Mathematical simulation of the large scale electric fields and currents in the Earth's atmosphere

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There exists an idea that some features of quasi-static electric fields in the ionosphere measured by spacecrafts like DEMETER can be earthquake precursors. We have done a detailed analysis of such a possibility and have got a negative result in contrast with previous models.

The Earth's atmosphere is studied as the way for the electric field penetration from the Earth's surface into the ionosphere and back. We calculate the spatial distribution of the electric conductivity tensor in the ionosphere using the empirical models IRI, MSISE, IGRF. Such a model is also necessary for the simulation of the ionospheric electric fields and currents generated by neutral winds and by currents from the magnetosphere.

The electric fields which penetrate from ground into the ionosphere in frame of our model are much smaller than those in the models that do not take ionospheric conductivity into account, but much larger than those in the models which are based on the infinite Pedersen conductivity in the upper ionosphere. We show that those models are not adequate because some unproved boundary conditions are used.

The electric conductivity equation

The electric conductivity equation for the electric potential V is

$$-\mathbf{div}\,(\hat{\sigma}\mathbf{grad}\,V) = q, \tag{1}$$

where $\hat{\sigma}$ - conductivity tensor, $-q$ - divergence of extrinsic currents, if those exists. It is possible to neglect the Earth's surface curvature for local events. We use Cartesian coordinates x, y, z with vertical z axis and $z = 0$ at ground.

The problem is simplified much if the magnetic field is vertical and conductivity depends only of the height z, since in such a case Hall conductivity σ_H does not matter and the only Pedersen σ_P and field-aligned $\sigma_\parallel$ conductivities are involved in the equation

$$-\sigma_P(z)\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) - \frac{\partial}{\partial z}\left(\sigma_\parallel(z)\frac{\partial V}{\partial z}\right) = q. \tag{2}$$

We have created the model *Denisenko et al., 2008 a* **to calculate the components σ_P, σ_H, $\sigma_\parallel$ of the conductivity tensor $\hat{\sigma}$ above 90 km, that is based on the empirical models IRI, MSISE, IGRF. We use the empirical model** *Molchanov, Hayakawa, 2008* **below 60 km and smooth interpolation between 60 and 90 km.**

