Unit C1 Core Mathematics 1

AS compulsory unit for GCE AS and GCE Mathematics, GCE AS and GCE Pure Mathematics

C1.1 Unit description

Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration.

C1.2 Assessment information

Preamble

Construction and presentation of rigorous mathematical arguments through appropriate use of precise statements and logical deduction, involving correct use of symbols and appropriate connecting language is required. Students are expected to exhibit correct understanding and use of mathematical language and grammar in respect of terms such as 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient', and notation such as $\therefore$, $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$.

Examination

The examination will consist of one 1½ hour paper. It will contain about ten questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

For this unit, students may **not** have access to any calculating aids, including log tables and slide rules.

Formulae

Formulae which students are expected to know are given below and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Quadratic equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

function	derivative
x^n	nx^{n-1}

Integration

function	integral
x^n	$\frac{1}{n+1}x^{n+1} + c, n \neq -1$

1 Algebra and functions

What students need to learn:

Laws of indices for all rational exponents.	The equivalence of $a^{m/n}$ and $\sqrt[n]{a^m}$ should be known.
Use and manipulation of surds.	Students should be able to rationalise denominators.
Quadratic functions and their graphs.	
The discriminant of a quadratic function.	
Completing the square. Solution of quadratic equations.	Solution of quadratic equations by factorisation, use of the formula and completing the square.
Simultaneous equations: analytical solution by substitution.	For example, where one equation is linear and one equation is quadratic.
Solution of linear and quadratic inequalities.	For example, $ax + b > cx + d$, $px^2 + qx + r \geq 0$, $px^2 + qx + r < ax + b$.

Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation.	Students should be able to use brackets. Factorisation of polynomials of degree n, $n \leq 3$, eg $x^3 + 4x^2 + 3x$. The notation $\mathrm{f}(x)$ may be used. (Use of the factor theorem is *not* required.)
Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.	Functions to include simple cubic functions and the reciprocal function $y = \frac{k}{x}$ with $x \neq 0$. Knowledge of the term asymptote is expected.
Knowledge of the effect of simple transformations on the graph of $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$.	Students should be able to apply one of these transformations to any of the above functions (quadratics, cubics, reciprocal) and sketch the resulting graph. Given the graph of any function $y = \mathrm{f}(x)$ students should be able to sketch the graph resulting from one of these transformations.

2 Coordinate geometry in the (x, y) plane

What students need to learn:

Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$.

To include:

(i) the equation of a line through two given points

(ii) the equation of a line parallel (or perpendicular) to a given line through a given point. For example, the line perpendicular to the line $3x + 4y = 18$ through the point $(2, 3)$ has equation $y - 3 = \frac{4}{3}(x - 2)$.

Conditions for two straight lines to be parallel or perpendicular to each other.

3 Sequences and series

What students need to learn:

Sequences, including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$.

Arithmetic series, including the formula for the sum of the first n natural numbers.

The general term and the sum to n terms of the series are required. The proof of the sum formula should be known.

Understanding of Σ notation will be expected.

4 Differentiation

What students need to learn:

The derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.	For example, knowledge that $\frac{dy}{dx}$ is the rate of change of y with respect to x. Knowledge of the chain rule is not required. The notation $f'(x)$ may be used.
Differentiation of x^n, and related sums and differences.	For example, for $n \neq 1$, the ability to differentiate expressions such as $(2x + 5)(x - 1)$ and $\frac{x^2 + 5x - 3}{3x^{1/2}}$ is expected.
Applications of differentiation to gradients, tangents and normals.	Use of differentiation to find equations of tangents and normals at specific points on a curve.

5 Integration

What students need to learn:

Indefinite integration as the reverse of differentiation.	Students should know that a constant of integration is required.
Integration of x^n.	For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$ and $\frac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected. Given $f'(x)$ and a point on the curve, students should be able to find an equation of the curve in the form $y = f(x)$.

Unit C2 Core Mathematics 2

AS compulsory unit for GCE AS and GCE Mathematics, GCE AS and GCE Pure Mathematics

C2.1 Unit description

Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; trigonometry; exponentials and logarithms; differentiation; integration.

