DYNAMO THEORY

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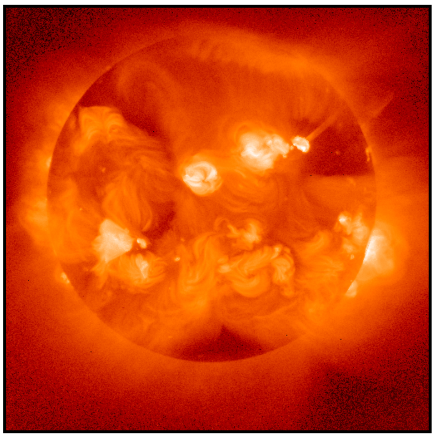


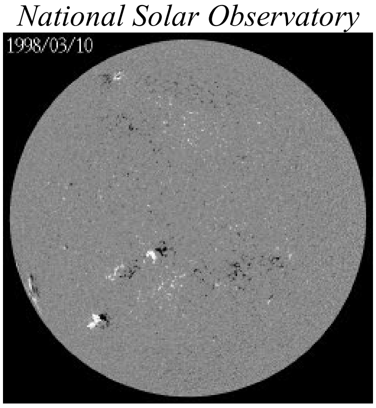
X-ray emission from solar corona

Thanks to D.W. Hughes, M.R.E. Proctor, S.M. Tobias

STFC ASSSP 2012 @ University of Warwick

Large-Scale Magnetic Field
($L>L_{turb}$)

National Solar Observatory



Magnetogram of solar surface shows line-of-sight component of the Sun's magnetic field.

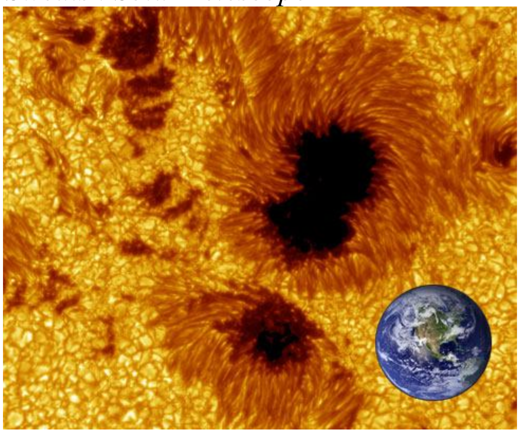
Active regions: Sunspot pairs and sunspot groups. Strong magnetic fields seen in an equatorial band within 30° of equator.

Rotate with sun differentially.

Median latitude varies in time.

Sunspots

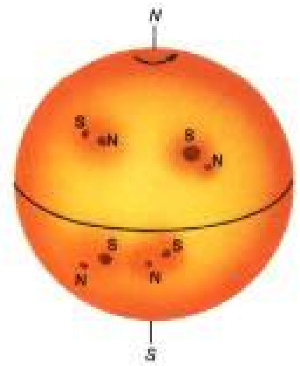
Swedish Solar Telescope



Dark spots on Sun cooler than surroundings ~3700K.

Sites of strong magnetic field (~3000G).

Last for several days (large ones for weeks).



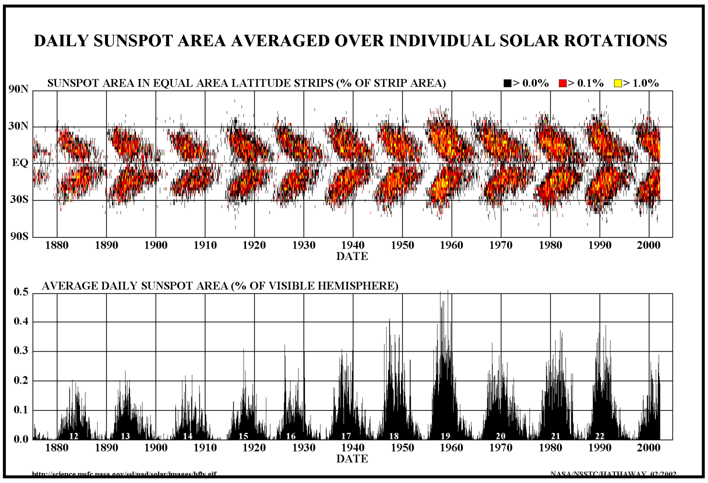
Hale's Law: Arise in pairs with opposite polarity in the North and South hemispheres.

Joy's Law: Axes of bipolar spots tilted by ~ 5-10^{o} with respect to the equator (leading spot closer to the equator).

Solar Cycle: Temporal Variation of Sunspots

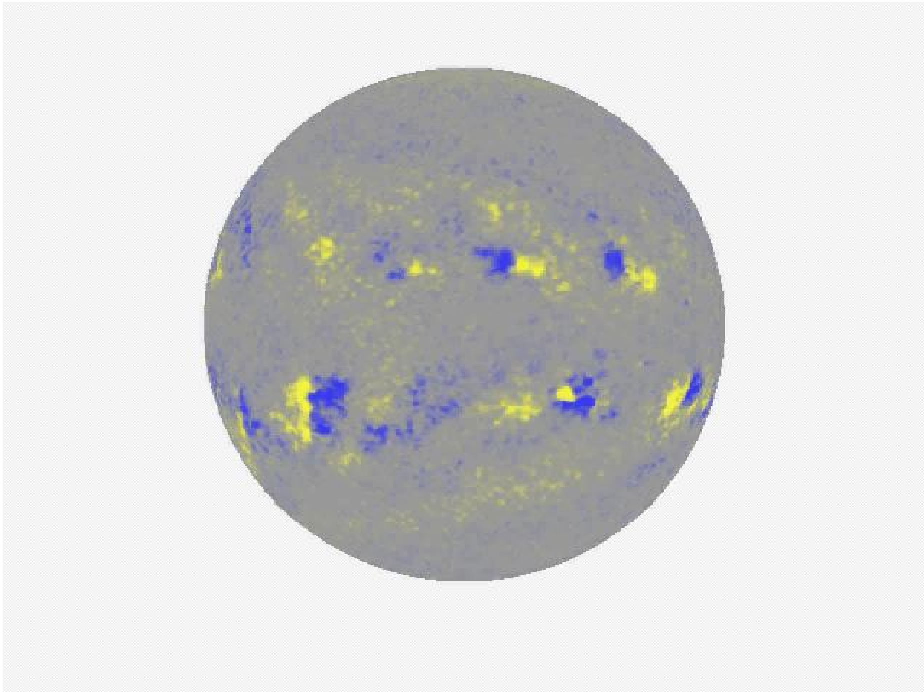
Location of sunspot appearance has cyclical component:

- Median latitude decreasing in time over an 11-year period,
- New cycle begins at higher latitude.



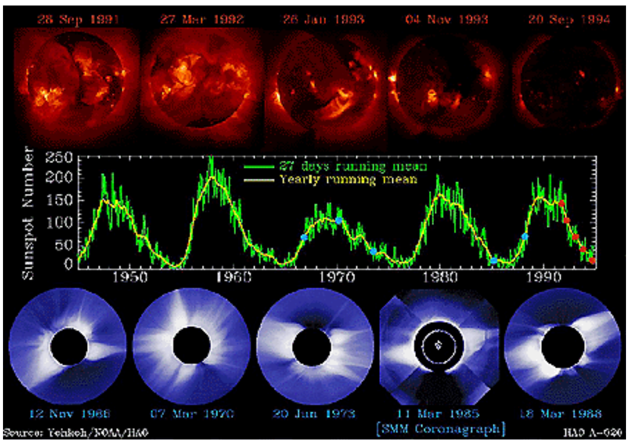
However, Butterfly Diagram is not symmetric and shows modulations.

