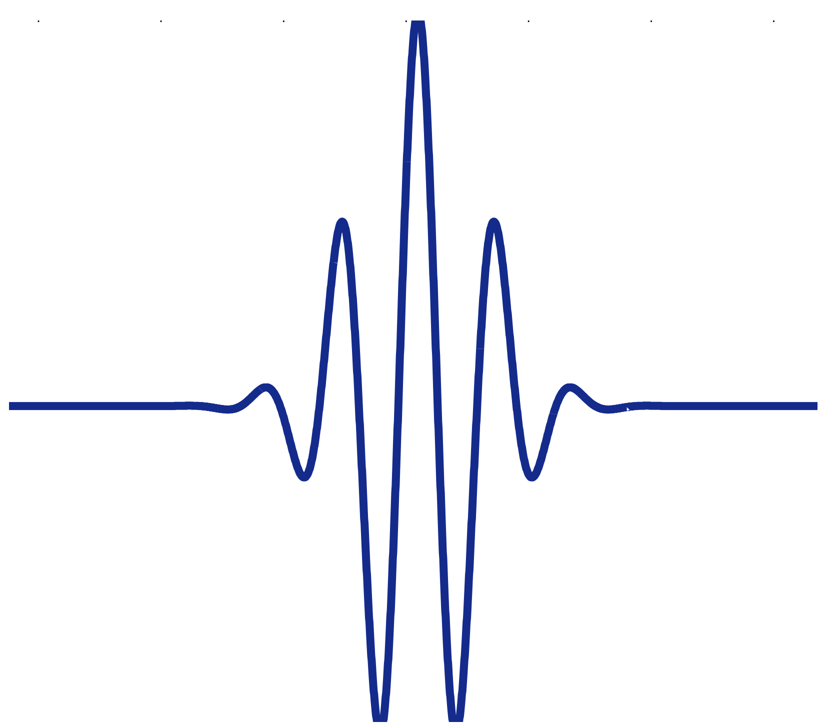
A multiscale approach to the analysis of anisotropic plasma turbulence

(Or studying turbulence wearing wavelet spectacles)

Khurom Kiyani

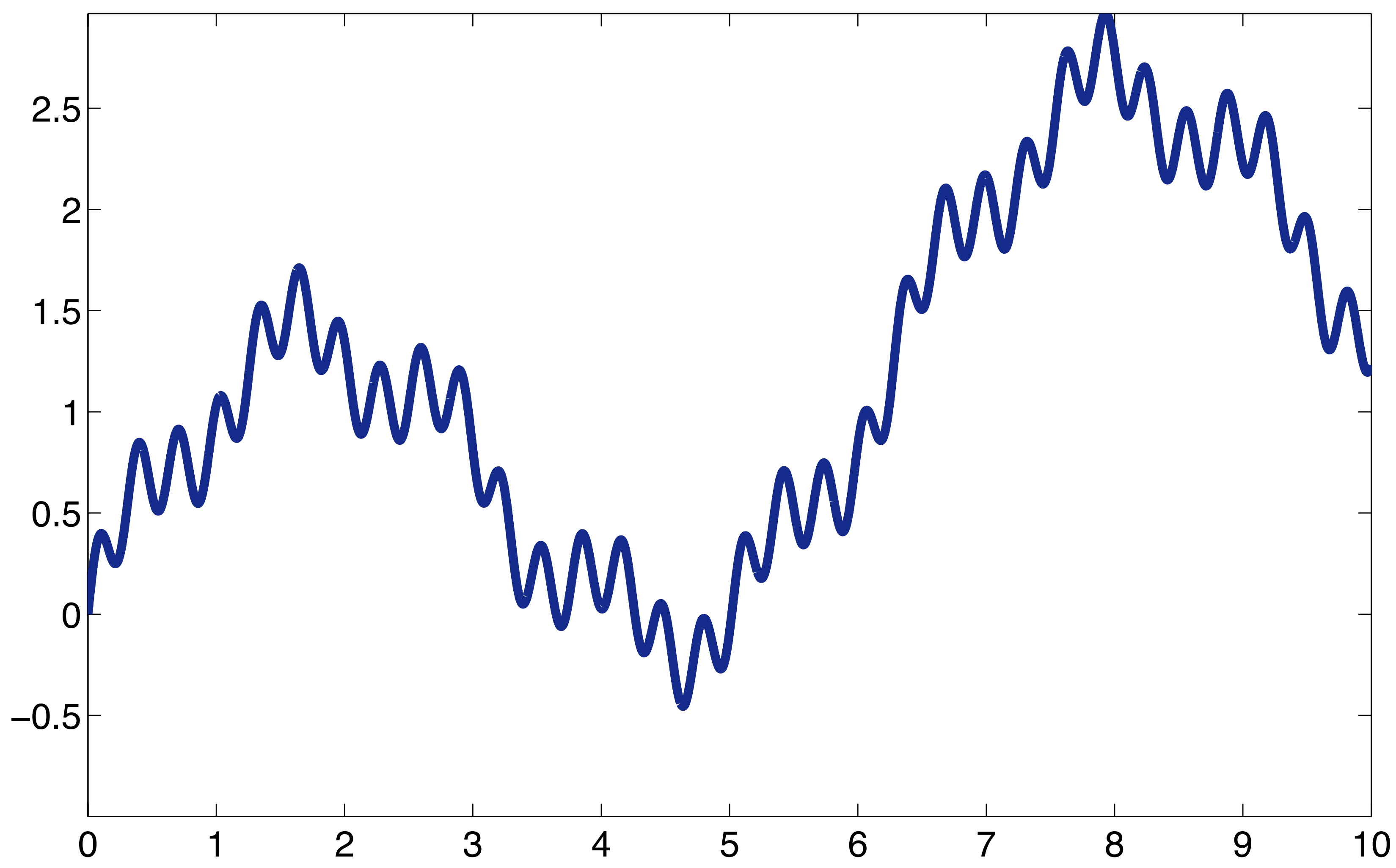


THE UNIVERSITY OF WARWICK

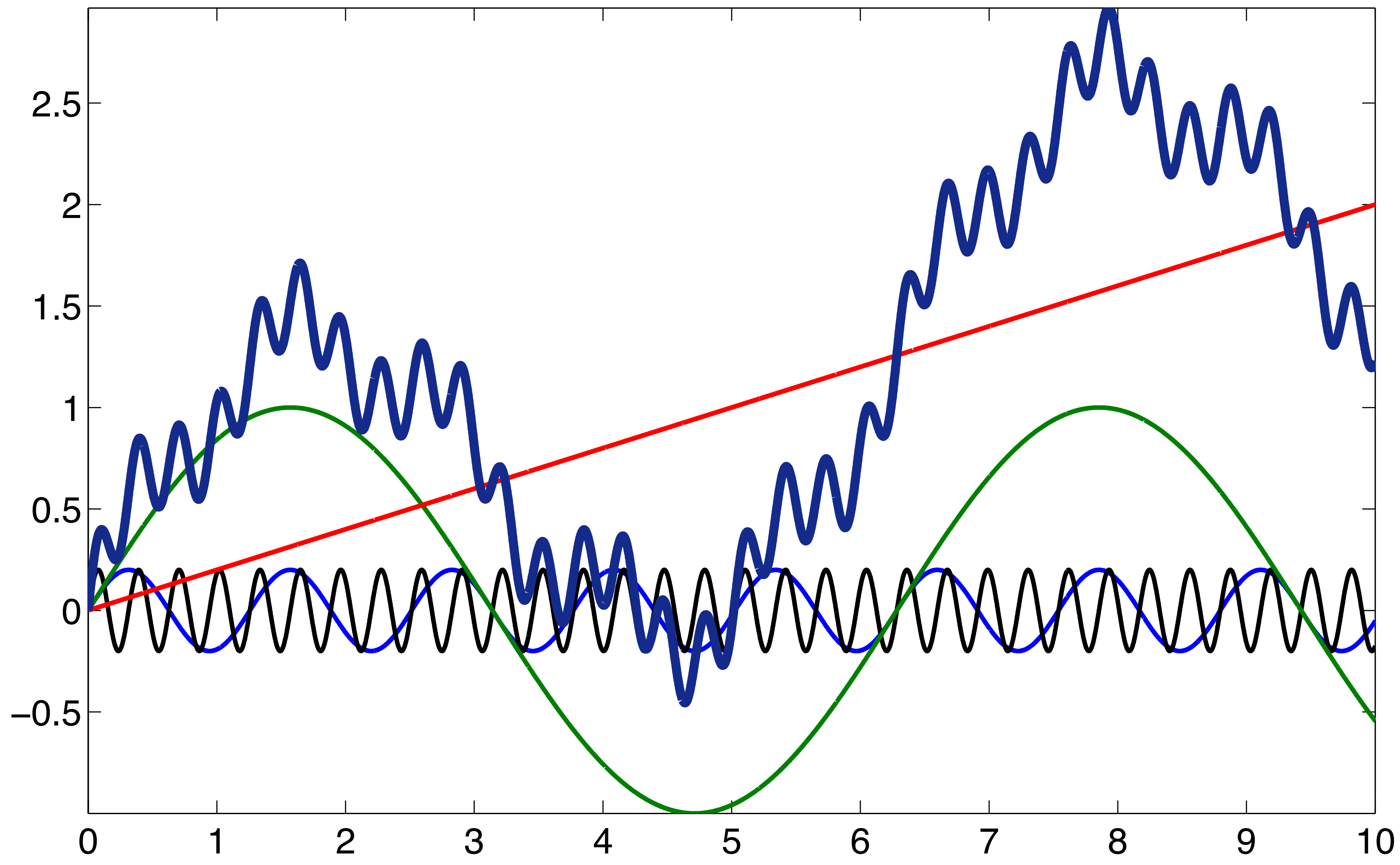
the aim

to compute theoretically relevant statistical quantities from a turbulent (stochastic) time-series