C2.2 Assessment information

Prerequisites	A knowledge of the specification for C1, its preamble and its associated formulae, is assumed and may be tested.
Examination	The examination will consist of one 1½ hour paper. It will contain about nine questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.
Calculators	Students are expected to have available a calculator with at least the following keys: $+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given below and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Laws of logarithms

$\log_a x + \log_a y = \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$

$k \log_a x = \log_a (x^k)$

Trigonometry

In the triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

area $= \frac{1}{2} ab \sin C$

Area

area under a curve $= \int_a^b y \, \mathrm{d}x \quad (y \geq 0)$

1 Algebra and functions

What students need to learn:

Simple algebraic division; use of the Factor Theorem and the Remainder Theorem.	Only division by $(x + a)$ or $(x - a)$ will be required. Students should know that if $f(x) = 0$ when $x = a$, then $(x - a)$ is a factor of $f(x)$. Students may be required to factorise cubic expressions such as $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$. Students should be familiar with the terms 'quotient' and 'remainder' and be able to determine the remainder when the polynomial $f(x)$ is divided by $(ax + b)$.

2 Coordinate geometry in the (x, y) plane

What students need to learn:

Coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$ and including use of the following circle properties: (i) the angle in a semicircle is a right angle; (ii) the perpendicular from the centre to a chord bisects the chord; (iii) the perpendicularity of radius and tangent.	Students should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa.

3 Sequences and series

What students need to learn:

The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\lvert r \rvert < 1$.	The general term and the sum to n terms are required. The proof of the sum formula should be known.
Binomial expansion of $(1 + x)^n$ for positive integer n. The notations $n!$ and $\binom{n}{r}$.	Expansion of $(a + bx)^n$ may be required.

4 Trigonometry

What students need to learn:

The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab \sin C$.	
Radian measure, including use for arc length and area of sector.	Use of the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ for a circle.
Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.	Knowledge of graphs of curves with equations such as $y = 3 \sin x$, $y = \sin\left(x + \frac{\pi}{6}\right)$, $y = \sin 2x$ is expected.
Knowledge and use of $\tan \theta = \frac{\sin\theta}{\cos\theta}$, and $\sin^2\theta + \cos^2\theta = 1$.	

Solution of simple trigonometric equations in a given interval.	Students should be able to solve equations such as $\sin\left(x + \frac{\pi}{2}\right) = \frac{3}{4}$ for $0 < x < 2\pi$, $\cos(x + 30°) = \frac{1}{2}$ for $-180° < x < 180°$, $\tan 2x = 1$ for $90° < x < 270°$, $6\cos^2 x + \sin x - 5 = 0$, $0° \le x < 360$, $\sin^2\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$ for $-\pi \le x < \pi$.

5 Exponentials and logarithms

What students need to learn:

$y = a^x$ and its graph.	
Laws of logarithms	To include $\log_a xy = \log_a x + \log_a y$, $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^k = k \log_a x$, $\log_a \frac{1}{x} = -\log_a x$, $\log_a a = 1$
The solution of equations of the form $a^x = b$.	Students may use the change of base formula.

6 Differentiation

What students need to learn:

Applications of differentiation to maxima and minima and stationary points, increasing and decreasing functions.

The notation $f''(x)$ may be used for the second order derivative.

To include applications to curve sketching. Maxima and minima problems may be set in the context of a practical problem.

7 Integration

What students need to learn:

Evaluation of definite integrals.

Interpretation of the definite integral as the area under a curve.

Students will be expected to be able to evaluate the area of a region bounded by a curve and given straight lines.

Eg find the finite area bounded by the curve $y = 6x - x^2$ and the line $y = 2x$.

$\int x \, dy$ will not be required.

Approximation of area under a curve using the trapezium rule.

For example,

evaluate $\int_0^1 \sqrt{(2x+1)} \, dx$

using the values of $\sqrt{(2x + 1)}$ at $x = 0$, 0.25, 0.5, 0.75 and 1.