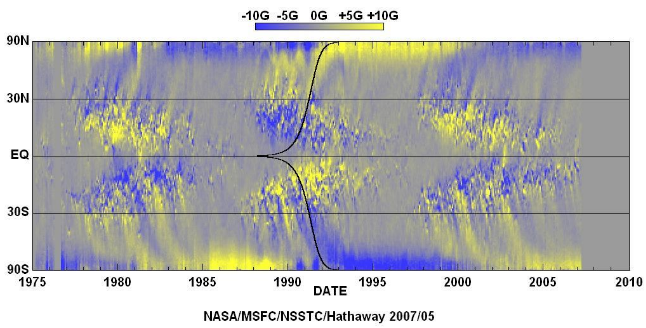
22 years of sunspots:



David Hathaway

Other Indicators of the Cycle

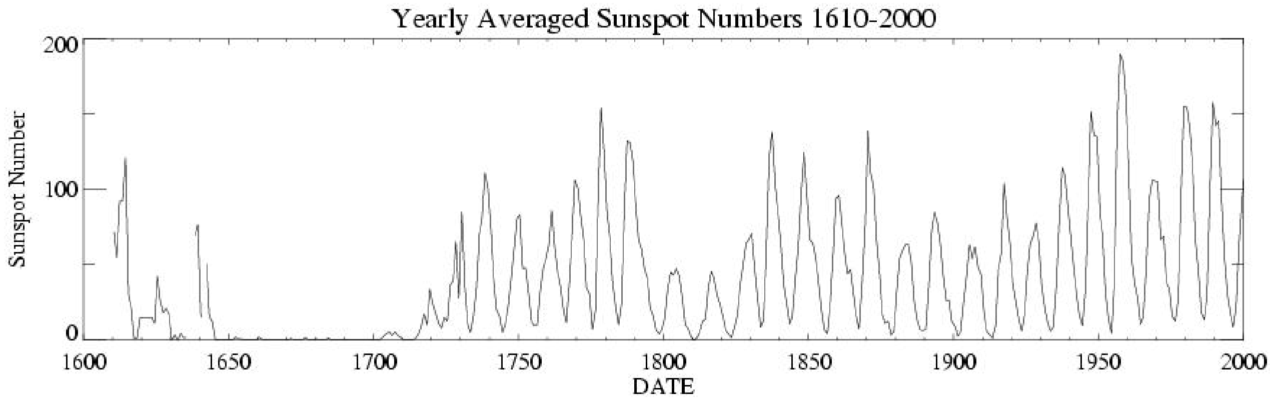




- Solar cycle not just visible in sunspots.
- Solar corona also modified as cycle progresses.
- Polar field is weak, has mainly one polarity at each pole (opposite polarities N. & S.) and reverses every 11 years – out of phase with the sunspot field.
- Global Magnetic field reversal every 11 years, leading to a 22-year cycle.

Modulation of the Cycle

Sunspot number: last 400 years



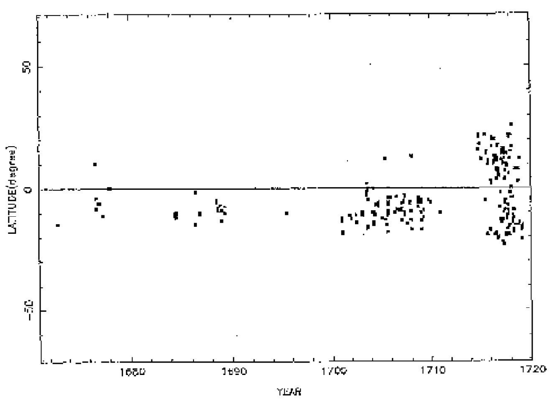
Modulation of basic cycle amplitude (and some modulation of frequency):

- Gleissberg Cycle: 88-year modulation.

- Maunder Minimum: virtually no sunspots from 1645 to 1715 (i.e. during a few cycles).

Butterfly Diagram as Sun Emerged from Minimum

(Ribes & Nesme-Ribes 1994)



- Asymmetry: Sunspots only seen in Southern Hemisphere.

No Longer Dipolar?

- Symmetry soon re-established.

Hence: (Anti)-Symmetric modulation when field is STRONG,
Asymmetric modulation when field is weak.

Observations: Solar (Proxy) I

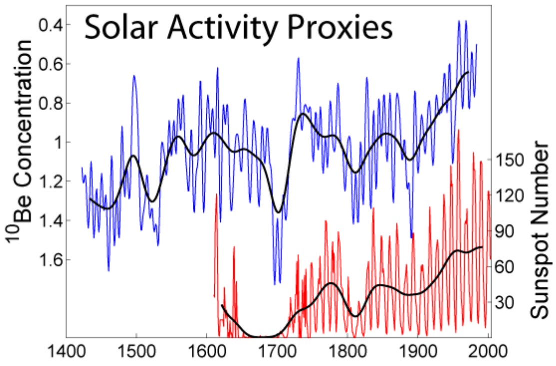
Proxy data of solar magnetic activity available.

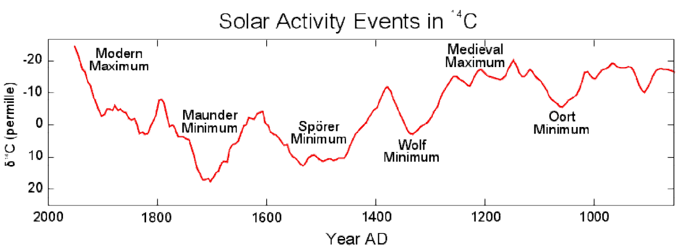
Solar magnetic field modulates the amount of cosmic rays reaching the Earth.

These are responsible for the production of terrestrial isotopes ^{10}Be and ^{14}C:

^{10}Be: stored in ice cores after 2 years in atmosphere,

^{14}C: stored in tree rings after ~30 years in atmosphere.





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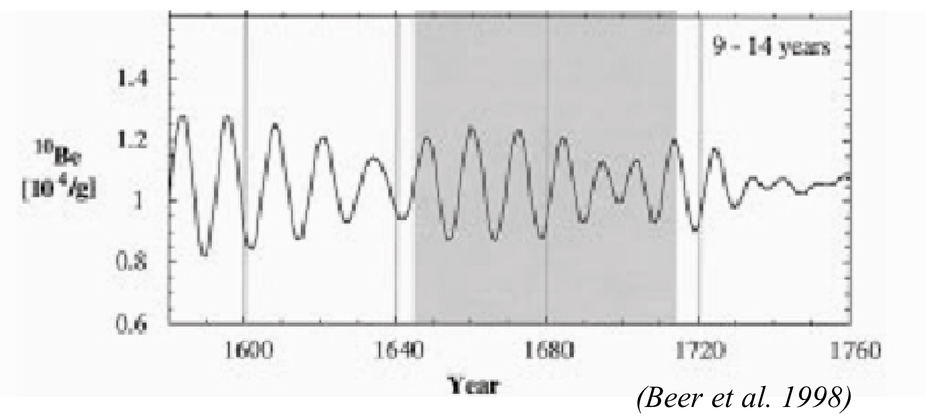
Observations: Solar (Proxy) II

Intriguingly, the ^{10}Be data shows clear evidence of the persistence of the *solar* cycle through the Maunder minimum, the period in which there is no evidence of a *sunspot* cycle.

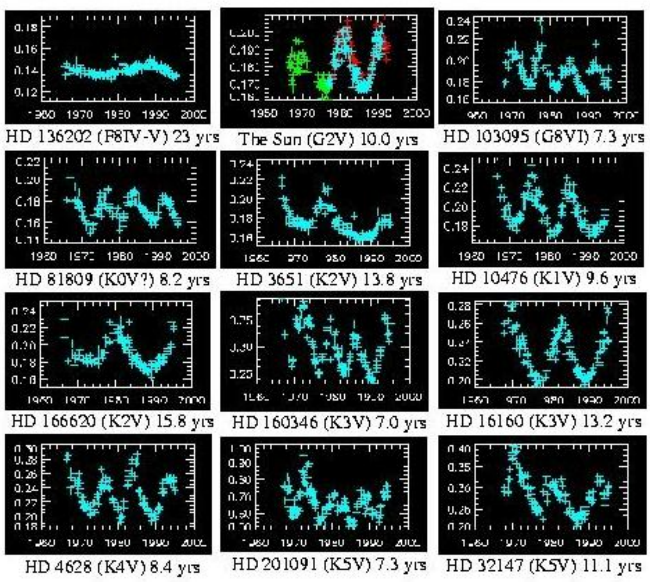


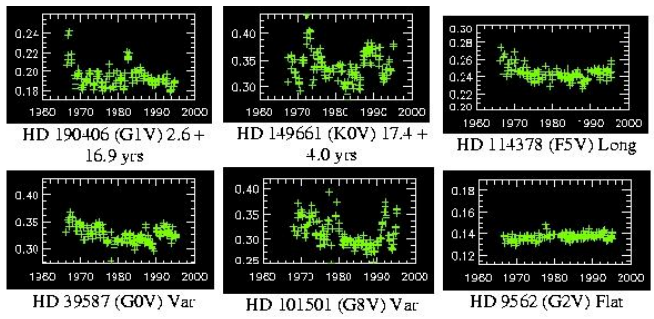
(Beer et al. 1998)

Magnetic Activity in Other Stars

Stellar Magnetic Activity can be inferred by amount of Chromospheric CaII emission in H and K.