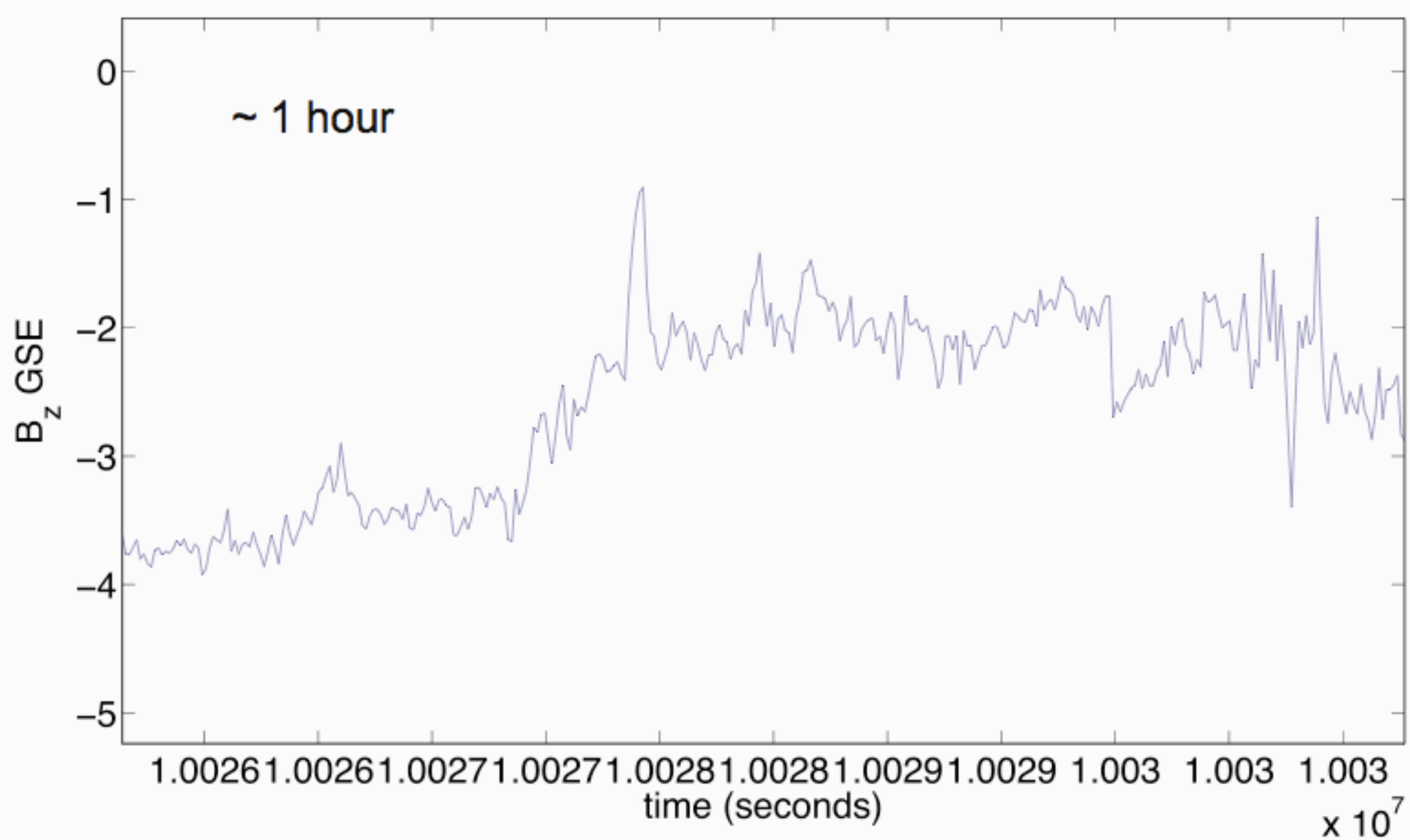
background guide field



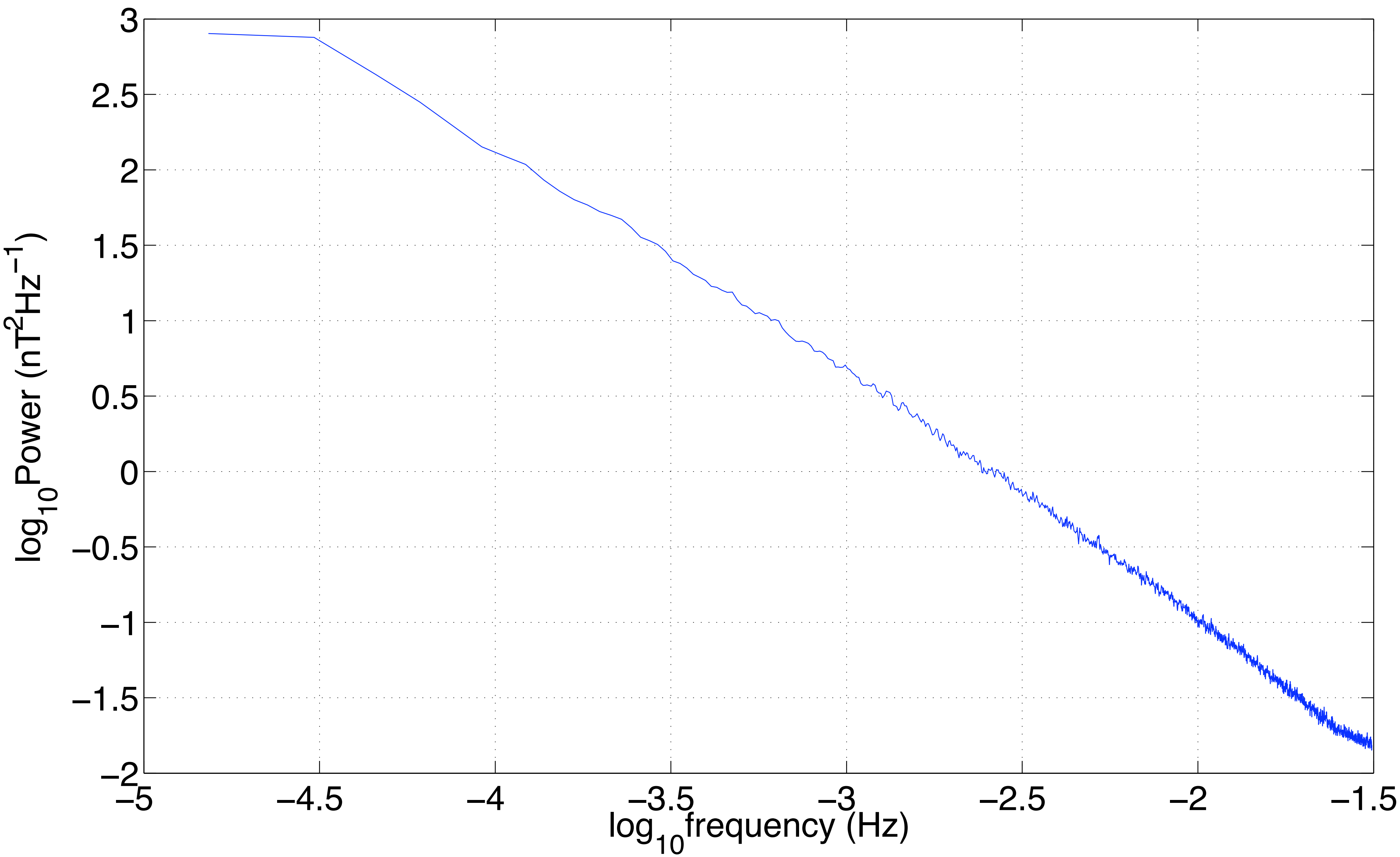
background guide field



background guide field

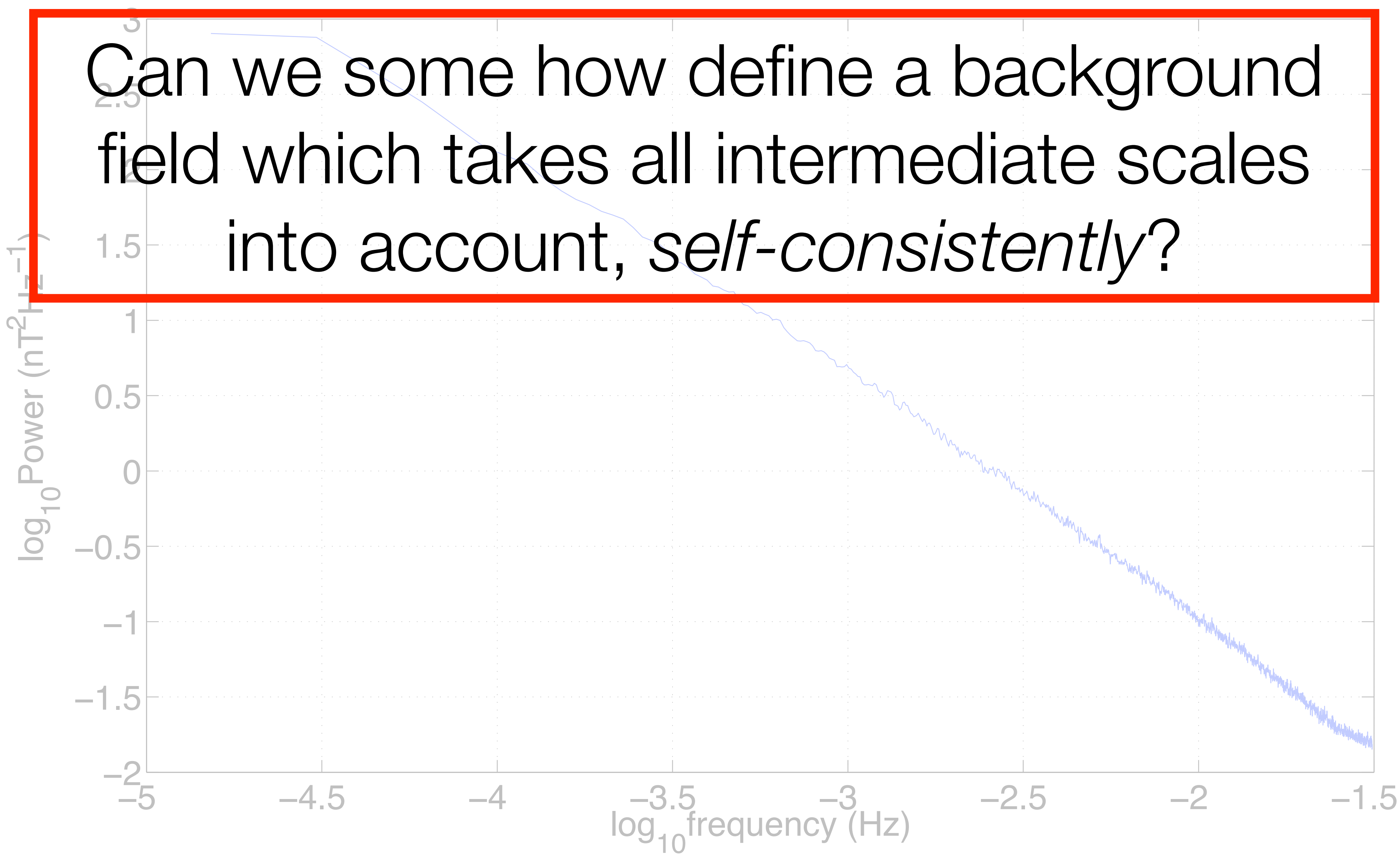


background guide field



question 1

Can we some how define a background field which takes all intermediate scales into account, *self-consistently*?



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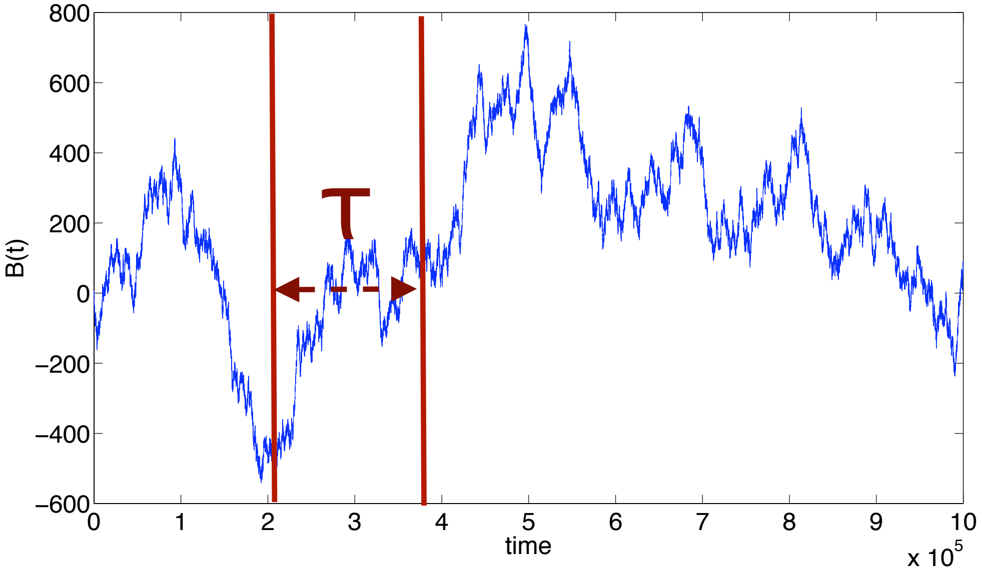
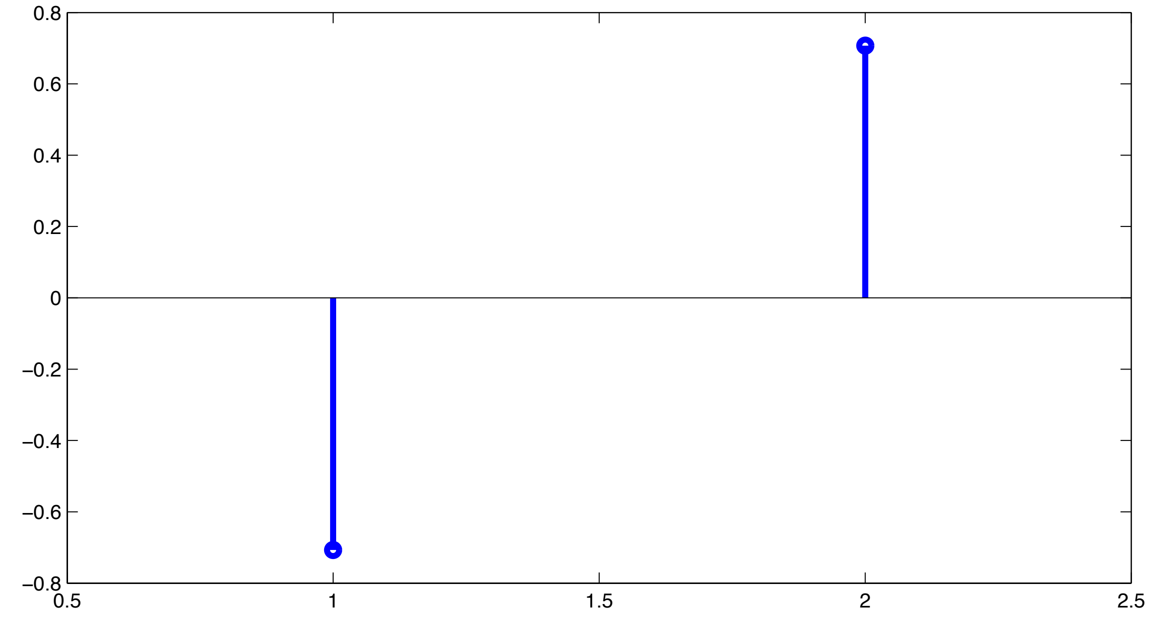
extreme time locality (Haar filter)