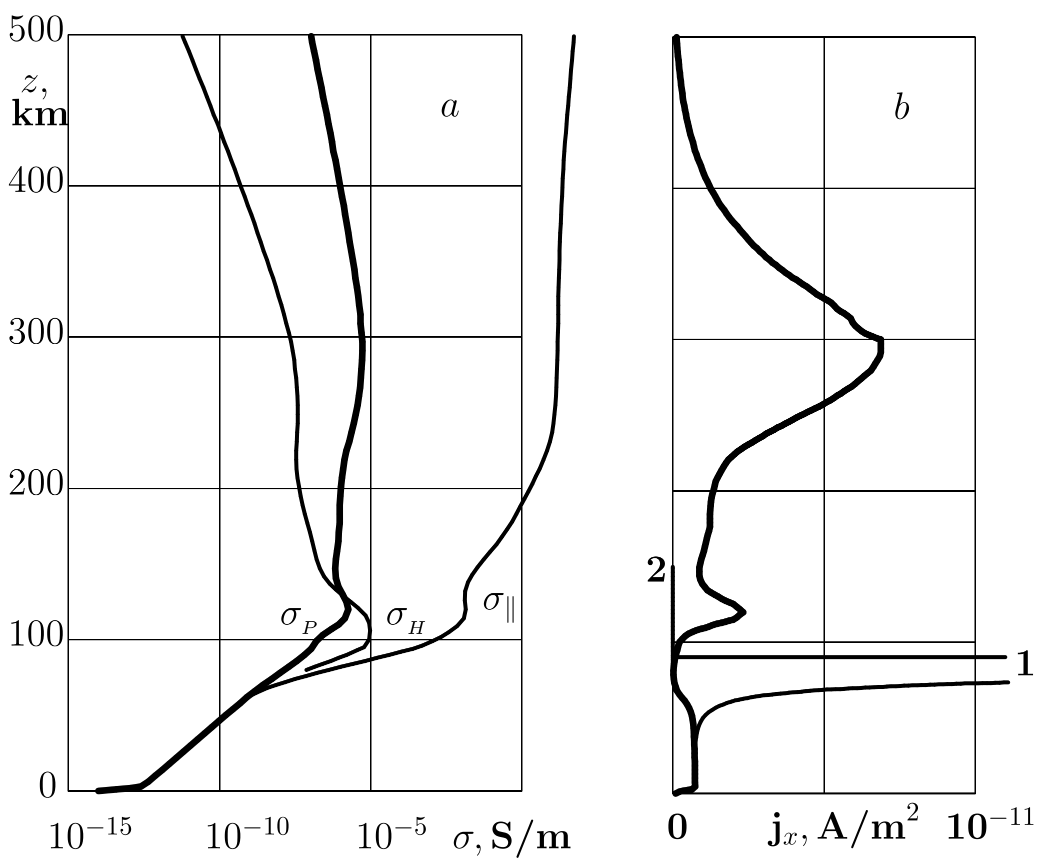

Figure 1: **a – Typical height distributions of the conductivity tensor $\hat{\sigma}$ components in middle latitudes. b – Height distributions of the horizontal current density, which are the result of our calculations and the models (***Pulinets et al., 2003***) (curve 1),** *Grimalsky et al., 2003* **(curve 2).**

2-D model of the ionospheric conductor

Let us consider only the ionosphere below $z < z_\infty$. For example the layer above 500 km adds less 1% to the integrated parameters of interest. The vertical current density can be given at this height

$$\sigma_\parallel(z_\infty)\frac{\partial V}{\partial z}\bigg|_{z=z_\infty} = j_\infty(x,y), \tag{3}$$

or the currents in far conductors, which are connected with this boundary by magnetic field lines, can be taken into account as it is described below.

We cut the upper ionosphere from the lower one by the plane $z = z_{up}$ and use the approximation $\sigma_\parallel = \infty$ above $z = z_{up}$. Hence the horizontal electric field components are independent of z and local Ohm law can be integrated over z to construct 2-D Ohm law with integral Pedersen and Hall conductivities Σ_P, Σ_H:

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \Sigma_P & -\Sigma_H \\ \Sigma_H & \Sigma_P \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}, \qquad \Sigma_P = \int_{z_{up}}^{z_\infty} \sigma_P \, dz \qquad \Sigma_H = \int_{z_{up}}^{z_\infty} \sigma_H \, dz. \tag{4}$$

Such a simplified model permits to construct the boundary condition at $z = z_{up}$: the currents, which enter this layer from below through the plane $z = z_{up}$, and given currents (3), which enter this layer from above through the plane $z = z_\infty$, are closed by the currents J in this layer

$$\mathbf{Div\ J} = j_z|_{z=z_{up}} + Q. \tag{5}$$

When the events of interest have horizontal scale much less then the ionospheric scale that equals thousands kilometers in middle latitudes, the values of σ_P, σ_H are independent of x, y, and so Σ_P, Σ_H are constants. The constant Σ_H can be omitted in (5) to obtain

$$-\Sigma_P\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right)\bigg|_{z=z_{up}} + \sigma_\parallel(z_{up})\frac{\partial V}{\partial z}\bigg|_{z=z_{up}} = Q. \tag{6}$$

The possibility to represent the ionospheric influence on the electric fields below $z = z_\infty$ by this boundary condition is tested by comparison of the solutions of the problem with this condition and the solutions in the whole ionosphere and atmosphere below $z = z_\infty$.

ELECTRIC FIELD PENETRATION FROM GROUND TO THE IONOSPHERE

The vertical component of the electric field at the ground is taken as given in many models

$$-\left.\frac{\partial V}{\partial z}\right|_{z=0} = E_0(x,y), \tag{7}$$

and we do the same. The function $E_0(x,y)$ is constructed on the base of published measurements and some general ideas. It would be better to say about vertical current density that is supported by some underground generator. Since conductivity of air is given, these conditions are equivalent.

We choose $z_\infty = 500$ km and the solutions of the form $f(z)\cos(x/x_0)$, where x_0 is the horizontal space scale. The equation (2) becomes the ordinary differential equation for the function $f(z)$. The boundary value problems with conditions which follow (7, 6) or (7, 3) can be solved numerically. We solve the problems with $x_0 =$ 100 km, $E_z(0,0) =$ 100 V/m..

The height distributions of the horizontal component of the electric field $E_x(\pi x_0/2, z)$ above the point $x = \pi x_0/2$, where it has maximal value.