Mount Wilson Survey (see e.g. Baliunas) of solar-type stars shows a variety of behaviour: cyclic, aperiodic, modulated, grand minima (?).



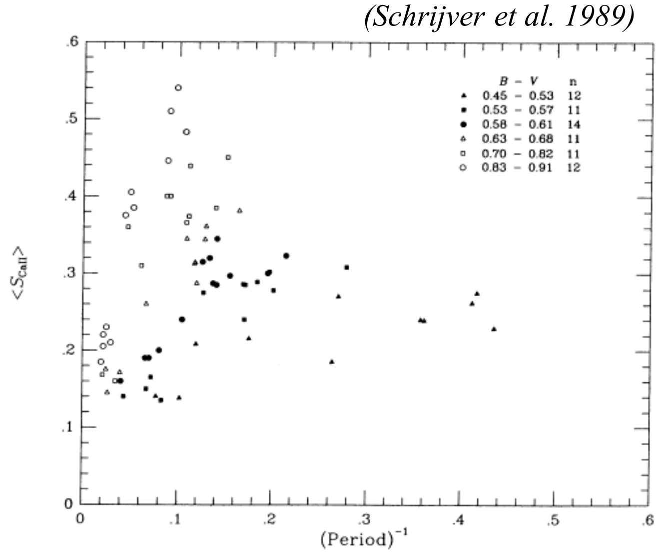


Activity and modulation increases with rotation rate.

Stellar CaII Flux

Stellar observations indicate a crucial dependence of the cycle on rotation.

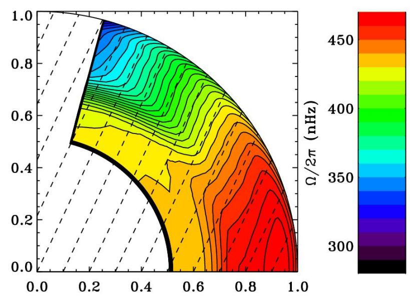
(Schrijver et al. 1989)

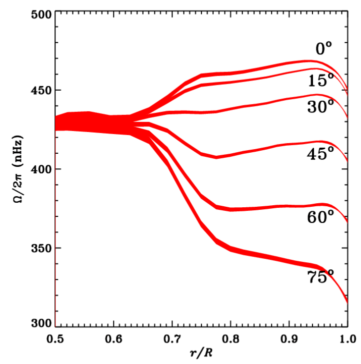


However, even non-rotating stars appear to have some residual emission – the so-called basal flux.

Differential Rotation in the Sun

At the surface, the period of rotation of the Sun is shorter at the equator (25 days) than at the poles (35 days)



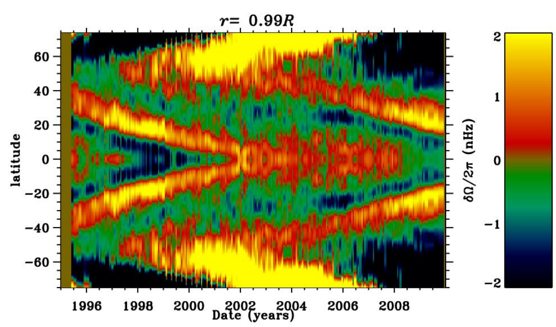


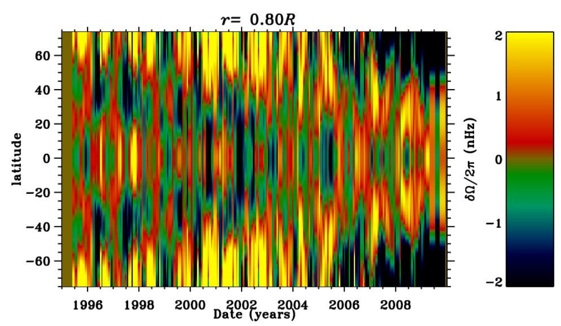
(Global Oscillation Network Group)

- Internal rotation profile inferred from Helioseismology
- Nearly solid body rotation in the radiative interior ($r < 0.7\ R_{Sun}$)
- Differential rotation approximately constant on radii (angle close to 25° to rotation axis) within the convection zone ($r > 0.7\ R_{Sun}$)
- Near-surface shear layer
- **Tachocline**: thin shear layer at the base of the convection zone

Torsional Oscillations

The solar differential rotation profile exhibits small modulations

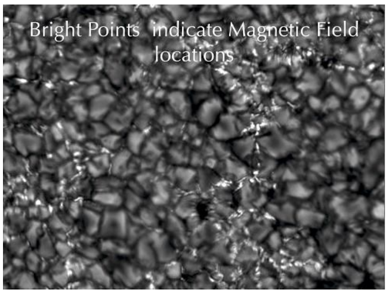




(Global Oscillation Network Group)

- These modulations penetrate through much or the convection zone and take the form of waves: torsional waves or oscillations
- They propagate towards the equator at lower latitudes and polewards at higher latitudes
- Torsional oscillations evolve on a 11-year period — not on the large-scale magnetic field 22-year cycle

Small-Scale Magnetic Field ($L \sim L_{turb}$)



Magnetic fields are also generated on small scales (meso-granular, supergranular scales), away from active regions.

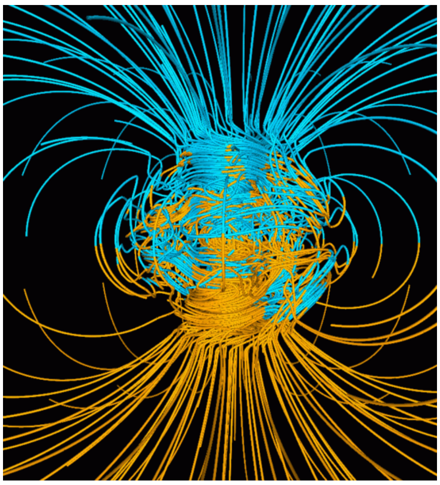
Unsigned flux appears continually in mini bipolar pairs.

These small-scale fields lead to many coronal events.

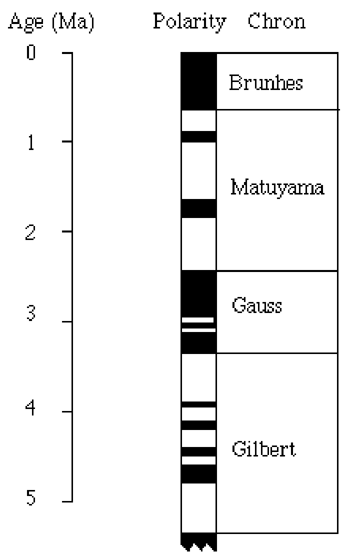
This *Magnetic Carpet* is generated largely independently of the solar cycle and generates fields of both polarities.

Could be relic of old active regions; but little or no net flux.

The Geomagnetic Field

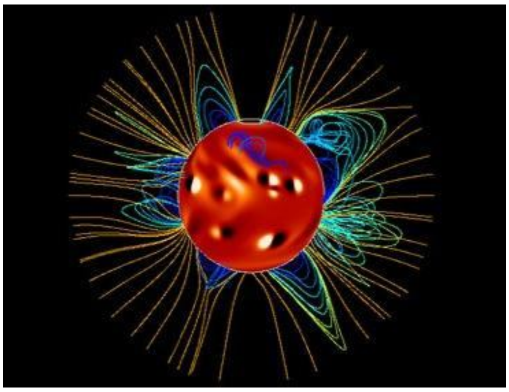


(Glatzmaier & Roberts 1995)



The Earth's magnetic field –approximately a magnetic dipole – has long periods of one sign, interspersed with relatively short intervals during which the field reverses.

How could a rotating body such as the Sun become a magnet? (Larmor 1919)



There are two ways of explaining the continued existence of a magnetic field in an astrophysical body.

The field has been there since the body was formed.

Or… it hasn't!

Why do we need dynamo theory?

In the absence of motion, magnetic field satisfies the diffusion equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B},$$

with Ohmic timescale of decay

$$\tau_\eta \approx L^2 / \eta.$$

For the Earth, $\tau_\eta \sim 10^4$ years, whereas the field has existed for 10^9 years. Thus the field cannot be a "fossil" field. Clearly, dynamo action is crucial for the maintenance of the field.

For the Sun, $\tau_\eta \sim 10^{10}$ years, comparable to the lifetime of the Sun! However, the cycle time is much less than τ_η.

Coherence of sunspot record suggests global mechanism operating at all longitudes and velocity data favours dynamo explanation over nonlinear oscillations involving *torsional waves*.

Dynamo theory addresses the issue of the maintenance of a magnetic field.