increments

$$y(t, \tau) = x(t + \tau) - x(t)$$



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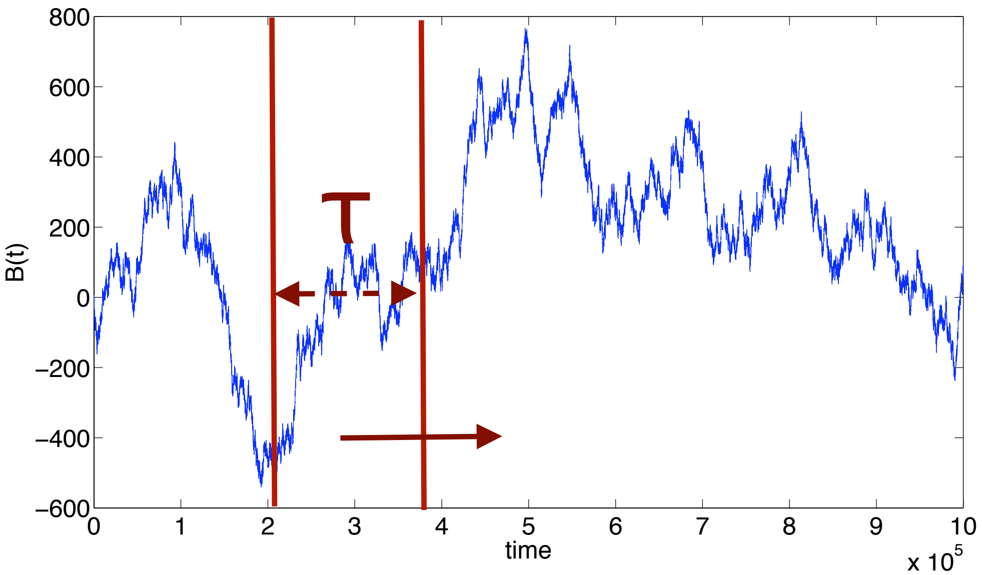

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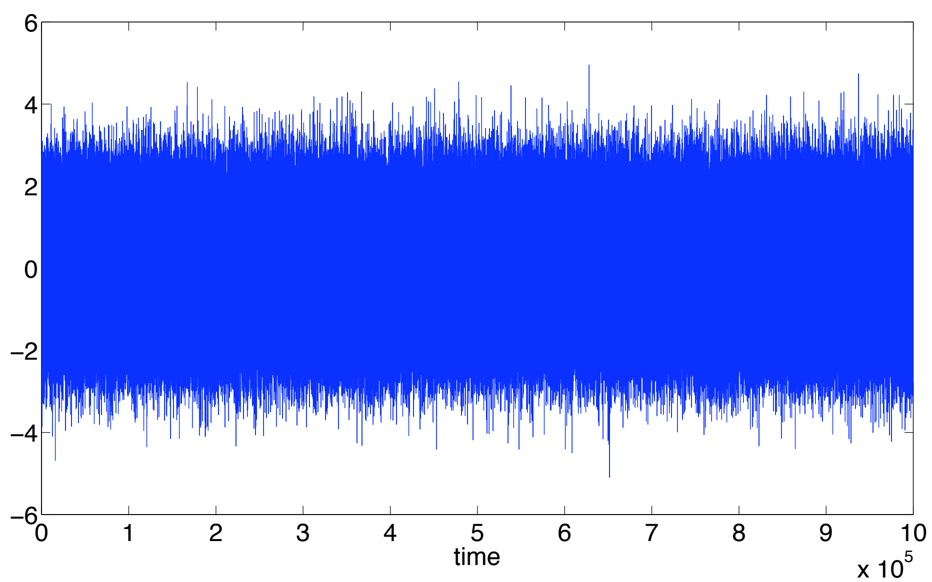
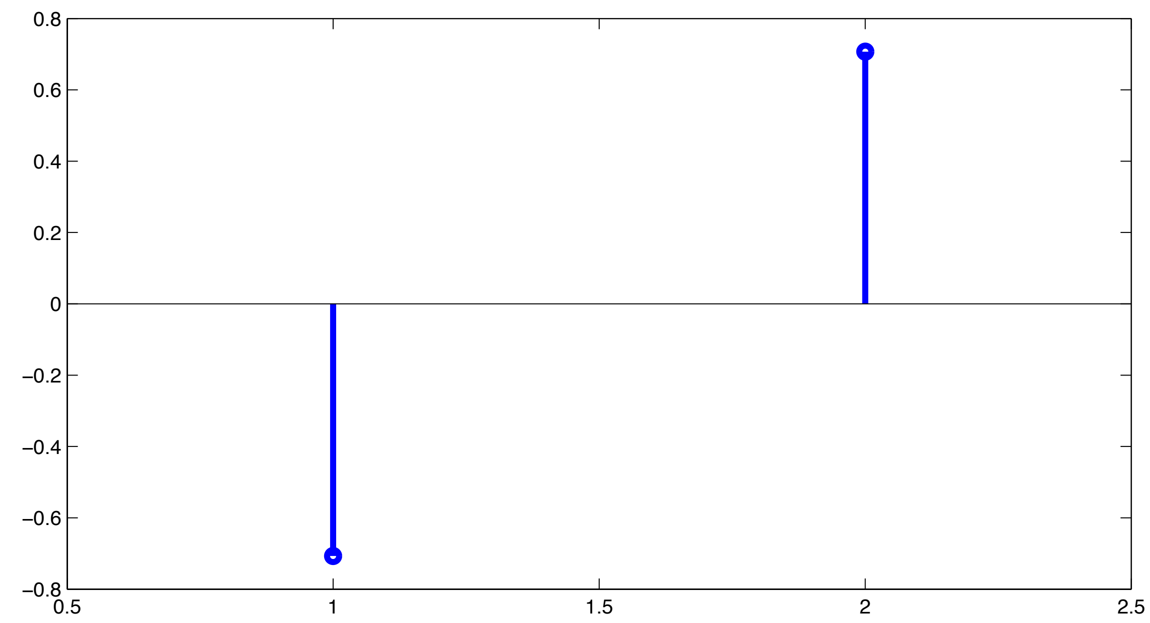




question 2

Can we obtain the best of both worlds and live 'well' in the time <u>and</u> frequency domains?

time (space) domain

better reference to detailed physics and structure

Fourier domain

much theory; normally coarse-grained picture

question 2

Can we obtain the best of both worlds and live ‘well’ in the time <u>and</u> frequency domains?

