Measuring coherence in macroscopic quantum systems

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Quantum measurement: a dialog of big and small, Warwick

Quantum at the macroscopic scale?

- What evidence do we have that quantum mechanics holds at the macroscopic scale?

- Various explanations for emergence of classical physics:

  - Decoherence
    Zurek, RMP 2003

  - Our measurements aren't precise enough to see quantum behaviour
    Kofler + Brukner, PRL 2007
    Sekatski et. al., PRL 2011
    Raeisi et. al., PRL 2011

  - Intrinsic modifications to quantum theory
    Bassi et. al., RMP 2013

Quantum at the macroscopic scale?

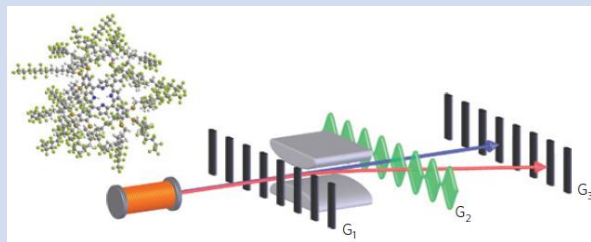
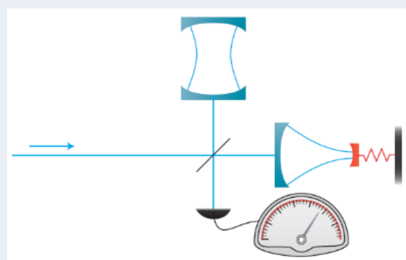
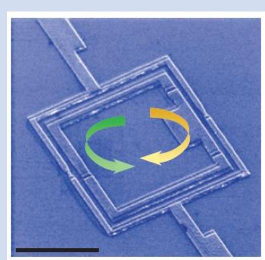
- How do we even define macroscopic quantum behaviour? Early questions addressed by Leggett – how can we capture the spirit of Schrödinger's cat?

  Leggett, Progress of Theoretical Physics Supplement 1980

- 3 general ways this is approached:
  - Rate of decoherence in collapse models
  - Model-independent tests (Bell / Leggett-Garg inequalities)
  - Quantum properties – 2 main questions: how quantum and how big?

Quantumness measured by coherence

- Experiments try to create large-scale superposition associated with some preferred observable – generally an extensive observable

Experiment		Preferred observable
Molecular interference		Centre of mass position / momentum
Mechanical oscillator		
Superconducting circuit		Current

Figures from Arndt and Hornberger, Nature Physics 2014

Quantumness measured by coherence

- Resource theory of coherence - Baumgratz et. al., PRL 2014
  - defined with respect to a preferred basis $\{|i\rangle\}$
  - aims to distinguish a superposition from a classical mixture

- Which states have zero coherence?

  Incoherent states: $$\rho = \sum_i p_i \, |i\rangle\langle i|$$

- Which operations cannot create coherence?

  Incoherent operations: incoherent state → incoherent state

Quantumness measured by coherence

- Measures of coherence must satisfy 2 main conditions:

    1) Zero for incoherent states

    2) Cannot increase under incoherent operations

- Typically some function of off-diagonal elements, e.g. $\sum_{i \neq j} |\rho_{i,j}|$

Coherence with a scale attached

There's a problem: can't distinguish between micro and macro!

Instead: fix an observable $A = \sum_i a_i |i\rangle\langle i|$

- Need to insert a concept of scale:
  - Introduce δ-coherence: superposition of $|i\rangle$ and $|j\rangle$ such that $a_i - a_j = \delta$

- The δ-coherence in a state ρ is a function of the elements $\rho_{i,j}$ associated with a superposition of scale δ

- Restrict the incoherent operations so they can't create δ-coherence – different δ are not interchangeable

BY + Vlatko Vedral, PRA 93, 022122 (2016)

Measuring macroscopic coherence

- For **pure states**, the variance is a good measure:

$$V(|\psi\rangle, A) = \langle\psi|A^2|\psi\rangle - \langle\psi|A|\psi\rangle^2$$

- Natural interpretation as coherent spread of wavefunction

- Given some reference coherence unit $|\phi\rangle$, we can also say that one copy of $|\psi\rangle$ is worth $\dfrac{V(|\psi\rangle, A)}{V(|\phi\rangle, A)}$ copies of $|\phi\rangle$