From an astrophysical perspective, it is pursued entirely within the framework of MHD – a good approximation for stellar interiors.

So the full set of governing equations is:

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{B})}_{\text{Induction}} + \eta \nabla^2 \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0,$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla \mathbf{u}\right) = -\nabla p + \underbrace{\mathbf{j} \times \mathbf{B}}_{\text{Lorentz force}} + \rho \mathbf{g} + \mathbf{F}_{viscous} + \mathbf{F}_{other},$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{u}) = 0,$$

$$\frac{D(p\rho^{-\gamma})}{Dt} = \text{loss terms},$$

$$p = R\rho T.$$

Nondimensional Parameters for the Sun

(Ossendrijver 2003)

	Base of CZ	Photosphere
$Ra \equiv g\Delta\nabla d^4 / \nu\chi H_P$	10^{20}	10^{16}
$\mathrm{Re} \equiv UL/\nu$	10^{13}	10^{12}
$Rm \equiv UL/\eta$	10^{10}	10^{6}
$\mathrm{Pr} = \nu/\chi$	10^{-7}	10^{-7}
$\beta = 2\mu_0 p / B^2$	10^{5}	1
$Pm = \nu/\eta$	10^{-3}	10^{-6}
$M = U/c_s$	10^{-4}	1
$Ro = U/2\Omega L$	0.1-1	10^{-3}-0.4

Equations of compressible MHD are *not* easy to solve. Even with today's fanciest computers it is impossible to solve these for astrophysically realistic parameter values.

Often the effects of compressibility are not crucial for an understanding of dynamo processes, and it is sufficient to work within incompressible MHD, governed by:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla \mathbf{u}\right) = -\nabla p + \mathbf{j} \times \mathbf{B} + \mathbf{F}_{viscous} + \mathbf{F}_{other},$$

$$\nabla.\mathbf{u} = 0, \quad \nabla.\mathbf{B} = 0.$$

Even though, it is still impossible to solve these for astrophysically realistic parameter values.

Large and Small-scale dynamos

LARGE SCALE

- Sunspots,
- Butterfly Diagram,
- 11-yr activity cycle,
- Coronal Poloidal Field,
- Systematic reversals,
- Periodicities.

Field generation on scales > L_{TURB} .

SMALL SCALE

- Magnetic Carpet,
- Field Associated with granular and supergranular convection,
- Magnetic network.

Field generation on scales ~ L_{TURB} .

Magnetic fields and flows

Interaction of magnetic fields and flows due to induction (kinematics) and body forces (dynamics).

Hydrodynamic dynamos:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

$$\Leftrightarrow \frac{\mathrm{D}\mathbf{B}}{\mathrm{D}t} = \frac{\partial \mathbf{B}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{u} + \eta \nabla^2 \mathbf{B}.$$

- Induction – leads to growth of energy through extension of field lines
- Dissipation – leads to decay of energy into heat through Ohmic loss.

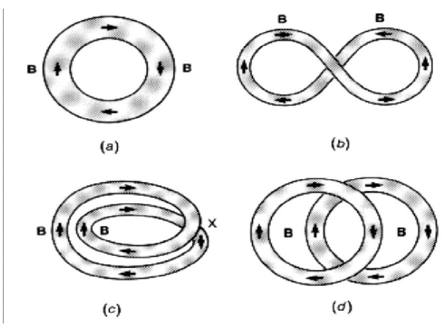
Hydrodynamic dynamo works if induction by the fluid motions overpowers the dissipative losses.

Sufficiently vigorous flows convert mechanical into magnetic energy if Magnetic Reynolds number large enough.

Kinematic Dynamos

Can we find a velocity field $\mathbf{u}(\boldsymbol{x},t)$ such that the magnetic field $\mathbf{B}(\boldsymbol{x},t)$ – *governed solely by the induction equation, linear* – grows?

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Rm} \nabla^2 \mathbf{B},$$



Stretch, twist and fold mechanism.

For large Rm energy grows on advective time but is accompanied by folding of field lines. Large gradients appear down to dissipation scale $\ell_\eta \sim Rm^{-1/2}$.

E.g., for *2-D planar fields and flows*, folding and dissipation **always** win! and fields ultimately decay even for very large Rm.

Anti-Dynamo Theorems

Not all flows and fields work as dynamos: simple situations (too much symmetry, slow flows…) cannot lead to growth.

1934: Cowling proves that an axisymmetric magnetic field cannot be maintained by dynamo action.

1957: Zeldovich theorem. Magnetic fields cannot be maintained by two-dimensional planar motions. (Also true for motions on a spherical surface, but *not* for motions on a cylindrical surface.)

1958: Backus' necessary condition: field cannot grow for arbitrarily small velocities: $Rm \equiv \max\left(|\nabla \mathbf{u}| L^2/\eta\right) > \pi$.

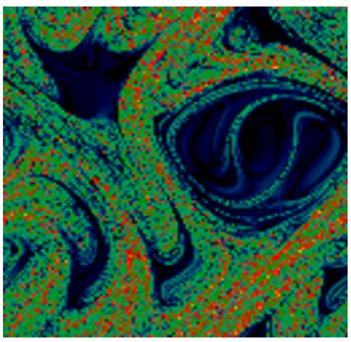
Fast and Slow Kinematic Dynamos

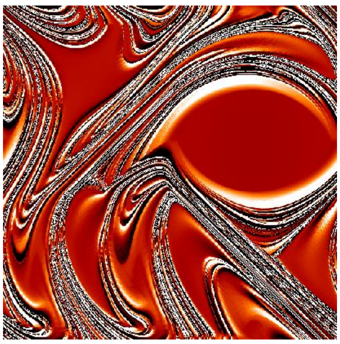
Two time scales at large *Rm*:

Turnover time: $\tau_a \approx L/U$ and *diffusion time* $\tau_\eta \approx L^2/\eta = Rm \times \tau_a$.

<u>Slow dynamo</u>: from Faraday, flux frozen as $Rm \to \infty$, so growth rate tends to 0. (Growth time $\sim \tau_a \times fn(Rm) >> \tau_a$.)

<u>Fast dynamo</u>: however, STF produces small length scales. So, diffusion is always important and growth time $\sim \tau_a$ as $Rm \to \infty$.





For fast dynamos exponential stretching of field lines needed: flow must be chaotic.

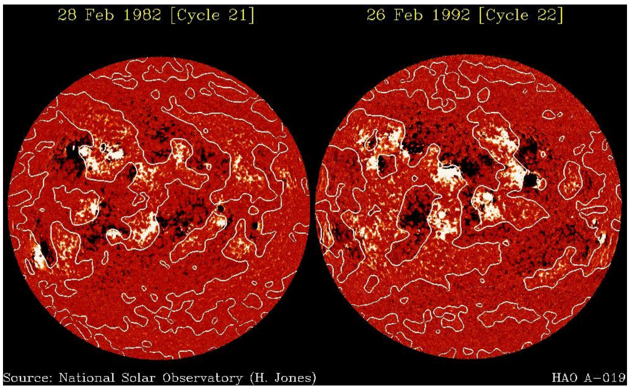
In most astrophysical contexts *Rm* typically is huge.
$Rm \sim 10^8$ in the solar convection zone.

Large-scale or Small-scale Dynamos

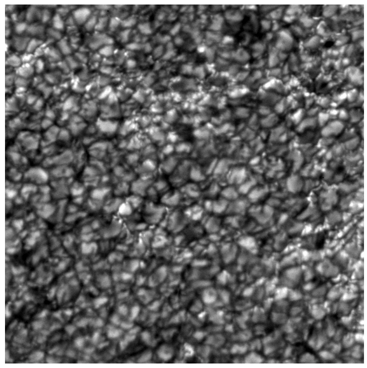
If the magnetic field generated has sizeable energy on scales much larger than those of the driving flow then we say that the dynamo is a *large-scale* dynamo.

If, on the other hand, the field exists on scales comparable to and smaller than those of the flow then we say that the dynamo is a *small-scale* dynamo.

The Sun might well have two sorts of dynamo operating:



Deep-seated large-scale dynamo responsible for solar cycle field.



Small-scale surface dynamo generating photospheric fields.

Modelling Approaches

- Because of the extreme nature of the parameters in the Sun and other stars there is no obvious way to proceed.
- Modelling has typically taken one of three forms:

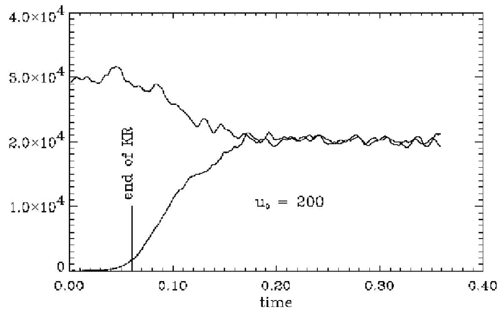
– Mean Field Models
  - Derive equations for the evolution of the mean magnetic field (and perhaps velocity field) by parametrising the effects of the small scale motions.
  - The role of the small-scales can be investigated by employing local computational models.