C3.1 Unit description

Algebra and functions; trigonometry; exponentials and logarithms; differentiation; numerical methods.

C3.2 Assessment information

Prerequisites and preamble

Prerequisites

A knowledge of the specifications for C1 and C2, their preambles, prerequisites and associated formulae, is assumed and may be tested.

Preamble

Methods of proof, including proof by contradiction and disproof by counter-example, are required. At least one question on the paper will require the use of proof.

Examination

The examination will consist of one 1½ hour paper. It will contain about seven questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: $+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given below and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Trigonometry

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2 \tan A}{1 - \tan^2 A}$

Differentiation

function	derivative
$\sin kx$	$k \cos kx$
$\cos kx$	$-k \sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\dfrac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)\,g(x)$	$f'(x)\,g(x) + f(x)\,g'(x)$
$f(g(x))$	$f'(g(x))\,g'(x)$

1 Algebra and functions

What students need to learn:

Simplification of rational expressions including factorising and cancelling, and algebraic division.	Denominators of rational expressions will be linear or quadratic, eg $\frac{1}{ax+b}$, $\frac{ax+b}{px^2+qx+r}$, $\frac{x^3+1}{x^2-1}$.
Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.	The concept of a function as a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. The notation $\mathrm{f}: x \mapsto$ and $\mathrm{f}(x)$ will be used. Students should know that fg will mean 'do g first, then f'. Students should know that if f^{-1} exists, then $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x) = x$.
The modulus function.	Students should be able to sketch the graphs of $y = \lvert ax+b \rvert$ and the graphs of $y = \lvert \mathrm{f}(x) \rvert$ and $y = \mathrm{f}(\lvert x \rvert)$, given the graph of $y = \mathrm{f}(x)$.
Combinations of the transformations $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$.	Students should be able to sketch the graph of, for example, $y = 2\mathrm{f}(3x)$, $y = \mathrm{f}(-x) + 1$, given the graph of $y = \mathrm{f}(x)$ or the graph of, for example, $y = 3 + \sin 2x$, $y = -\cos\left(x + \frac{\pi}{4}\right)$. The graph of $y = \mathrm{f}(ax + b)$ will *not* be required.

2 Trigonometry

What students need to learn:

Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.	Angles measured in both degrees and radians.
Knowledge and use of $\sec^2 \theta = 1 + \tan^2 \theta$ and $\text{cosec}^2 \theta = 1 + \cot^2 \theta$.	
Knowledge and use of double angle formulae; use of formulae for $\sin (A \pm B)$, $\cos (A \pm B)$ and $\tan (A \pm B)$ and of expressions for $a \cos \theta + b \sin \theta$ in the equivalent forms of $r \cos (\theta \pm \alpha)$ or $r \sin (\theta \pm \alpha)$.	To include application to half angles. Knowledge of the t $(\tan \frac{1}{2}\theta)$ formulae will *not* be required. Students should be able to solve equations such as $a \cos \theta + b \sin \theta = c$ in a given interval, and to prove simple identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.

3 Exponentials and logarithms

What students need to learn:

The function e^x and its graph.	To include the graph of $y = e^{ax+b} + c$.
The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x.	Solution of equations of the form $e^{ax+b} = p$ and $\ln (ax + b) = q$ is expected.

4 Differentiation

What students need to learn:

Differentiation of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$ and their sums and differences.

Differentiation using the product rule, the quotient rule and the chain rule.

Differentiation of $\operatorname{cosec} x$, $\cot x$ and $\sec x$ are required. Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos x^2$ and $\tan^2 2x$.

The use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.

Eg finding $\frac{dy}{dx}$ for $x = \sin 3y$.

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5 Numerical methods

What students need to learn:

Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.

Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.

Solution of equations by use of iterative procedures for which leads will be given.

Unit C4 Core Mathematics 4

A2 compulsory unit for GCE Mathematics and GCE Pure Mathematics

C4.1 Unit description

Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration; vectors.