  (variance is the unique asymptotic measure)

Measuring macroscopic coherence

- For **general (mixed) states**, the correct extension is given by the quantum Fisher information (QFI)

- The QFI is $\mathcal{F}(\rho, A) := 4\,\partial_t^2 D_B^2(\rho, e^{-itA}\rho e^{itA})$

  where D_B is the Bures distance

- Can think of the QFI as a "quantum variance"

- $\dfrac{\mathcal{F}(\rho, A)}{4V(|\phi\rangle, A)}$ = value of ρ in units of $|\phi\rangle$ (cost of preparation)

Measuring macroscopicity

We can identify two interesting measures:

1. **Extensive measure** $$N_{\text{ext}}(\rho, A) = \frac{\mathcal{F}(\rho, A)}{4A_0^2}$$

   - Total amount of coherence, measured in suitable atomic-scale units, e.g. for momentum, take $P_0 = \hbar / a_0$

2. **Effective size** $$N_{\text{eff}}(\rho, A) = \frac{\mathcal{F}(\rho, A)}{4\sum_{i=1}^{N} V(\rho, A_i)}$$

   - Choose a set of N subsystems
   - Measures total coherence relative to size of subsystems
   - Tells us about number of quantum-correlated subsystems (multipartite entanglement witness)

BY et al., in preparation

Measuring macroscopicity

Estimates for real and hypothetical systems:

	N_{ext}		N_{eff}	
Experiment	Q	P	Q	P
Molecular diffraction	$10^{14} - 10^{15}$	$10^{-9} - 10^{-8}$	$30 - 300$	1
SQUID	-	$10^{4} - 10^{6}$	1	$10^{7} - 10^{9}$
Proposed SQUID	-	10^{9}	1	10^{12}
Proposed micromirror	10^{25}	-	10^{9}	-
Actual cat	10^{46}	-	10^{26}	-

$Q = MX_{CM}$

P = total momentum

$$N_{\text{ext}}(\rho, A) = \frac{\mathcal{F}(\rho, A)}{4A_0^2}$$

$$N_{\text{eff}}(\rho, A) = \frac{\mathcal{F}(\rho, A)}{4\sum_{i=1}^{N} V(\rho, A_i)}$$

Molecular diffraction: S. Gerlich et al., Nature Communications 2, 263 (2011)

SQUID: J. R. Friedman et al., Nature 406, 43 (2000)

Hypothetical experiments as described in: S. Nimmrichter and K. Hornberger, PRL 110, 160403 (2013).

Measuring macroscopicity in the lab

1. Interference patterns
   - Measure response of system to a small perturbation (inverse of metrology!)
   - Basic idea: high QFI given by high-visibility and high-frequency fringes
   - Analysis of spin and photonic systems in preparation by Florian Fröwis: effective sizes up to 70

2. Multi-copy interferometry
   - Have to do a controlled swap on two copies of system – not easy!

   Girolami, PRL 113, 170401 (2014)

3. Linear response
   - Perturb system at different frequencies and measure response
   - E.g. Can probe condensed matter systems with neutron scattering

   Hauke et. al., Nature Physics 12, 8 (2016)

Relating back to other concepts…

- Actually a close connection with collapse models:

  QFI can be related to decoherence rate in large class of models (those in Nimmrichter + Hornberger, PRL 2012)

  BY + Vlatko Vedral, PRA 93, 022122 (2016)

- Also a connection with Leggett-Garg inequalities:

  Violation of an LGI requires the system to be disturbed by an intermediate measurement

  Suppose this is a noisy measurement with uncertainty Δ. Then LGI violation can be observed only if $\mathcal{F} \gtrsim \Delta^2$

  Fröwis et al., PRL 116, 090802 (2016)

Conclusion

- Coherence is a natural language for assessing experiments testing the superposition principle
- Have a mathematical framework for defining measures
- Quantum Fisher information is a notable measure with “coherence cost” interpretation
- Gives 2 different ways of quantifying macroscopicity – extensive measures and effective sizes
- QFI is observable – though it is not clear which detection technique is the most easily scalable