnn

➤ q along X



➤ q along Y



L. Udvardi and L. Szunyogh, Phys. Rev. Lett. **102**, 207204 (2009)

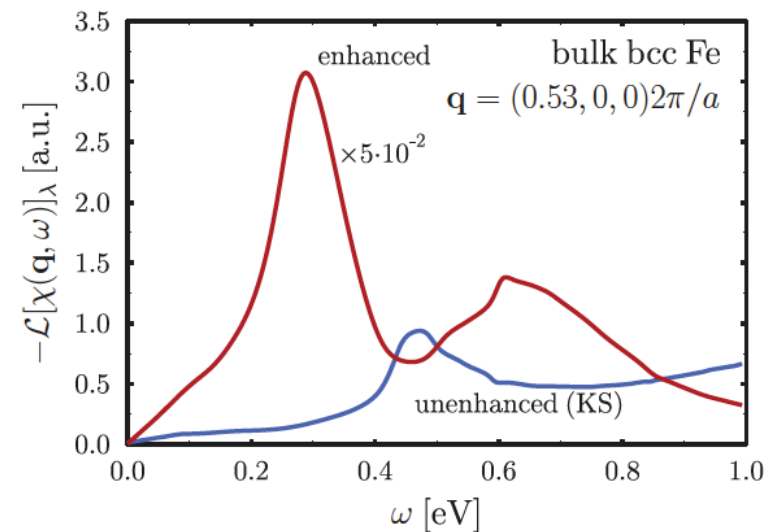
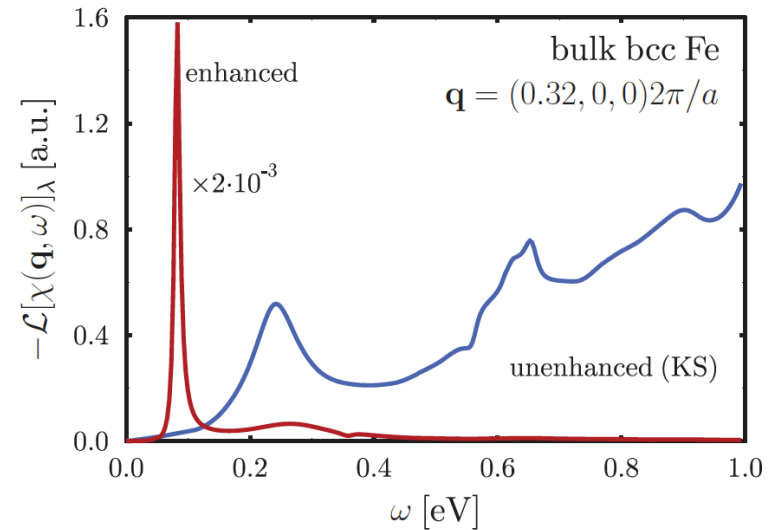
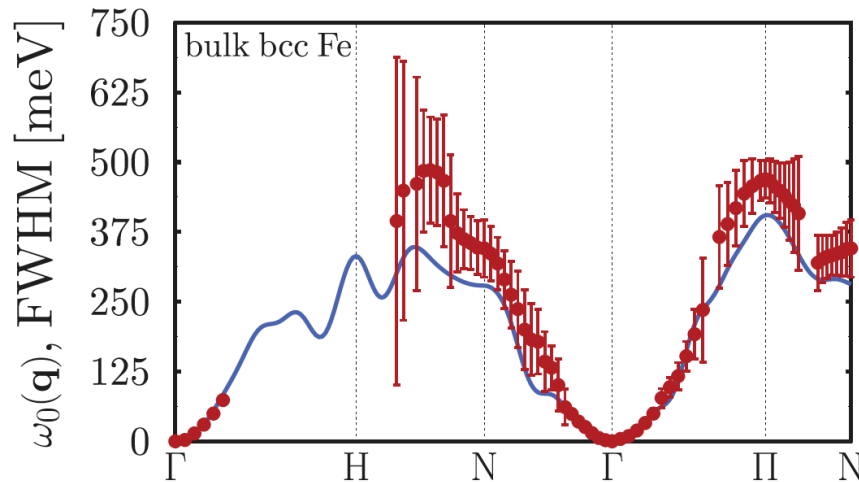
Spin waves from TD-DFT

- ◆ Dynamical magnetic susceptibility:

$$\chi(\omega) = \chi^{\text{KS}}(\omega) + \chi^{\text{KS}}(\omega) \mathcal{K}^{\text{xc}} \chi(\omega)$$

- ◆ Example: beloved bcc Fe

- ◆ What is a spin wave peak and what isn't?



P. Buczek et al, Phys. Rev. B **84**, 174418 (2011)

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- ◆ DFT-KKR as powerful and versatile tool for magnetic properties
 - ◆ Magnetic hamiltonians: how many terms do we need and make sense to include?
 - ◆ Check calculations are converged before writing the PRL – **k-points!**
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- Start here: S. Blundell, *Magnetism in Condensed Matter*, OUP (2001)
 - Overview: R. Skomski, *Simple Models of Magnetism*, OUP (2008)
 - DFT: J. Kübler, *Theory of Itinerant Electron Magnetism*, OUP (2000)
 - Many other nice books around, lots of KKR references also – ask me