Mock Diagnostic Test

This paper is intended to give you a feel for the kind of topics that will be covered in the diagnostic test, and the standard of the questions (all topics are within the A-level Pure Mathematics syllabus). In the real test each section will be considered separately and ultimately you will be expected to pass all sections (the pass mark will be about 70%). If you fail a section, or sections, at the first attempt there will be further opportunities to take failed sections in the first few weeks of term.

We suggest that you try these questions, without looking at the answers, as soon as possible, and if you encounter difficulties you are strongly advised to do some revision from your old school/college notes and any text books you may have. You should note that calculators and formulae sheets will not be permitted in the test. You should simplify your answers as much as possible, but leave them as exact values. We rarely want a numerical approximation (as provided by a calculator) as such an answer will often obscure what is going on: $\sqrt{2}-1$ is much more meaningful than 0.4142135 and $\pi/3$ tells you much more about what is going on than 1.0471976. The absence of a calculator means that you will need to remember the values of the trigonometric functions of standard angles such as $\pi/2, \pi/3$ etc.

Trigonometry

1. Give the value in radians of the angle 60°.

2. A sector of a circle has a length 10 cm and subtends an angle of 2 radians at its centre. What is the radius of the circle?

3. Which of the following is equal to cos 210°? (a) –cos 30°, (b) –cos 60°, (c) cos 30°, (d) cos 60°.

4. Consider an equilateral triangle ABC of side 2 units. What is the value of cosec(A)?

5. Suppose that $3\sin\theta + 4\cos\theta$ is expressed in the form $R\sin(\theta + \phi)$. Give the values of R and $\tan\phi$.

6. Evaluate $\cos\left(\dfrac{3\pi}{2}\right) - \sin\left(\dfrac{3\pi}{2}\right)$.

Algebra

7. Rearrange the expression $\dfrac{1+\sqrt{5}}{3+\sqrt{5}}$ into the form $p + q\sqrt{5}$ where p and q are rational numbers (fractions).

8. Rearrange the following formula to give d in terms of D and f.
$$\frac{2}{D-d} + \frac{2}{D+d} = \frac{1}{f}$$

9. Find the roots of the quadratic equation, $2x^2 - 4x - 1 = 0$

10. Given that $y = \log_e\left(Ae^{-Bx}\right)$, find x in terms of y, A and B.

11. Suppose that $(1-x)^8$ is expanded and arranged in increasing powers of x. What are the first three terms of the expansion?

12. Solve the following simultaneous equations

$$\frac{d}{2} + \frac{e}{3} = \frac{17}{6}$$

$$1 + \frac{e}{2} = d$$

Differentiation

13. Differentiate $y = e^{4x}$ with respect to x.

14. Differentiate $y = x \ln x$ with respect to x

15. Differentiate $y = \ln(3x^2 + 1)$ with respect to x

16. Differentiate $y = (x^2 + x)^{-2}$ with respect to x.

17. By differentiating the relation $x^2 + y^2 = e^k$ (where k is a constant) with respect to x, find dy/dx.

18. Find the slope of the tangent to the curve with equation $y = x^2 - 2x$ at the point with co-ordinates (-1,3).

Integration

19. Evaluate $\int e^{3x} dx$

20. Evaluate $\int \frac{dx}{x}$ for $x > 0$.

21. By using the substitution $z = \sin x$, or otherwise, evaluate the integral $\int_0^{\pi/2} \sin^3 x \cos x \, dx$.

22. By using partial fractions, or otherwise, evaluate the integral $\int \frac{dx}{(x-1)(x+2)}$.

23. Use integration by parts to evaluate the integral $\int x \cos x \, dx$.

24. Use integration by parts to evaluate $\int_0^1 x e^{3x} dx$.

Answers (but not solutions) are provided on the next sheet. Don't look at then until you have attempted the questions!