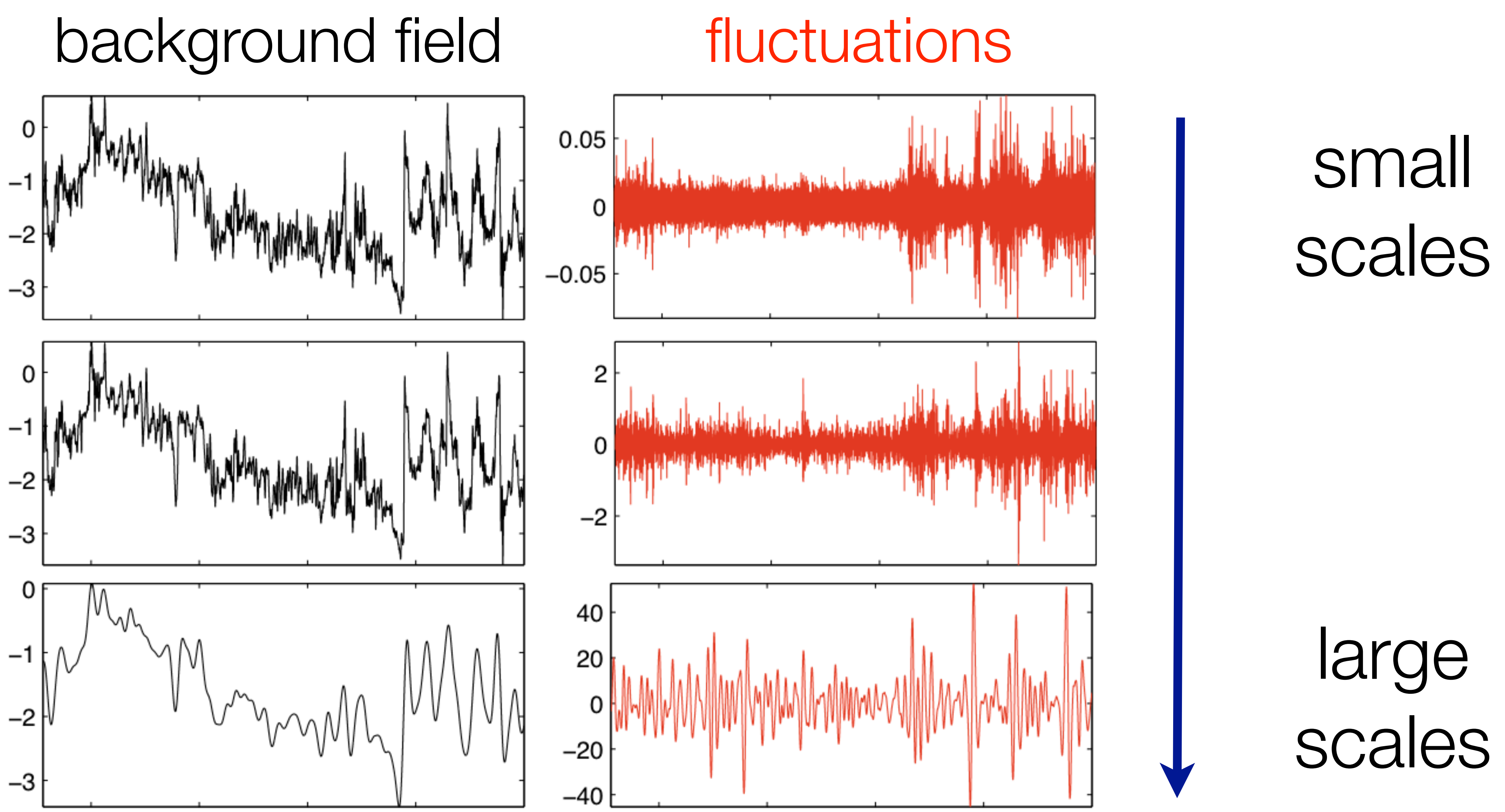
time domain

Fourier domain

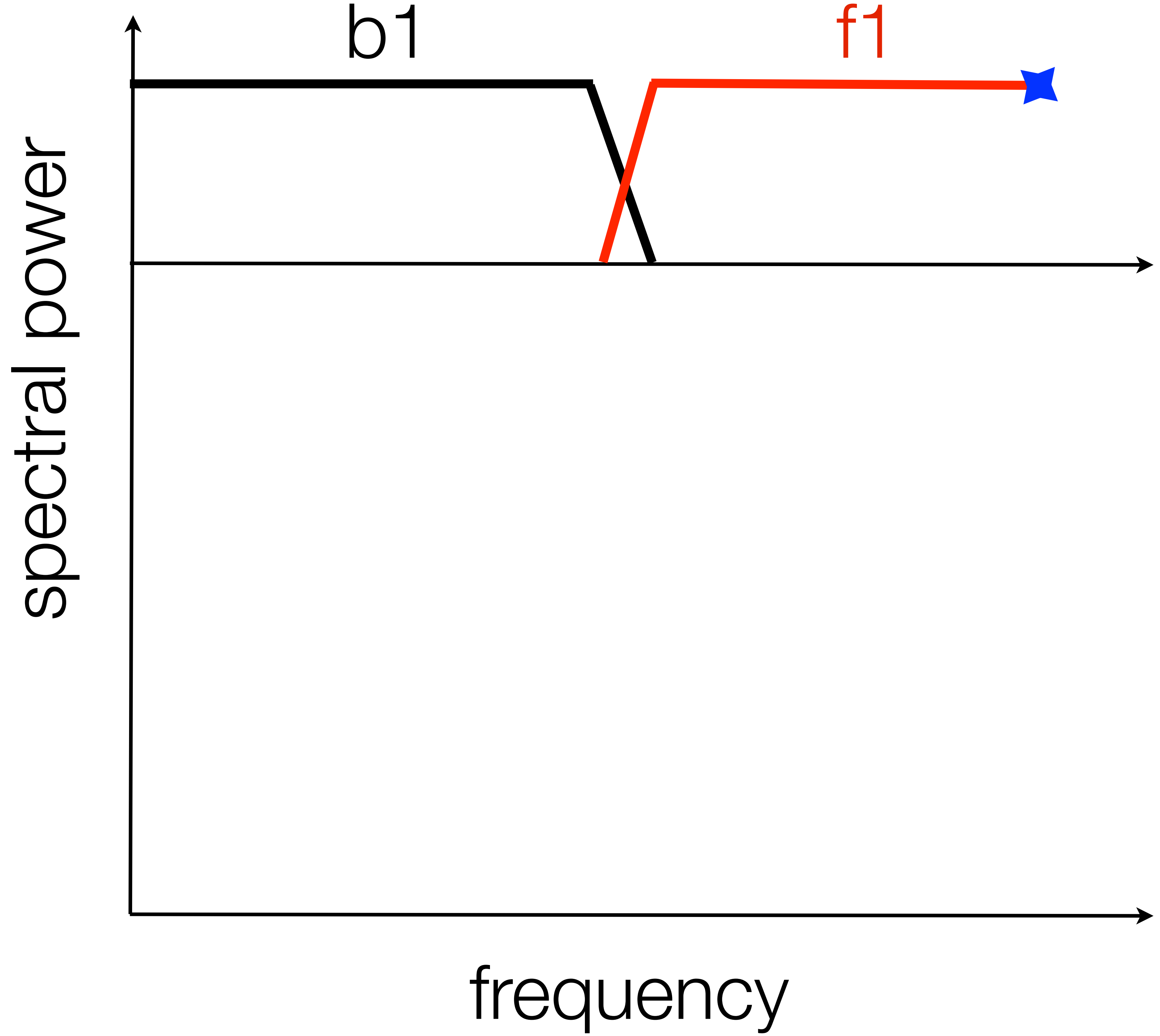
outline

- Discrete wavelets as band-pass digital filters
- Picking and designing an appropriate wavelet/transform
- Some results on separating anisotropic signatures in plasma turbulence in the solar wind

adaptive local scale-dependent fluctuations **and** background guide field



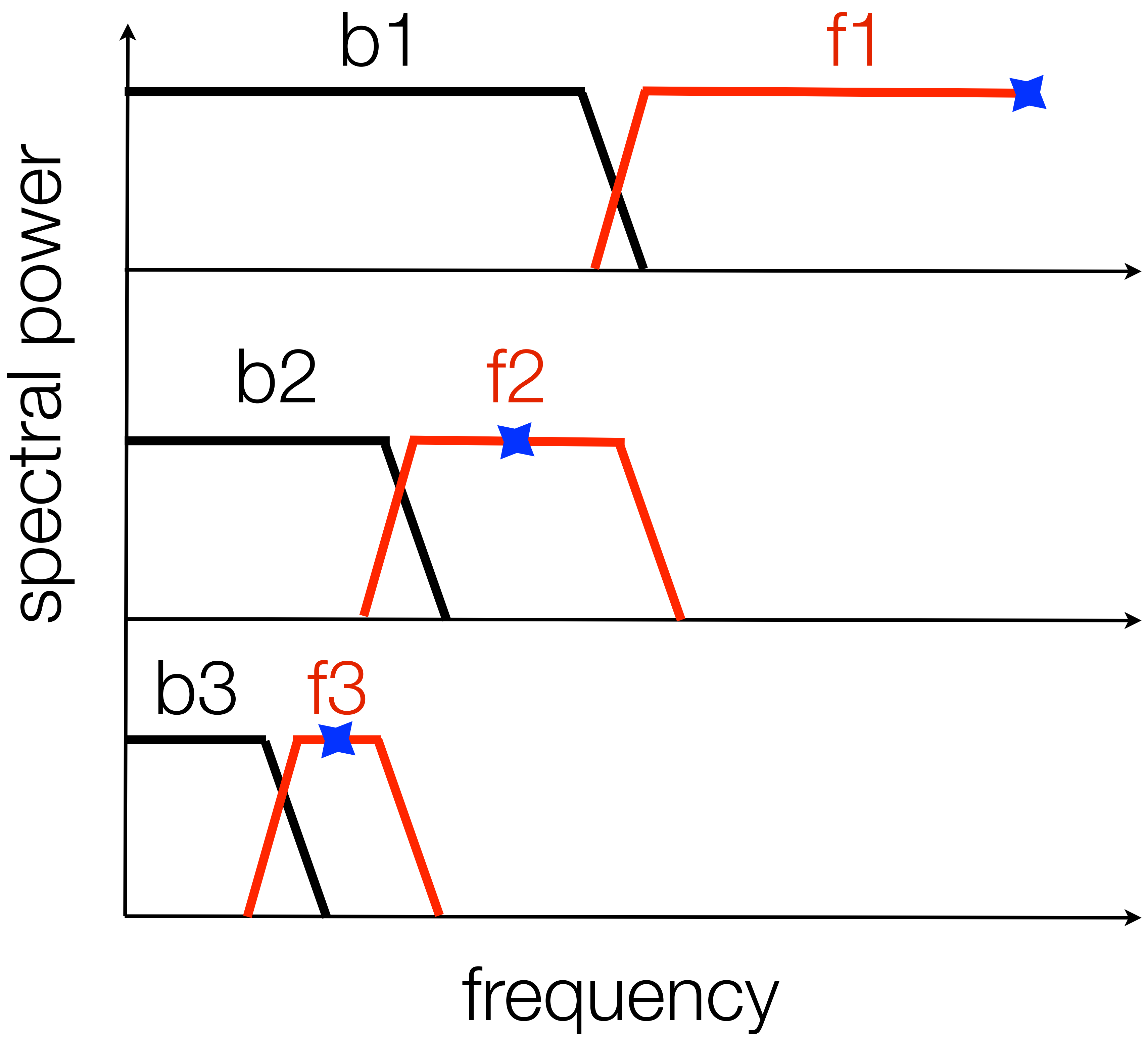
local scale-dependent fluctuations *and* background field using low/high pass filters



fluctuations

background field

local scale-dependent fluctuations *and* background field using low/high pass filters