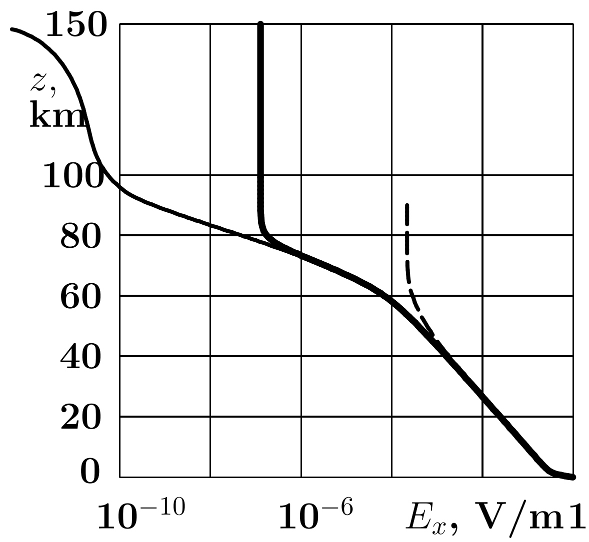

Figure 2: Height distributions of the horizontal component of the electric field in the night-time ionosphere for $x_0 = 100$ km. Bold line - our model. Dashed line - model (*Pulinets et al., 2003*). Thin line - model (*Grimalsky et al., 2003*).

The height $z_{up} =$ 90 km as it was done in the models *Denisenko et al., 2008*, *Ampferer et al., 2010* adds only 1% error to the ionospheric value of E_x if x_0 exceeds 3 km.

The solution with boundary condition that is used in the model *Pulinets et al., 2003*,

$$\left.\frac{\partial V}{\partial z}\right|_{z=90} = 0, \tag{8}$$

is plotted by dashed line. Thin line corresponds to the boundary condition

Grimalsky et al., 2003

$$V|_{z=150} = 0. \tag{9}$$

The last would be valid if an ideal conductivity in horizontal directions exists above 150 km. The condition (8) means no vertical current from the atmosphere at 90 km. It would be valid if the medium above 90 km has zero conductivity at least in horizontal directions. The conditions (8, 9) can be derived from ours (6) when Σ_P equals zero or infinity.

As it is shown in Fig. 2 the neglecting the ionospheric conductivity (8) increases ionospheric E_x about thousand times. The approximation $\Sigma_P = \infty$ decreases E_x a few thousands times at $z = 100$ km and makes it exactly zero above 150 km.

It can be mentioned that if we add conductivity of the adjoint ionosphere, that means twice larger Σ_P, then E_x in the ionosphere would be twice less. If a process in the auroral zone is under analysis then the conductivity of the plasma layer Σ_P about 100 S aught be added and E_x becomes 140 times less. Nevertheless it stays much larger than in the model *Grimalsky et al., 2003*. If we take into account the decrease of the effective σ_P, that describes the ionospheric conductor after its 1 hour acceleration by Ampere force E_x would be 2.5 times larger, but it stays much less than E_x in *Pulinets et al., 2003*.

If the magnetic field B is inclined from vertical by the angle χ, the tensor $\hat{\Sigma}$ aught be modified. In our test problem the parameter Σ_P in the upper boundary condition (6) aught be substituted with $\Sigma_P / \cos(\chi)$ or $\Sigma_P / \cos^2(\chi)$ when B is in y, z or x, z planes. Therefore the result E_x in the ionosphere decreases in comparison with those presented in Fig. 2a by the factor $\cos(\chi)$ or $\cos^2(\chi)$. Some more complicated model than (5) is necessary for the equatorial ionosphere *Denisenko et al., 2008 a*.

Extrinsic currents

The the model (*Sorokin et al., 2006, 2001*) differs from three models analyzed above by inclusion of extrinsic currents.

The authors suppose that the vertical extrinsic current exists due to the diffusion of charged particles of aerosol. In their estimations these particles density equals to $N_+ = 4 \cdot 10^9/\text{m}^3$ near ground, the current density decreases with height, the value near ground is $j_s = 3 \cdot 10^{-9}\text{A/m}^2$, and the decrement equals $1/H = 1/(2\,\text{km})$. Since the charge of each particle is supposed to be equal to the electron charge e, one can calculate the flux density as j_s/e. It is proportional to the density gradient because it is due to diffusion

$$\frac{j_s}{e} = K \frac{\partial N_+}{\partial z}, \tag{10}$$

where K is the coefficient of vertical turbulent diffusion in the near ground atmosphere. If we suppose that the decrease of this current with height corresponds to decreasing of the charged aerosol particles concentration, then gradient approximately equals N_+/H.