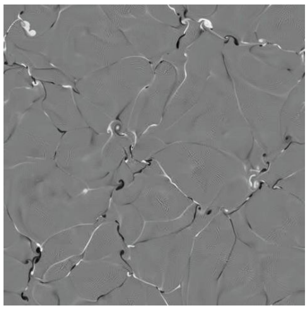
– Global Computations
  - Solve the relevant equations on a massively-parallel machine.
  - Either accept that we are at the wrong parameter values or claim that parameters invoked are representative of their turbulent values.
  - Maybe employ some "sub-grid scale modelling.

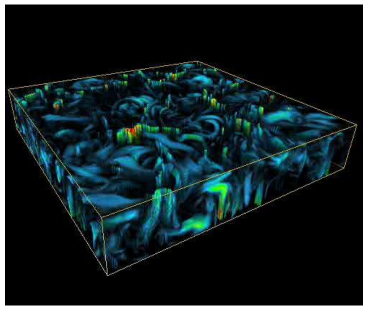
– Low-order models
  - Try to understand the basic properties of the equations with reference to simpler systems (cf. Lorenz equations and weather prediction).

Small-Scale Dynamos

Dynamo produced by Boussinesq convection (Emonet & Cattaneo)







Fields and flows on same scale.

Magnetic and kinetic energies comparable.

Harder to obtain dynamo action if *Pm* is small but can be found for sufficiently vigorous motions. (Recall $Pm \sim 10^{-6}$ at the top of CZ.)

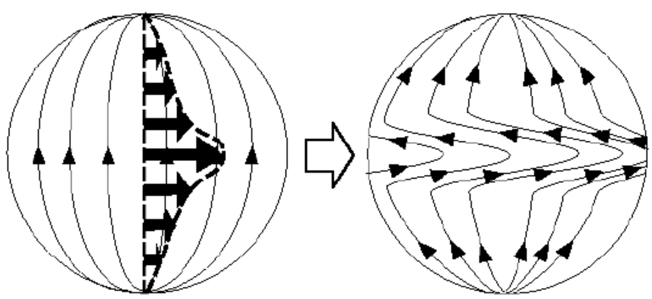
Ra (1e6)	P	Pm	S=P/Pm	Re	Rm	Em/Ek
0.5	1	5	0.2	200	1000	0.20
2	1	1	1	428	428	0.045
4	1	1	1	550	550	0.06 (+)
16	1	0.2	5	1200	240	1e-06 (-)
16	1	0.5	2	1200	600	1e-06 (+)
16	0.5	0.5	1	1900	950	1e-05 (+)
2	0.25	0.5	0.5	1100	550	0.01

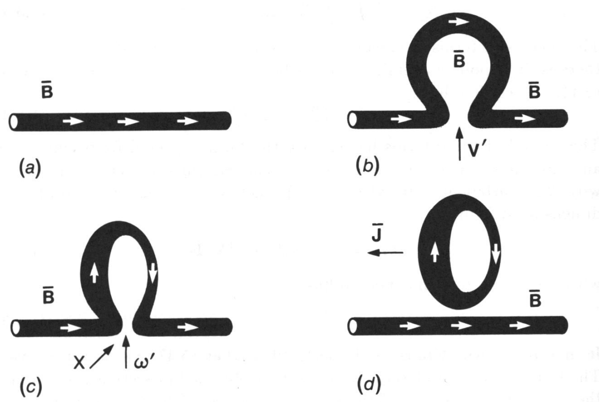
Modeling the Large-Scale Dynamo

Mean field electrodynamics: fundamental physics of the α-ω dynamo

The Ω effect

Conversion of poloidal to toroidal field by differential rotation.





ω-effect – poloidal → toroidal

α-effect – toroidal → poloidal
poloidal → toroidal

Kinematic Mean Field Theory

Starting point is the magnetic induction equation of MHD:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

where **B** is the magnetic field, **U** is the fluid velocity and η is the magnetic diffusivity (assumed constant for simplicity).

Assume *scale separation* between large- and small-scale field and flow:

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{b}, \quad \mathbf{U} = \mathbf{U}_0 + \mathbf{u},$$

where **B** and **U** vary on some large length scale *L*, and **u** and **b** vary on a much smaller scale *l*.

$$\langle \mathbf{B} \rangle = \mathbf{B}_0, \quad \langle \mathbf{U} \rangle = \mathbf{U}_0,$$

where averages are taken over some intermediate scale $l \ll a \ll L$.

For simplicity, for the moment *let's ignore the large-scale flow*.

Induction equation for mean field: $$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times \mathcal{E} + \eta \nabla^2 \mathbf{B}_0,$$

where mean emf is $\mathcal{E} = \langle \mathbf{u} \times \mathbf{b} \rangle$.

This equation is exact, but is only useful if we can relate $\mathcal{E}$ to $\mathbf{B}_0$.

Induction equation for fluctuating field:

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}_0) + \nabla \times \mathbf{G} + \eta \nabla^2 \mathbf{b},$$ where $\mathbf{G} = \mathbf{u} \times \mathbf{b} - \langle \mathbf{u} \times \mathbf{b} \rangle$.

Traditional approach is to assume that the fluctuating field is driven solely by the large-scale magnetic field: in the absence of mean field the fluctuating field decays. There is no small-scale dynamo (not really appropriate for high *Rm* turbulent fluids).

Under this assumption, the relation between $\mathbf{b}$ and $\mathbf{B}_0$ (and hence between $\mathcal{E}$ and $\mathbf{B}_0$) is linear and homogeneous.

Postulate an expansion of the form:

$$E_i = \alpha_{ij} B_{0j} + \beta_{ijk} \frac{\partial B_{0j}}{\partial x_k} + \ldots$$

where α_{ij} and β_{ijk} are *pseudo*-tensors.

Simplest case is that of isotropic turbulence, for which $\alpha_{ij} = \alpha\delta_{ij}$ and $\beta_{ijk} = \beta\varepsilon_{ijk}$.
Then mean induction equation becomes:

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\alpha \mathbf{B}_0) + (\eta + \beta)\nabla^2 \mathbf{B}_0.$$

α: regenerative term, responsible for large-scale dynamo action.
Since **E** is a polar vector whereas ***B*** is an axial vector then α can be non-zero only for turbulence lacking reflexional symmetry (i.e. possessing handedness).

β: turbulent diffusivity.

Cowling's theorem does not apply to the *mean* induction equation – allows axisymmetric solutions.

Re-inserting the large scale flow gives the mean induction equation as:

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\mathbf{U}_0 \times \mathbf{B}_0) + \nabla \times (\alpha \mathbf{B}_0) + (\eta + \beta)\nabla^2 \mathbf{B}_0.$$

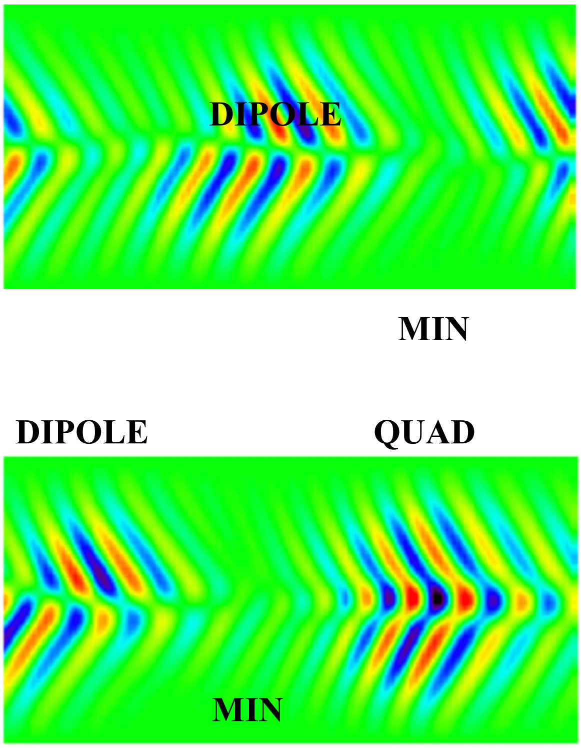
ω-effect **α-effect**

The ω-effect, arising from the differential rotation, is responsible for the generation of toroidal field from poloidal field.