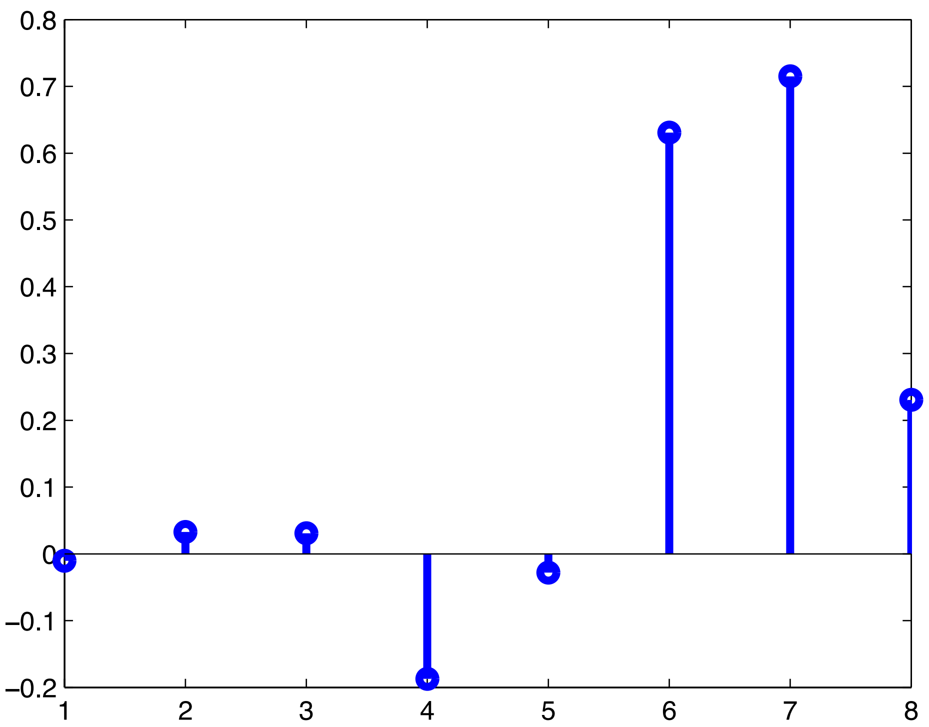


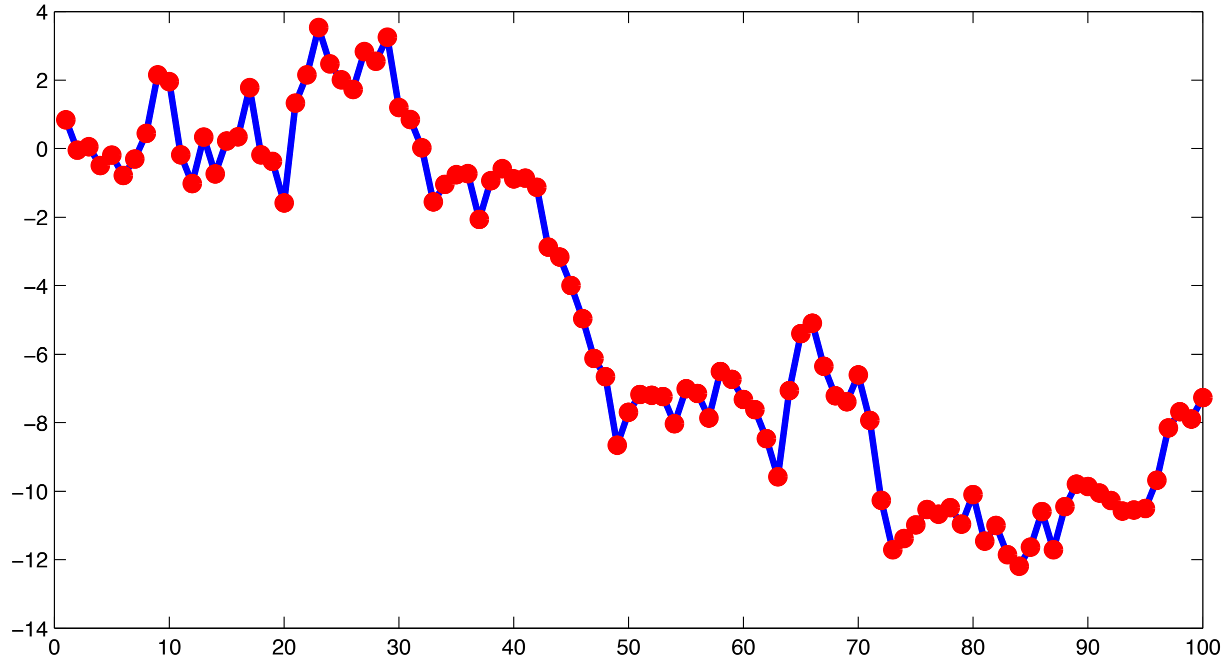
fluctuations

background field

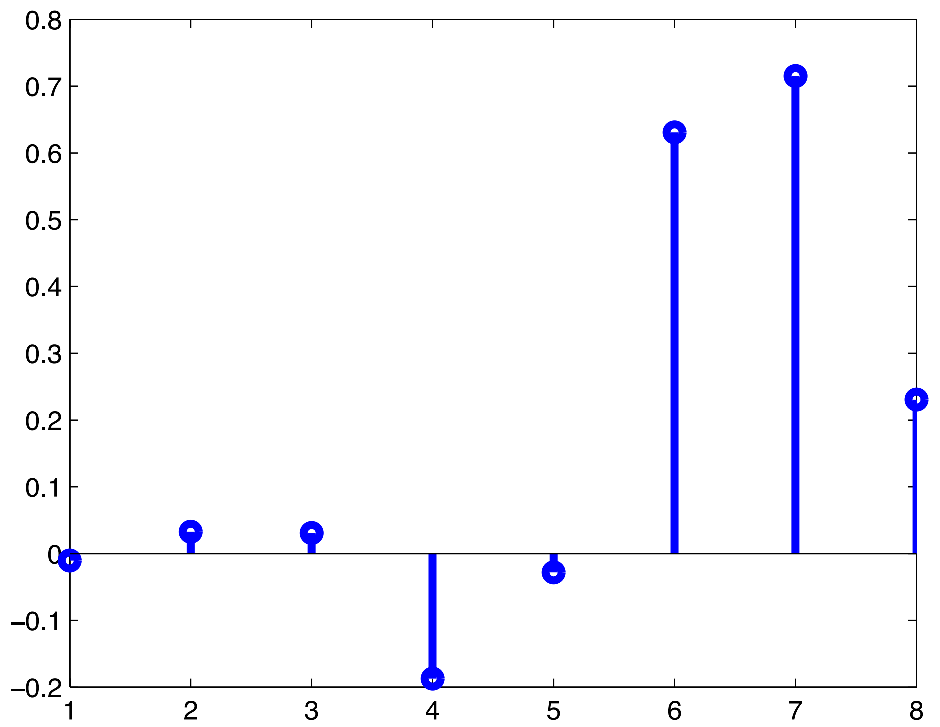
filter bank implementation using quadrature mirror filters

discrete wavelet transform (dwt)





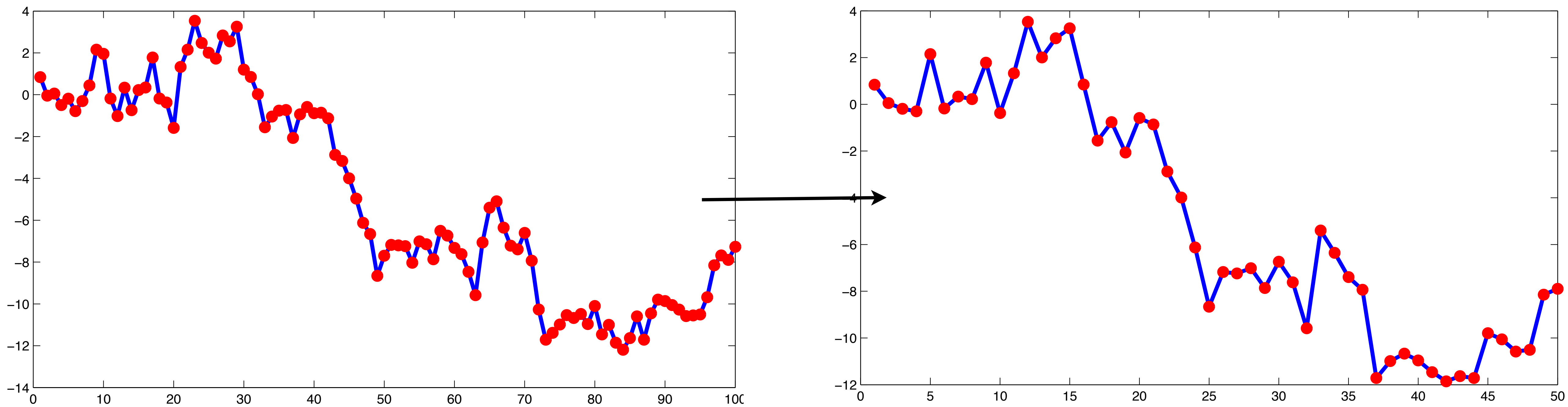
discrete wavelet transform (dwt)



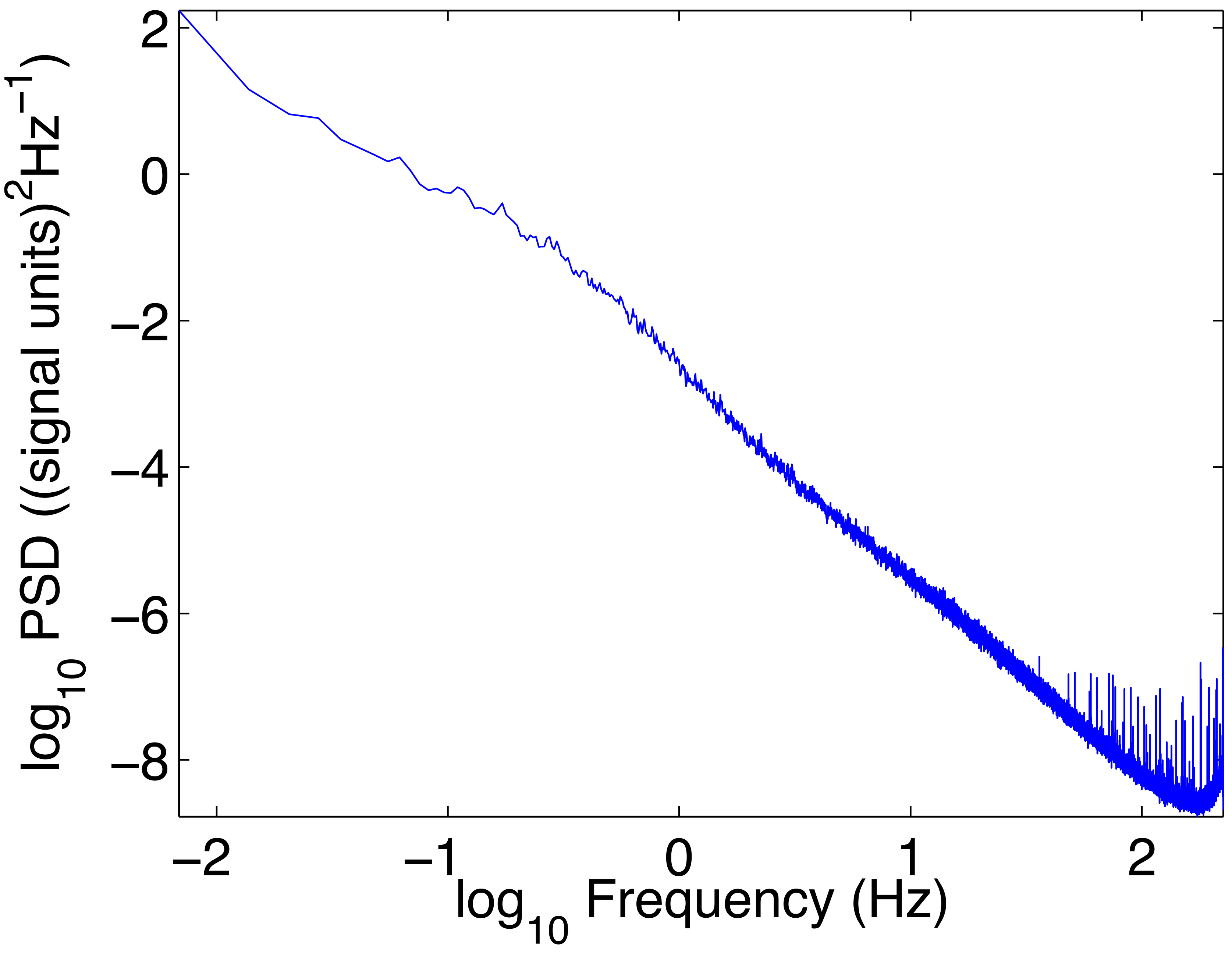
<u>DWT steps:</u>

Leave filter as it is

Decimate signal by 2



let's see it working



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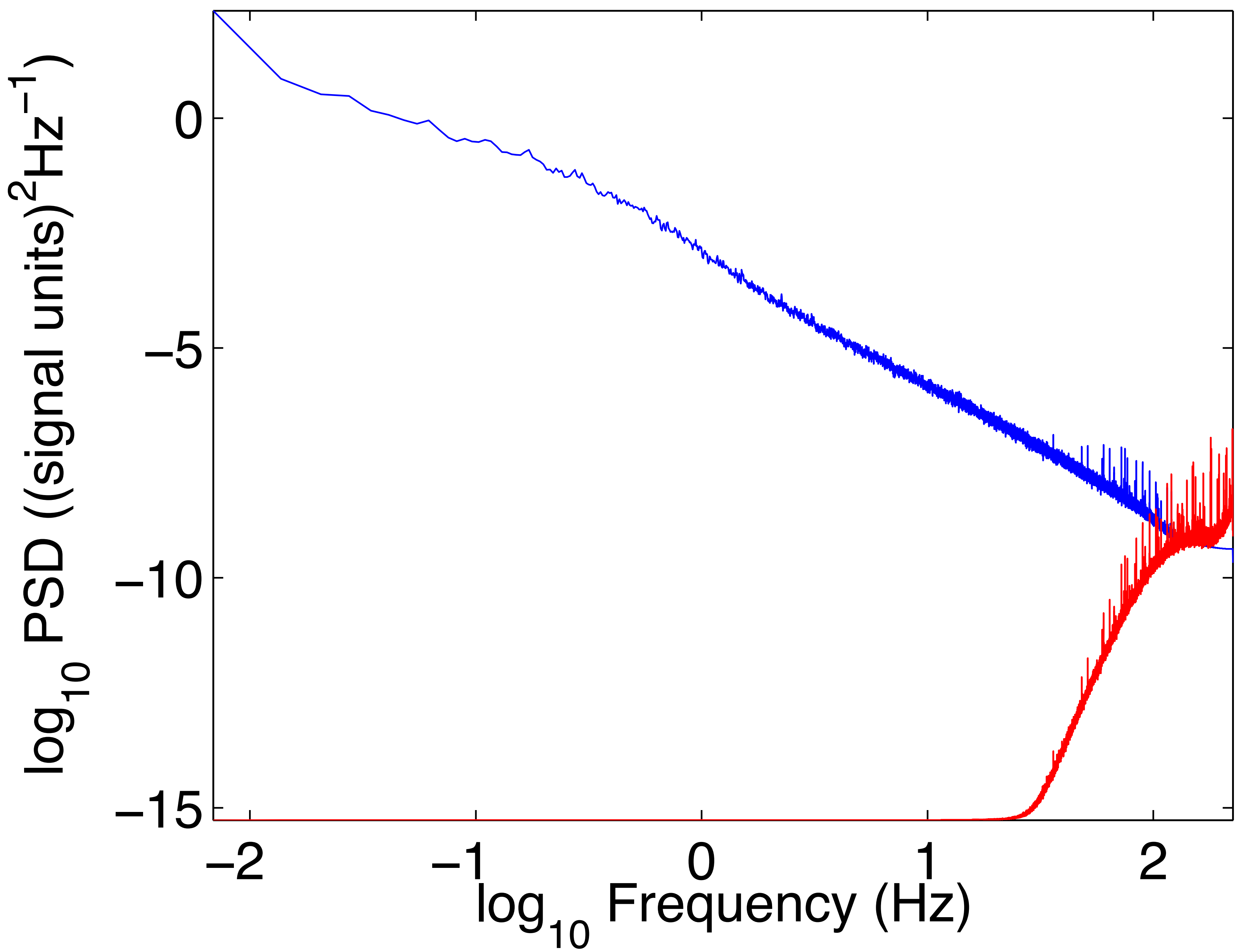