C4.2 Assessment information

Prerequisites	A knowledge of the specifications for C1, C2 and C3 and their preambles, prerequisites and associated formulae, is assumed and may be tested.
Examination	The examination will consist of one 1½ hour paper. It will contain about seven questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.
Calculators	Students are expected to have available a calculator with at least the following keys: $+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.
Formulae	Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage. This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Integration

function	integral
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\frac{1}{x}$	$\ln \lvert x \rvert + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))\, g'(x)$	$f(g(x)) + c$

Vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$$

1 Algebra and functions

What students need to learn:

Rational functions. Partial fractions (denominators not more complicated than repeated linear terms).

Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$.

The degree of the numerator may equal or exceed the degree of the denominator. Applications to integration, differentiation and series expansions.

Quadratic factors in the denominator such as $(x^2 + a)$, $a > 0$, are *not* required.

2 Coordinate geometry in the (x, y) plane

What students need to learn:

Parametric equations of curves and conversion between Cartesian and parametric forms.	Students should be able to find the area under a curve given its parametric equations. Students will *not* be expected to sketch a curve from its parametric equations.

3 Sequences and series

What students need to learn:

Binomial series for any rational n.	For $\lvert x \rvert < \frac{b}{a}$, students should be able to obtain the expansion of $(ax + b)^n$, and the expansion of rational functions by decomposition into partial fractions.

4 Differentiation

What students need to learn:

Differentiation of simple functions defined implicitly or parametrically.	The finding of equations of tangents and normals to curves given parametrically or implicitly is required.
Exponential growth and decay.	Knowledge and use of the result $\frac{\mathrm{d}}{\mathrm{d}x}(a^x) = a^x \ln a$ is expected.
Formation of simple differential equations.	Questions involving connected rates of change may be set.

5 Integration

What students need to learn:

Integration of e^x, $\frac{1}{x}$, $\sin x$, $\cos x$.	To include integration of standard functions such as $\sin 3x$, $\sec^2 2x$, $\tan x$, e^{5x}, $\frac{1}{2x}$. Students should recognise integrals of the form $\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c$. Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$.
Evaluation of volume of revolution.	$\pi \int y^2 \, dx$ is required, but *not* $\pi \int x^2 \, dy$. Students should be able to find a volume of revolution, given parametric equations.
Simple cases of integration by substitution and integration by parts. These methods as the reverse processes of the chain and product rules respectively.	Except in the simplest of cases the substitution will be given. The integral $\int \ln x \, dx$ is required. More than one application of integration by parts may be required, for example $\int x^2 e^x \, dx$.

Simple cases of integration using partial fractions.

Integration of rational expressions such as those arising from partial fractions, eg $\frac{2}{3x+5}$, $\frac{3}{(x-1)^2}$.

Note that the integration of other rational expressions, such as $\frac{x}{x^2+5}$ and $\frac{2}{(2x-1)^4}$ is also required (see above paragraphs).

Analytical solution of simple first order differential equations with separable variables.

General and particular solutions will be required.

Numerical integration of functions.

Application of the trapezium rule to functions covered in C3 and C4. Use of increasing number of trapezia to improve accuracy and estimate error will be required. Questions will not require more than three iterations.

Simpson's Rule is *not* required.

6 Vectors

What students need to learn:

Vectors in two and three dimensions.

Magnitude of a vector.

Students should be able to find a unit vector in the direction of $\mathbf{a}$, and be familiar with $|a|$.

Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.

Position vectors.	$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.
The distance between two points.	The distance d between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$.
Vector equations of lines.	To include the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t(\mathbf{d} - \mathbf{c})$. Intersection, or otherwise, of two lines.
The scalar product. Its use for calculating the angle between two lines.	Students should know that for $\overrightarrow{OA} = \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a} . \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ and $\cos \angle AOB = \dfrac{\mathbf{a} . \mathbf{b}}{\lvert\mathbf{a}\rvert \lvert\mathbf{b}\rvert}$. Students should know that if $\mathbf{a} . \mathbf{b} = 0$, and that $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors, then $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.