The α-effect, arising from helical turbulence (e.g. cyclonic events in rotating convection), can generate poloidal from toroidal field, and also toroidal from poloidal field.

Consequently, depending on which terms are dominant, we speak of α^2, α-ω or $\alpha^2\omega$ dynamos.

Results from Interface Mean Field Models



Tobias (1996,1997) Beer et al. (1998)

- Interface models have proved very successful at reproducing large-scale structure of Solar magnetic field.
- Computer-generated Butterfly diagrams.
- Rich behaviour
- Large Modulation and Grand Minima.
- Emerges from Minima with asymmetry.
- Minima can trigger FLIPPING of parity.
- Just because Sun's field is dipolar now – might not have been in the past.
- Modulation increases as rotation and activity increase.

A Word of Warning

What are α and β ?

Mean field electrodynamics is, in some sense, very successful.
If one is free to choose the forms (spatial, temporal, functional) of α, β, ω then it is possible to reproduce a whole range of astrophysical magnetic behaviour.

However, α, β, ω are *not* free parameters.

They must really be determined from a self-consistent solution of the induction and momentum equations.

So what we really need is a theory of MHD turbulence!

Analytic progress possible *if we neglect the* ***G*** *term* ("first order smoothing").

This can be done if either *Rm* or the correlation time of the turbulence τ is small.

For the latter we get the result (assuming isotropy):

$$\alpha = -\frac{\tau}{3}\langle \mathbf{u}.\boldsymbol{\omega} \rangle$$

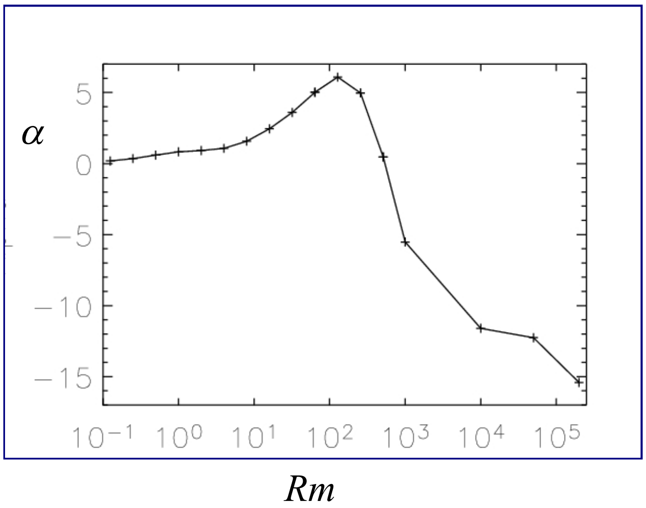
Correlations between ***u*** and ***b*** have been replaced by correlations between ***u*** and ***ω*,** the curl of the velocity field – the *vorticity.*

Is there a relation between α and helicity for general flows?

Computation of α for the "Galloway-Proctor" flow

One flow, but different magnetic Reynolds numbers.

(*Courvoisier et al.* 2006)



The α–effect is a very sensitive function of *Rm*.

Clearly α can in no way be related in a simple manner to the helicity of the flow.

Nonlinear Behaviour of α

- As the magnetic field grows it reacts back, via the Lorentz force, to suppress α.
- Key question: *How strong is the large scale field before this happens?*

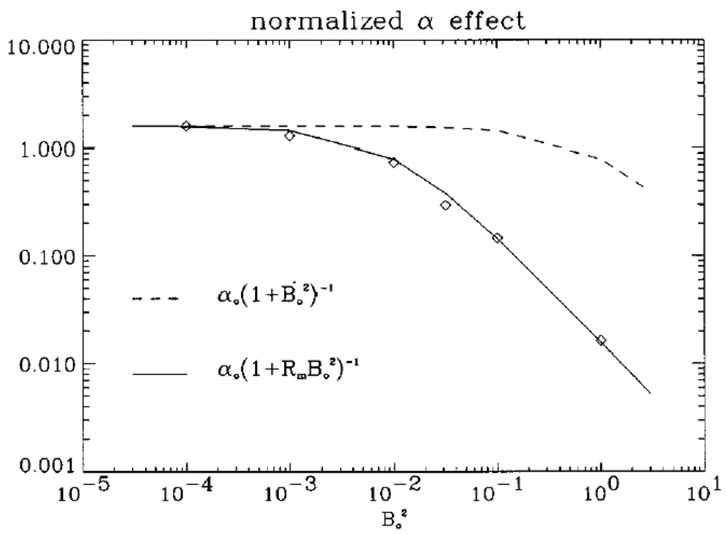
In a turbulent fluid at high *Rm* the energy in the small-scale magnetic field dominates that in the large-scale field.

From a simple scaling it follows that: $\langle B^2 \rangle \approx Rm^p \langle \mathbf{B} \rangle^2.$

where p is a flow and geometry dependent coefficient ($p > 0$)

This leads to an α-effect of the form: $\alpha = \dfrac{\alpha_0}{1 + Rm^\gamma B_0^2}$

How α and β are modified by the mean field in the nonlinear regime is a *crucial question* and still a *very controversial* issue.



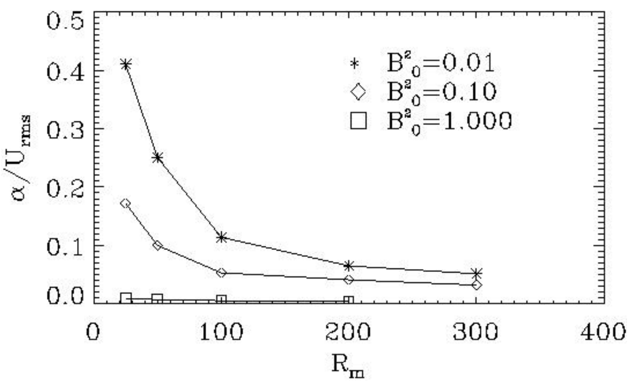
α versus B_0^2
(Cattaneo & Hughes 1996)

Suggestive of the formula:

$\alpha = \dfrac{\alpha_0}{1 + Rm^\gamma B_0^2}$ for γ = O(1).

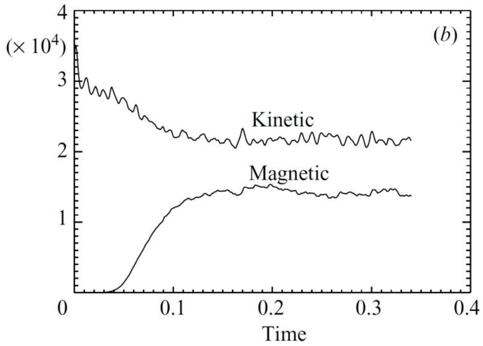
α versus *Rm*

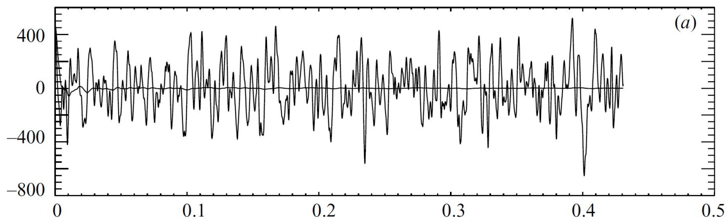
(Cattaneo, Hughes & Thelen 2002)



Incompressible Rotating Convection

When rotation is important the turbulent flow develops a well defined distribution of kinetic helicity





(Cattaneo & Hughes 1996)

- Convection at high Rm acts as an efficient small-scale dynamo: substantial magnetic energy is generated on turnover time

- α is extremely strongly fluctuating

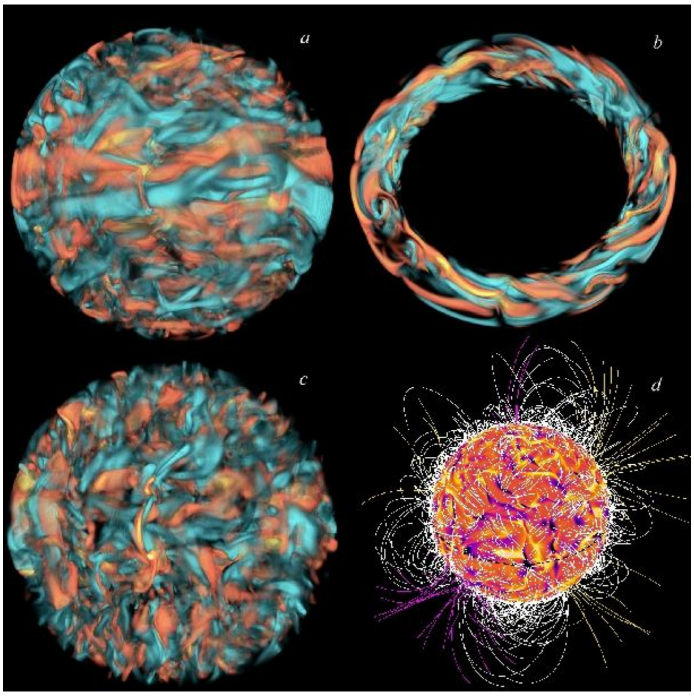
- Meaningful (converged) values of α can only be obtained after long time averaging