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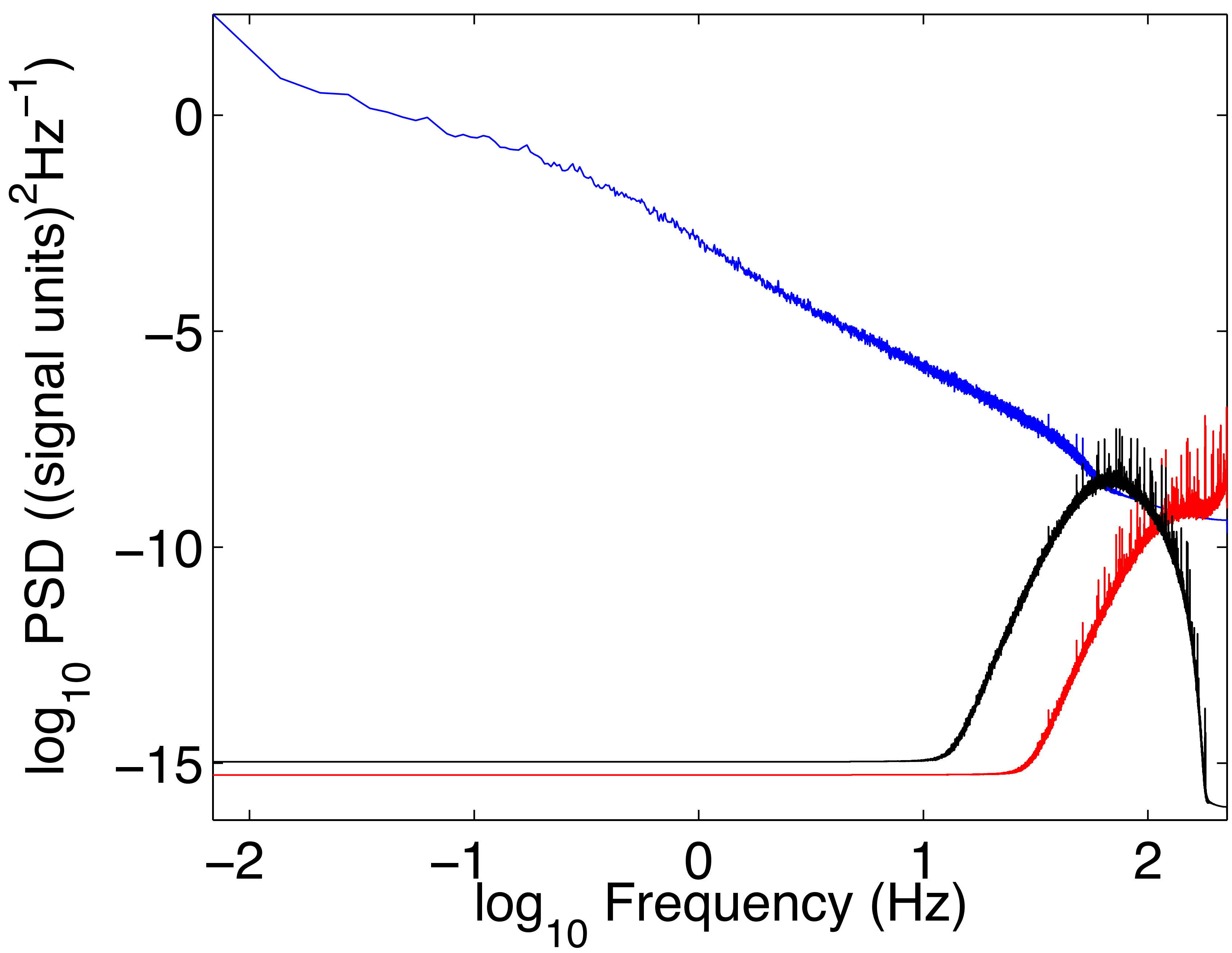

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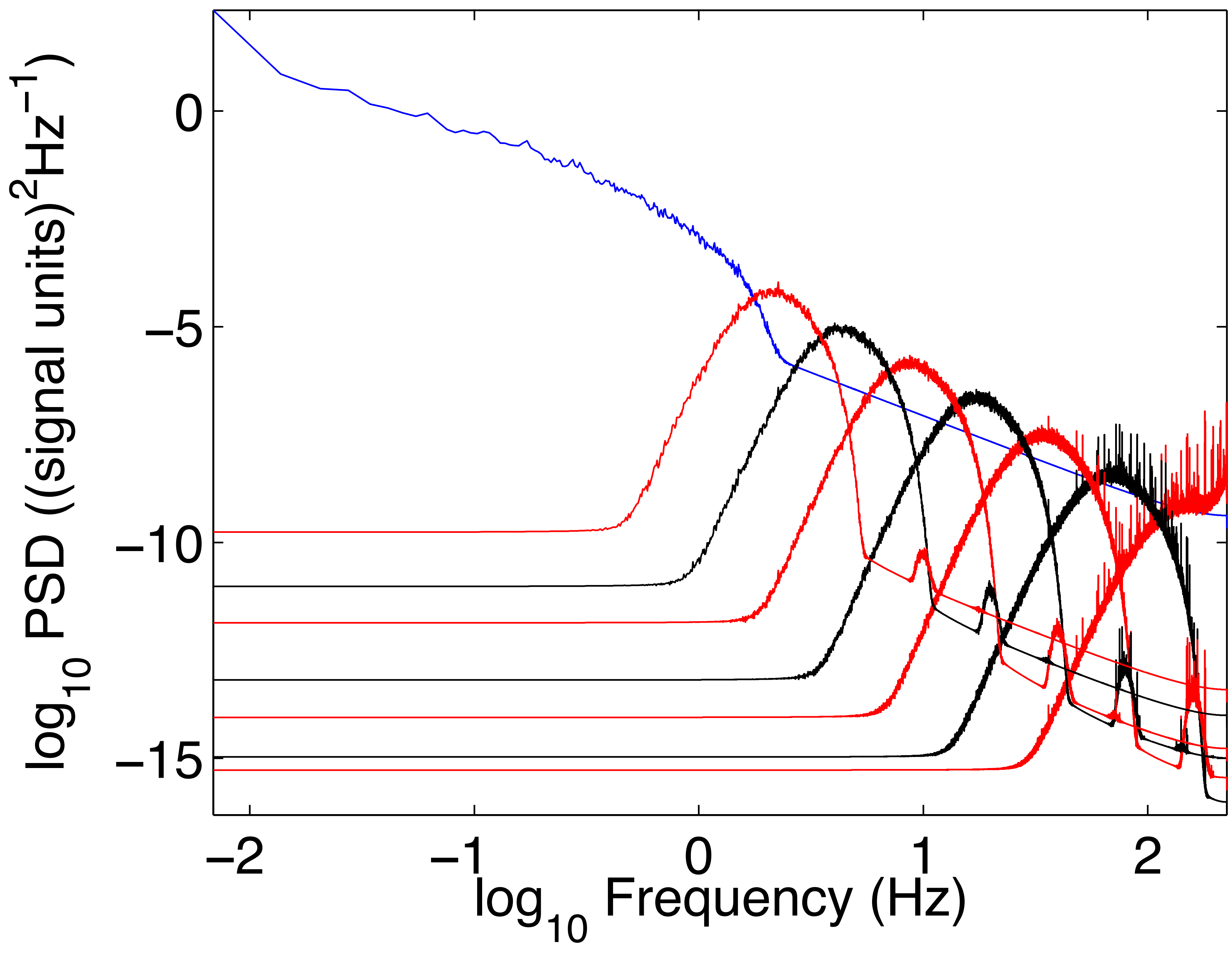
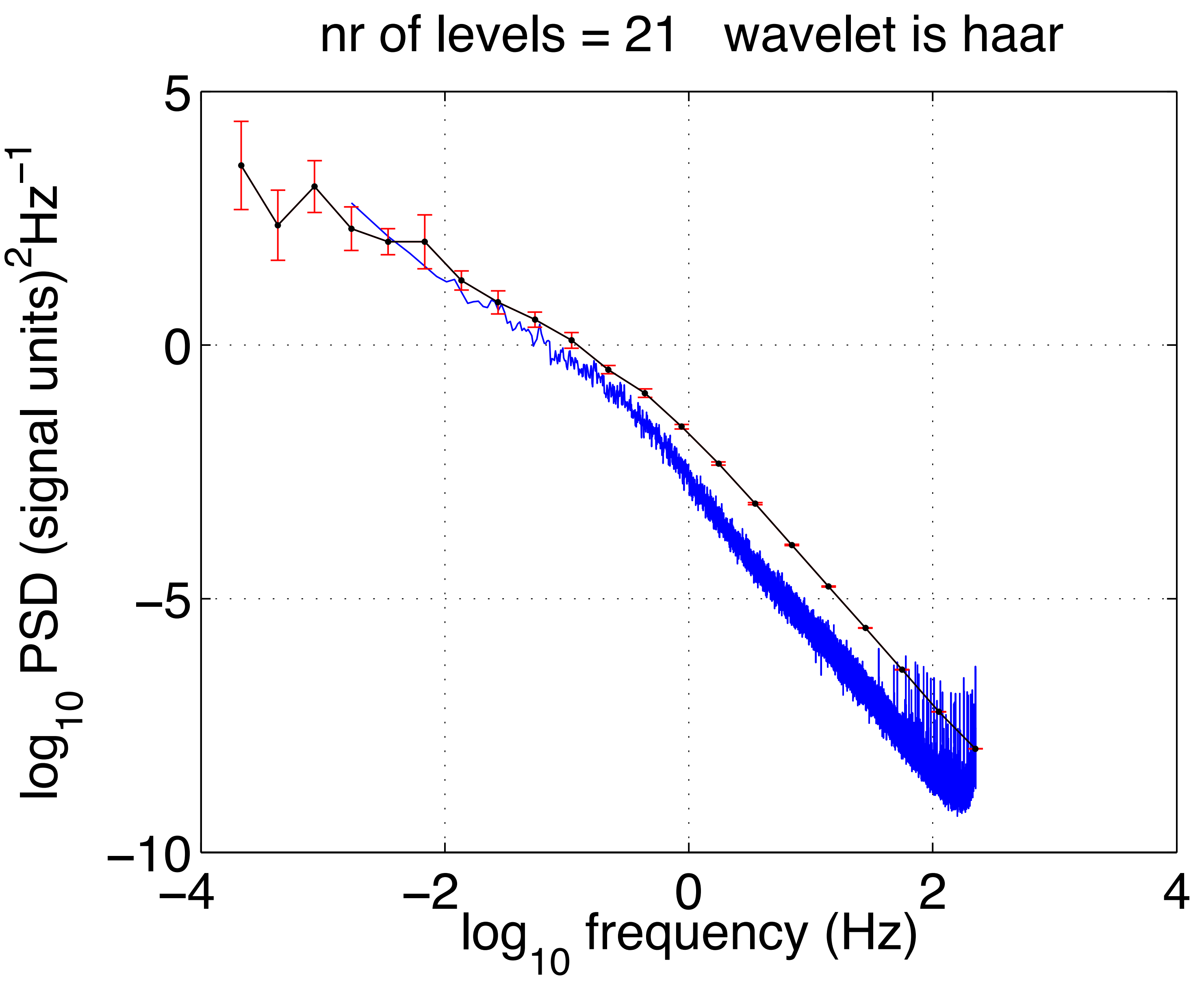
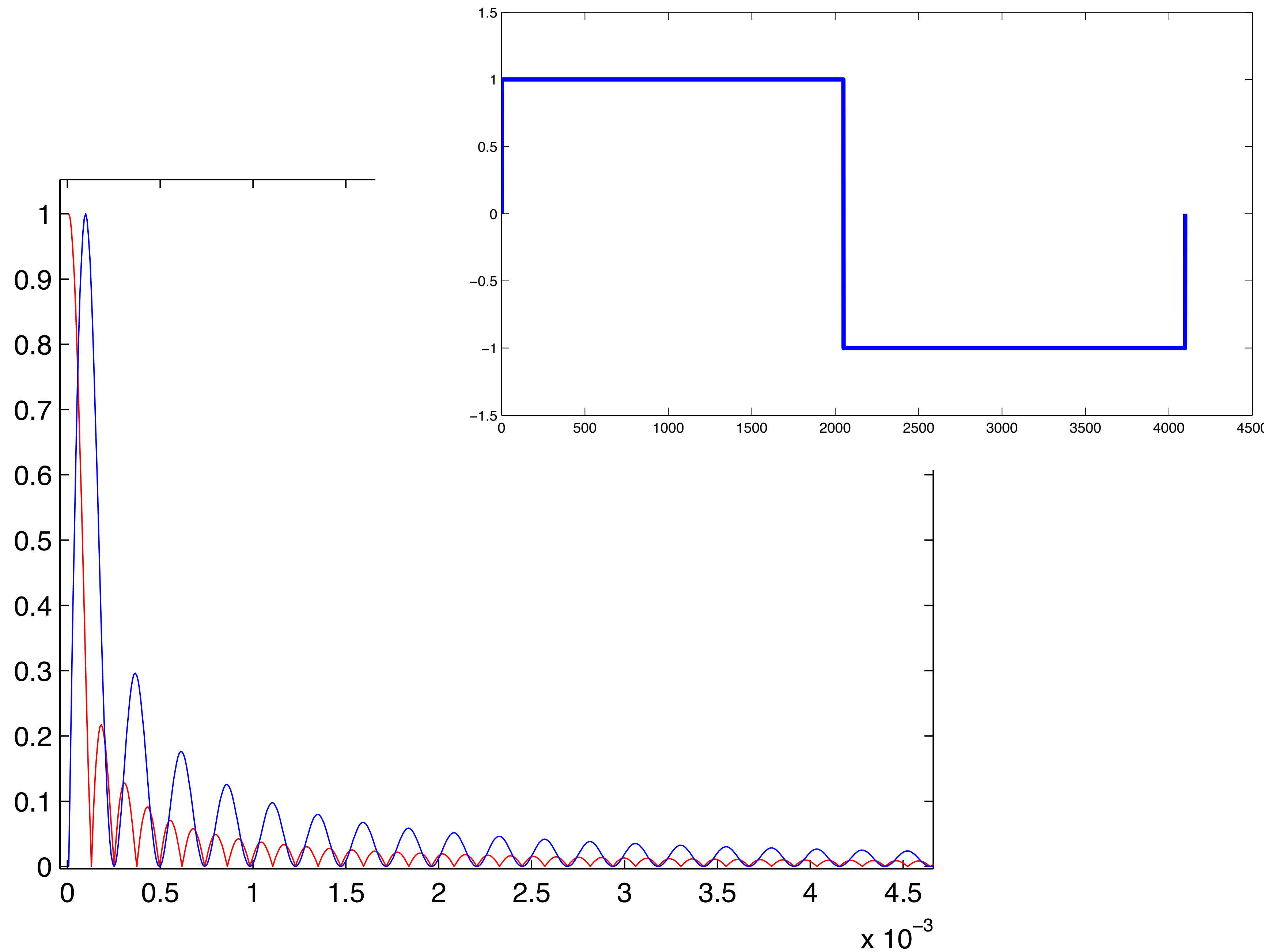