Using (10) we can estimate $K = j_s H/(eN_+) = 10^4 \mathrm{m}^2/\mathrm{sec}$, that is thousands times larger than it is possible in the Earths atmosphere.

It is also important that this extrinsic current exists as the transfer of uncompensated charges. If a charge is placed into a conducting medium, it is compensated with charges of other sign by conductivity current after typical time $\varepsilon_0/\sigma < 10$ minutes, since usually σ exceeds $2 \cdot 10^{-14}\mathrm{S/m}$. Then movement of neutral air means zero total current. So such an extrinsic current can not exist as a quasi stationary one even in a very turbulent atmosphere.

Conductivity variations

Fare weather currents can be disturbed if conductivity near ground is varied *Harrison et al., 2010*. If radon concentration increases 10 times then conductivity increases 3 times and decreases 3 times. If 0.25 mkm diameter aerosol concentration increases 10 times then conductivity decreases 4 times and increases 4 times. So δE_z near ground +70 V/m or -400 V/m could be observed with negligible variations in the ionosphere.

The ionospheric result *Harrison et al., 2010* appear since the vertical velocity of D layer equals to the variation of the current velocity of the electrons, that is not obvious. Also, if horizontal scale is less than 100 km, 1-D approach becomes not adequate and current density in the ionosphere decreases.

Conclusions on THE ELECTRIC FIELD PENETRATION TO THE IONOSPHERE

The new mathematical model is proposed to represent the ionospheric conductor by the boundary condition. This approximation is rather precise for large scale processes.

It is shown that two popular models of the electric field penetration into the ionosphere *Pulinets et al., 2003*, *Grimalsky et al., 2003* are not adequate in spite of that they give good results below 50 and 80 km respectively. Unproved upper boundary conditions are used in these models. In fact the good ionospheric conductor is excluded in *Pulinets et al., 2003* and unreal good conductor is added in *Grimalsky et al., 2003*. That is why our models *Denisenko et al., 2008*, *Ampferer et al., 2010* predict ionospheric electric fields not so large as the model *Pulinets et al., 2003* does, and not so small as the model *Grimalsky et al., 2003* does.

Other physical processes but electric conductivity of the atmosphere aught be analyzed to explain earthquake precursors in the ionosphere.

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ELECTRIC FIELD PENETRATION FROM THE IONOSPHERE TO THE NEAR GROUND ATMOSPHERE

Minimum of the energy functional $W(V) = \int \sigma(\mathbf{grad}\ V)^2\ d\Omega$.
3-D multirgid finite element method and proper software are created.

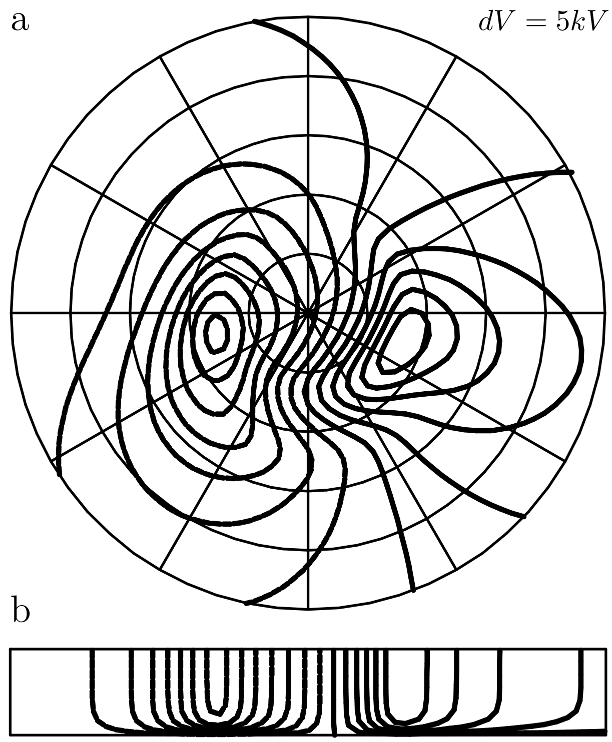

Figure 3: **a - Quiet time potential in the ionosphere, $\delta\theta = 10^o$ between the circles. b - Vertical cross-section $0 < h < 80$km through the points with max and min values of V.**

If the potential difference in the ionosphere equals ± 30 kV, the vertical electric field near ground is about $E_r = \pm 13$ V/m. It can be calculated in frame of 1-D model if horizontal scale exceeds 100 km. 3-D model must be used for calculation of the electric field in the upper ionosphere.