- α-effect is small and controlled by the magnetic diffusivity

Various Possible Scenarios for the Solar Dynamo

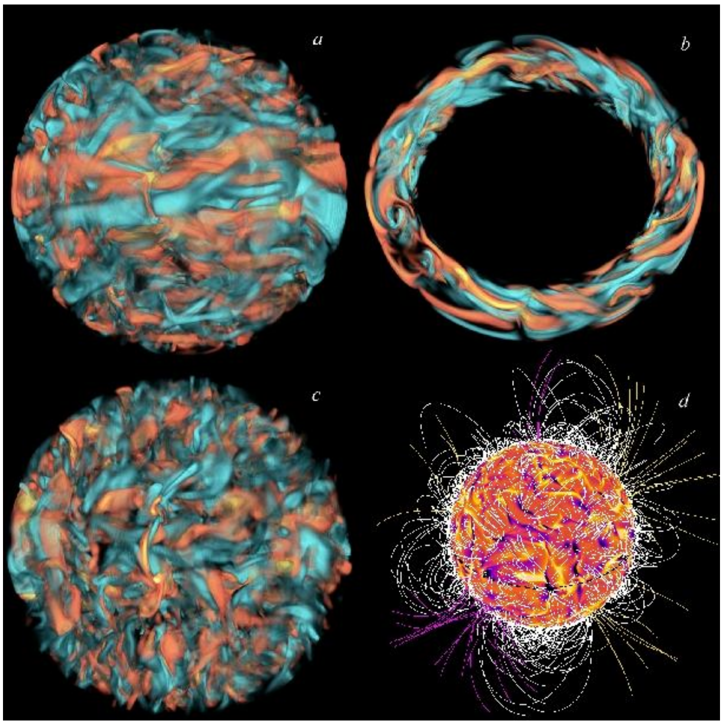
1. Dynamo action distributed throughout the convection zone.
2. α-effect located at surface, differential rotation of the tachocline.
3. α-effect located in convection zone, differential rotation of the tachocline.

Distributed Dynamo Scenario



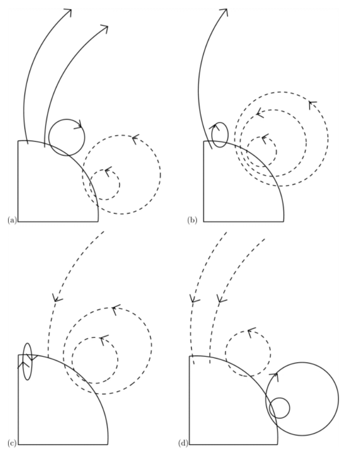
- Here the poloidal field is generated throughout the convection zone by the action of cyclonic turbulence.
- Toroidal field is generated by the latitudinal distribution of differential rotation.
- No role is envisaged for the tachocline
- Angular momentum transport would presumably be most effective by Reynolds and Maxwell stresses.

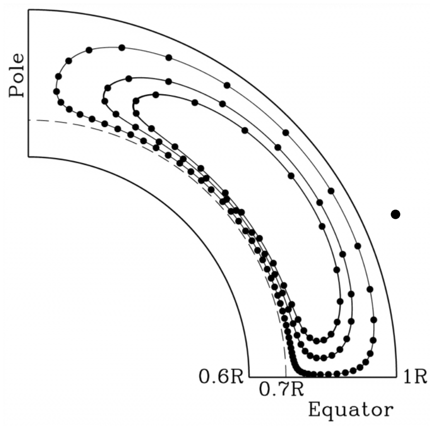
Distributed Dynamo Scenario



- PROS
  - Scenario is "possible" wherever convection and rotation take place together.
- CONS
  - Computations show that it is hard to get a large-scale field.
  - Mean-field theory shows that it is hard to get a large-scale field (catastrophic α-quenching).
  - Buoyancy removes field before it can get too large.

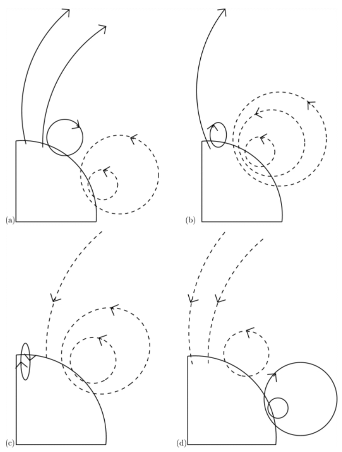
Flux Transport Scenario

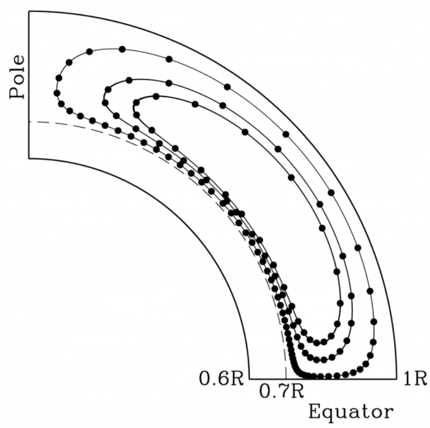




- Here the poloidal field is generated at the surface of the Sun via the decay of active regions with a systematic tilt (Babcock-Leighton Scenario) and transported towards the poles by the observed meridional flow.
- The flux is then transported by a conveyor belt meridional flow to the tachocline where it is sheared into the sunspot toroidal field.
- No role is envisaged for the turbulent convection in the bulk of the convection zone.

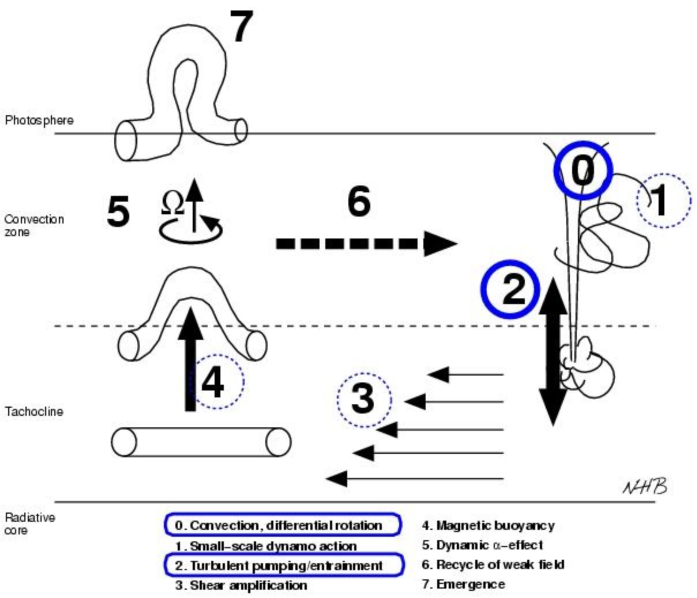
Flux Transport Scenario





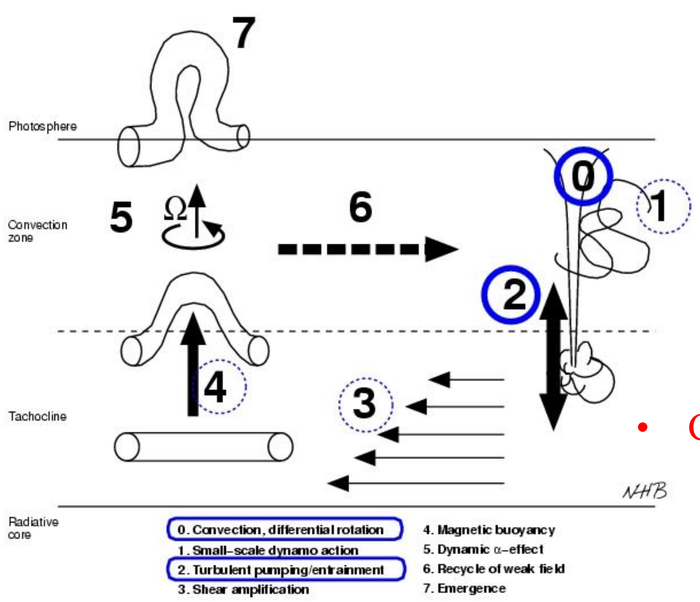
- PROS
  - Does not rely on turbulent α-effect therefore all the problems of α-quenching are not a problem.
  - Sunspot field is intimately linked to polar field immediately before.
- CONS
  - Requires strong meridional flow at base of CZ of exactly the right form.
  - Ignores all poloidal flux returned to tachocline via the convection
  - Effect will probably be swamped by "α-effects" closer to the tachocline.
  - Relies on existence of sunspots for dynamo to work (cf. Maunder Minimum).