importance of zero moments



importance of zero moments



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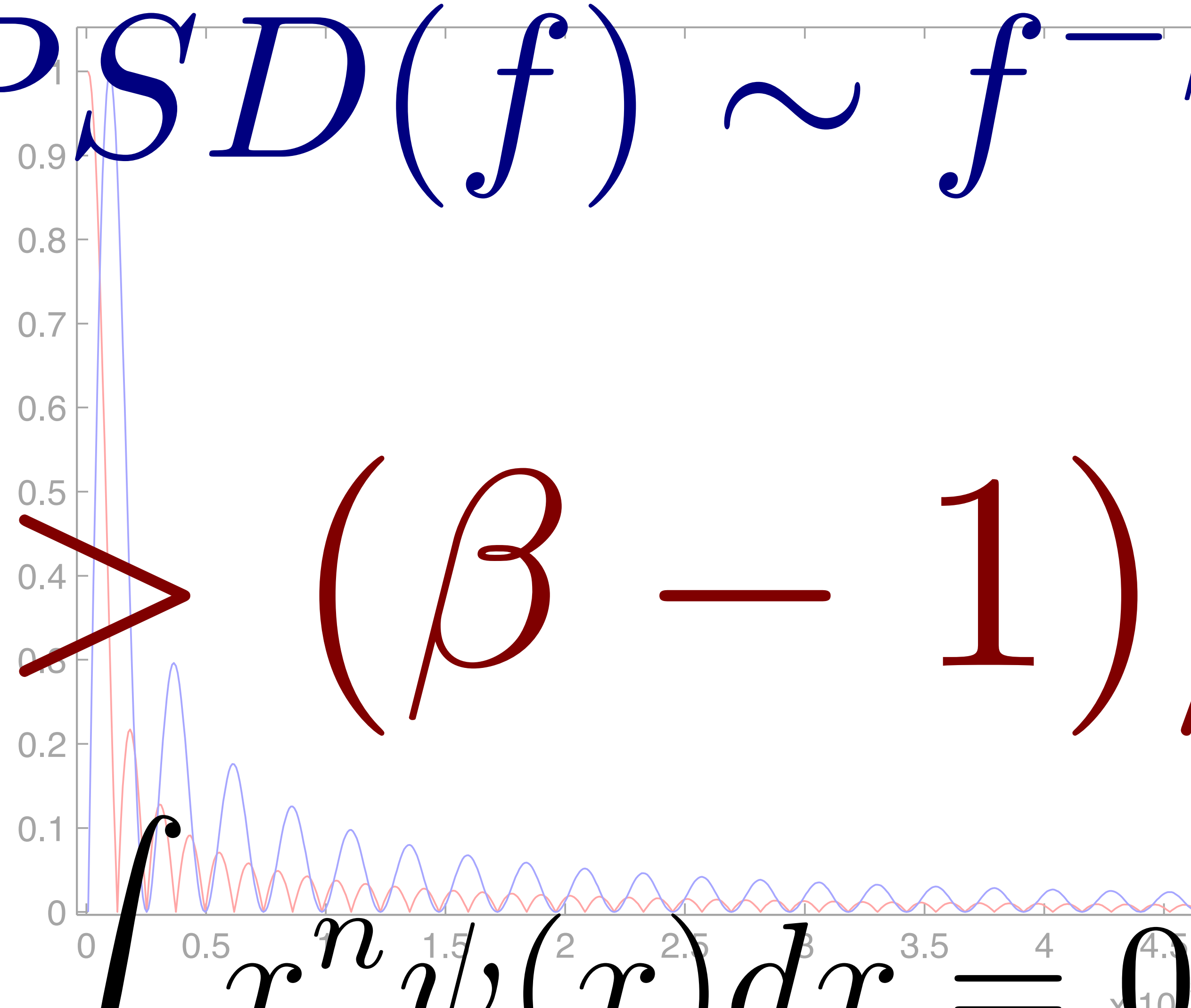
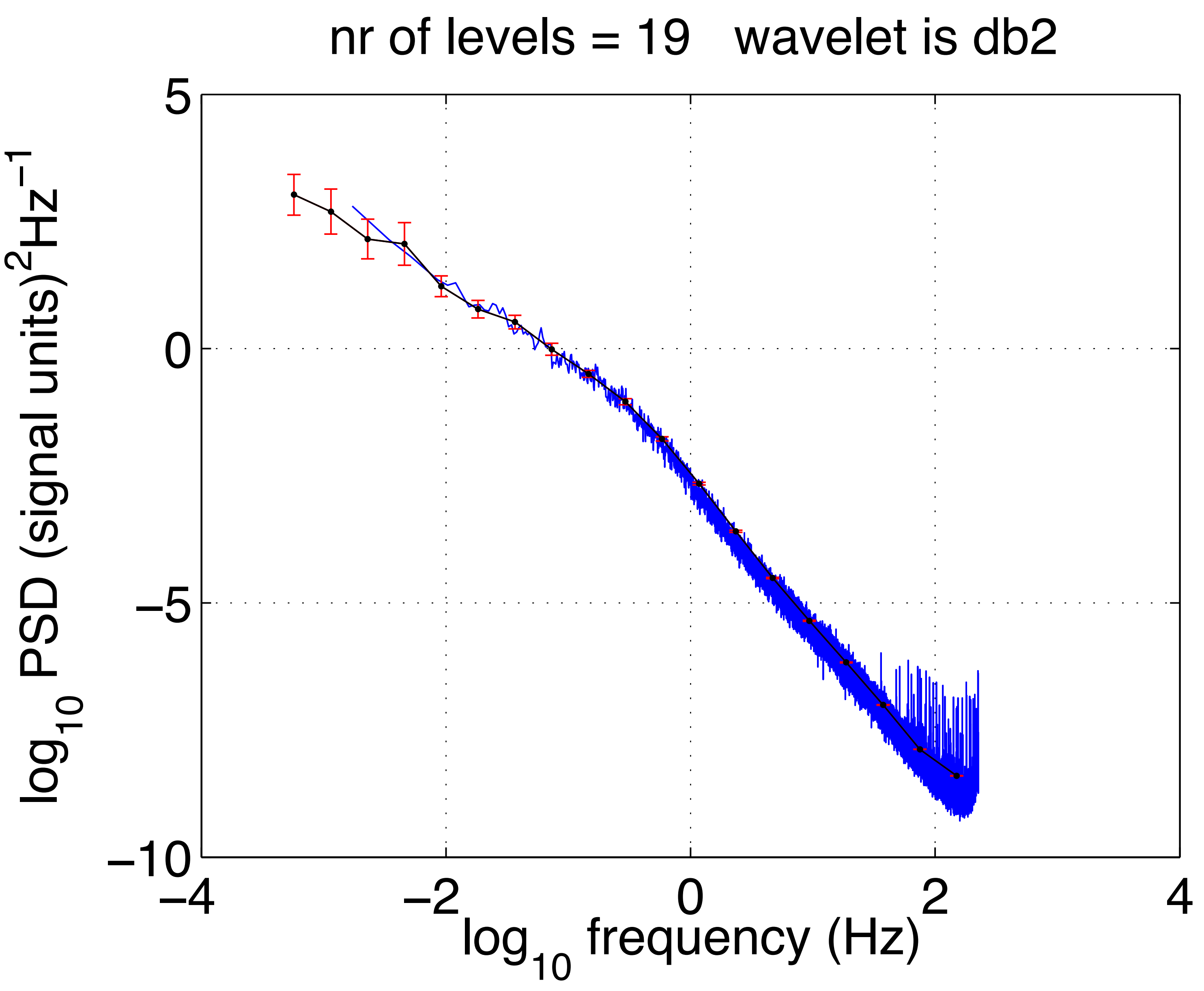