Interface Dynamo Scenario



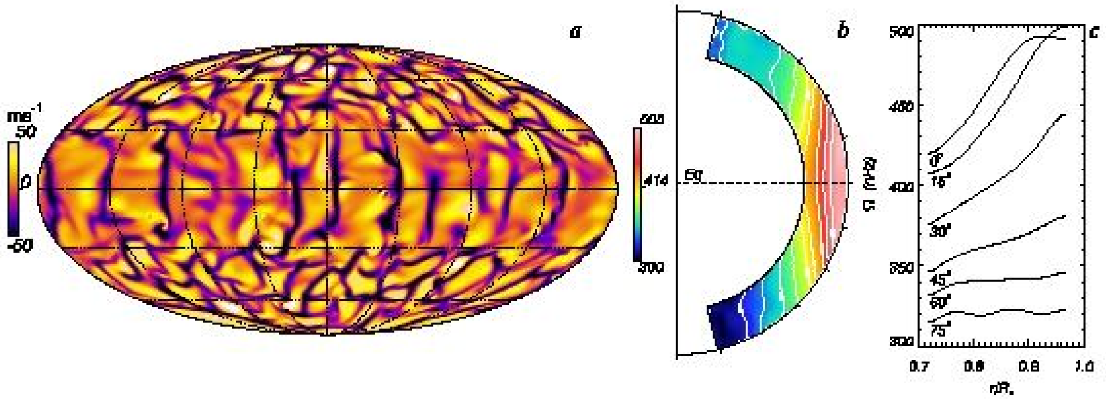
- The dynamo is thought to work at the interface of the convection zone and the tachocline.
- The mean toroidal (sunspot field) is created by the radial diffential rotation and stored in the tachocline.
- The mean poloidal field (coronal field) is created by turbulence (or perhaps by a dynamic α-effect) in the lower reaches of the convection zone.

Interface Dynamo Scenario



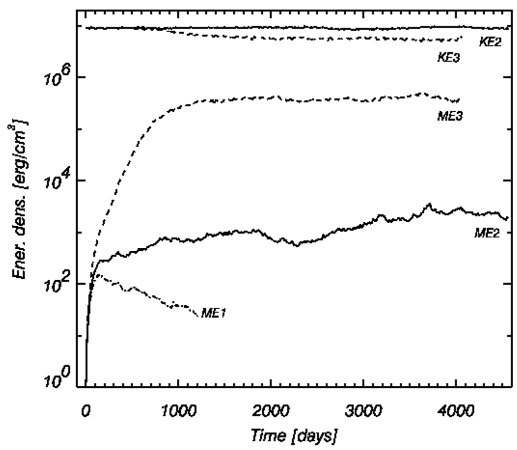
- PROS
  - The radial shear provides a natural mechanism for generating a strong toroidal field.
  - The stable stratification enables the field to be stored and stretched to a large value.
  - As the mean magnetic field is stored away from the convection zone, the α-effect is not suppressed.
- CONS
  - Relies on transport of flux to and from tachocline – how is this achieved?
  - Delicate balance between turbulent transport and fields.

Global Computations: Hydrodynamic State

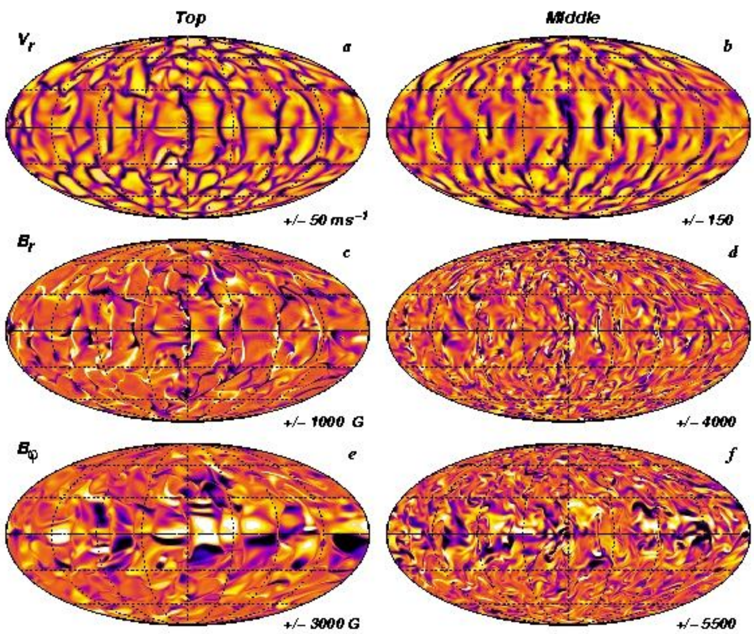


- Moderately turbulent Re ~ 150
- Low latitudes downflows align with rotation
- High latitudes more isotropic

Global Computations: Dynamo Action

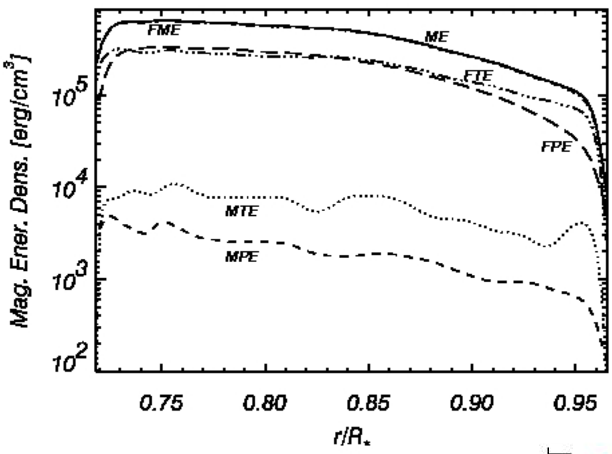


- For Rm > 300 dynamo action is sustained.
- ME ~ 0.07 KE



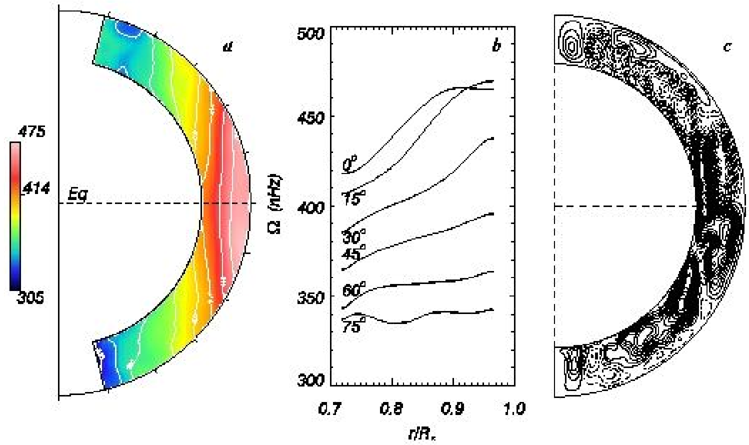
- B_r is aligned with downflows
- B_ϕ is stretched into ribbons

Global Computations: Saturation



- Magnetic energy is dominated by fluctuating field,
- Means are a lot smaller,
- $\langle B_T \rangle \sim 3 \langle B_P \rangle$.

- Dynamo equilibrates by extracting energy from the differential rotation.
- Small scale field does most of the damage!



Outstanding problems (1)

1. What is the role of the Lorentz force on the transport coefficients α and β?

2. What happens when the fluctuating field may exist of its own accord, independent of the mean field?

3. Hence, what is the functional dependence of α and β on the large-scale field B_0 and the magnetic Reynolds number Rm? e.g.

$$\alpha = \frac{\alpha_0}{1 + B_0^2 / U^2} \quad \text{or} \quad \alpha = \frac{\alpha_0}{1 + Rm^\gamma B_0^2 / U^2}$$

Outstanding problems (2)

4. What is the precise role of the tachocline in the solar dynamo? Is the α-effect spatially separated from the ω-effect?

5. How do dynamos operate in stars without tachoclines – such as fully convective stars?