$$PSD(f) \sim f^{-\beta}$$

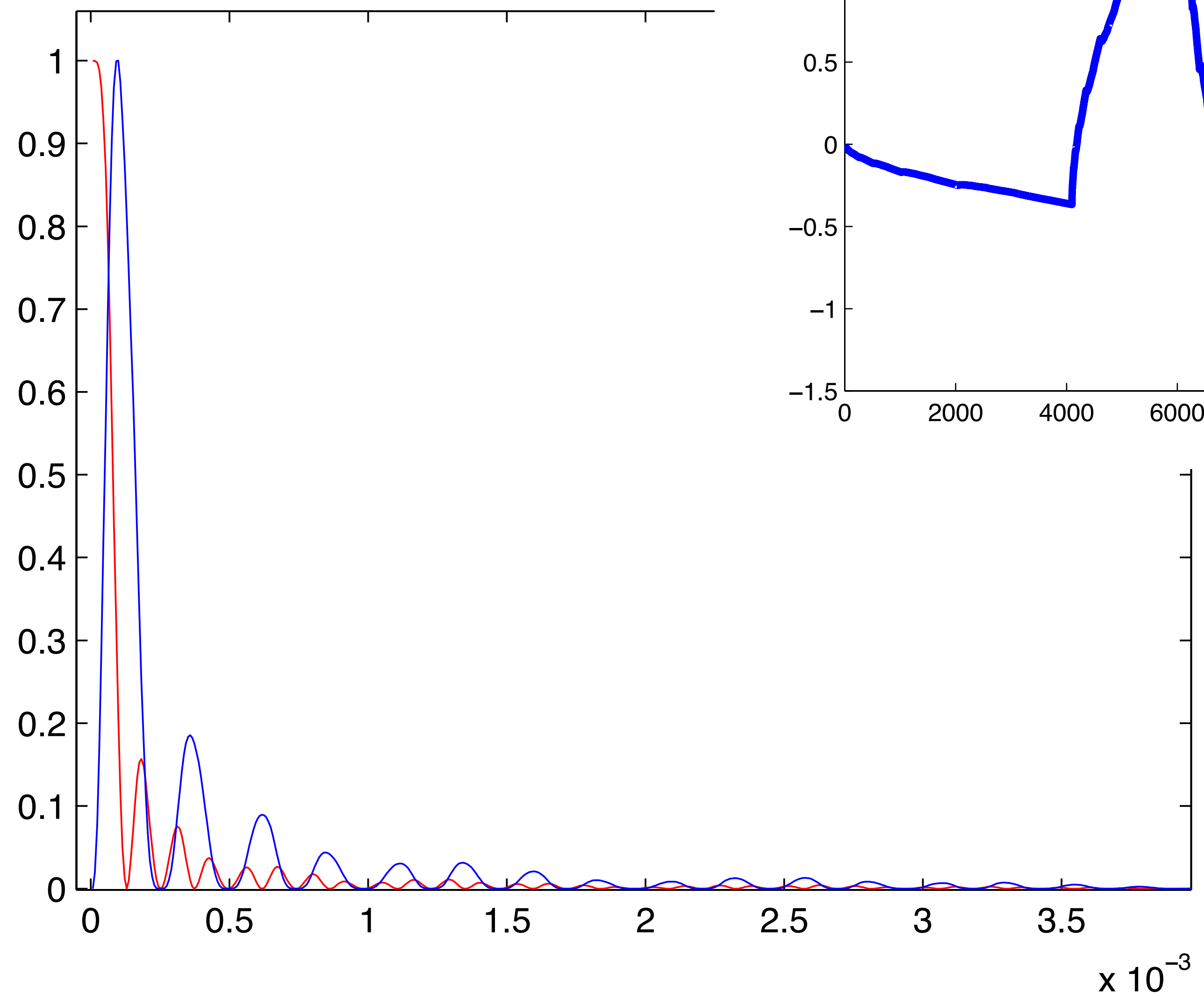
$$n > (\beta - 1)/2$$

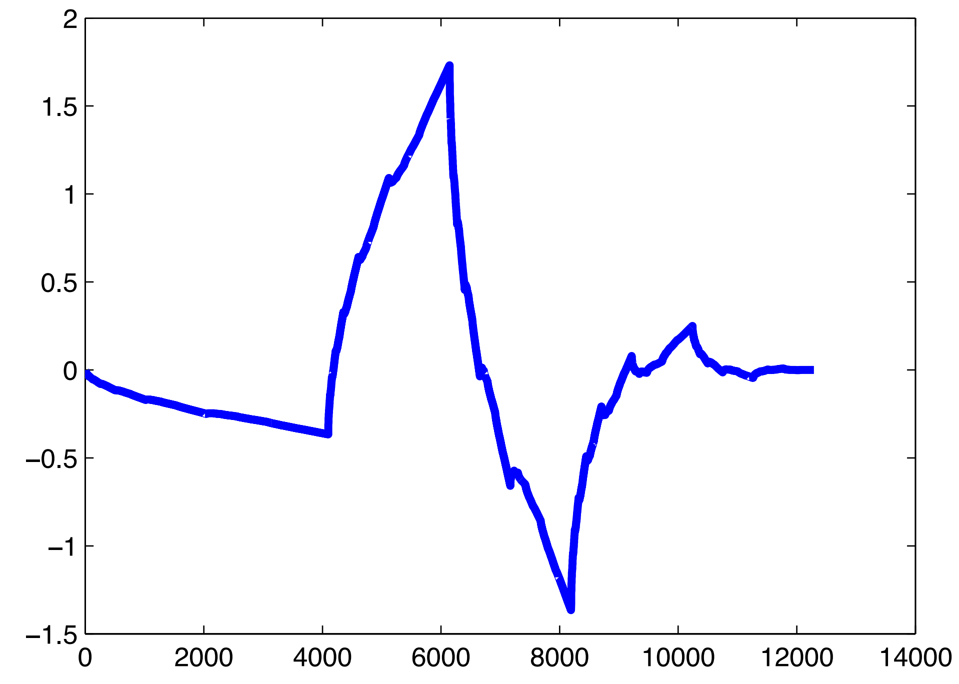
$$\int x^n \psi(x)\,dx = 0$$

importance of zero moments



importance of zero moments





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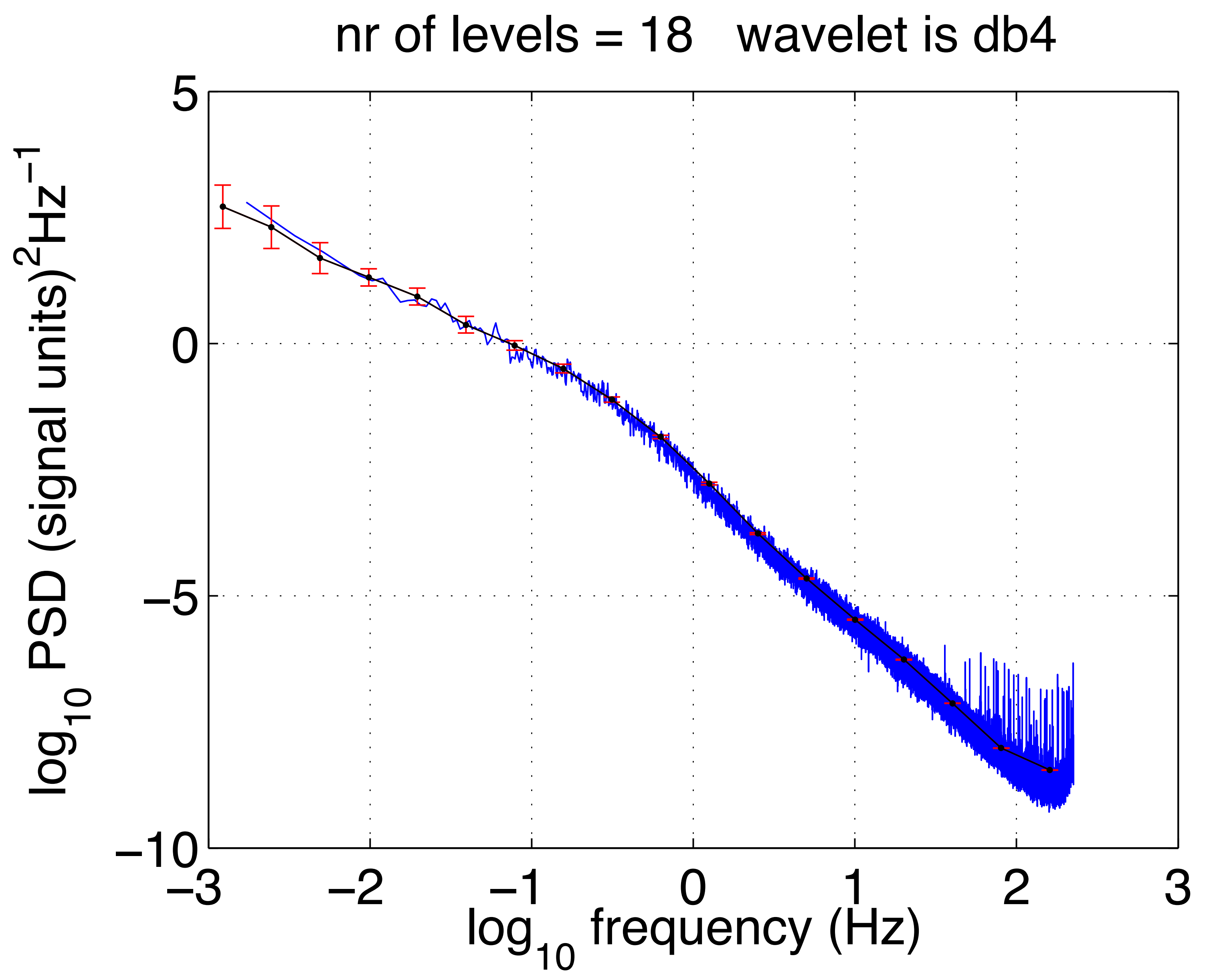

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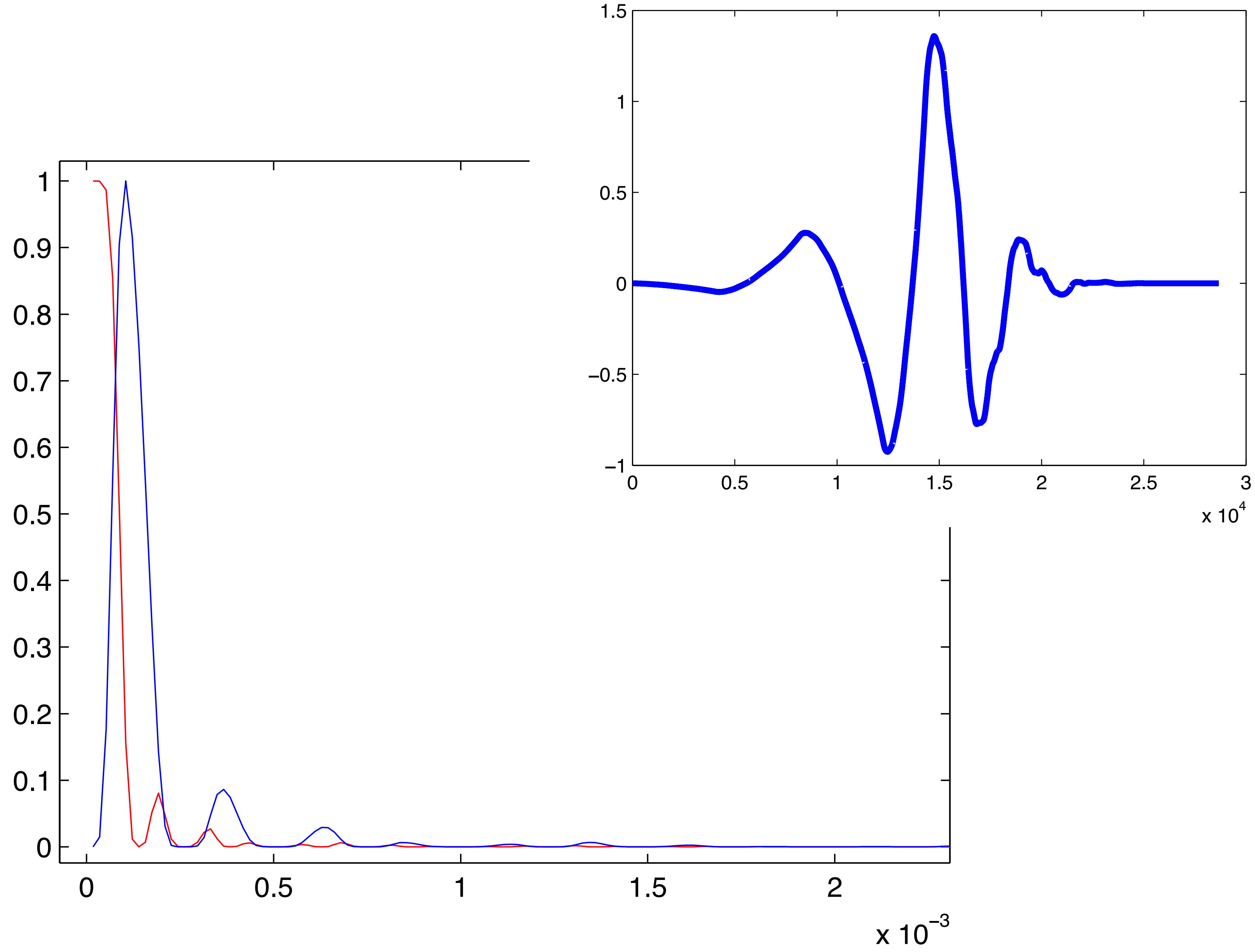

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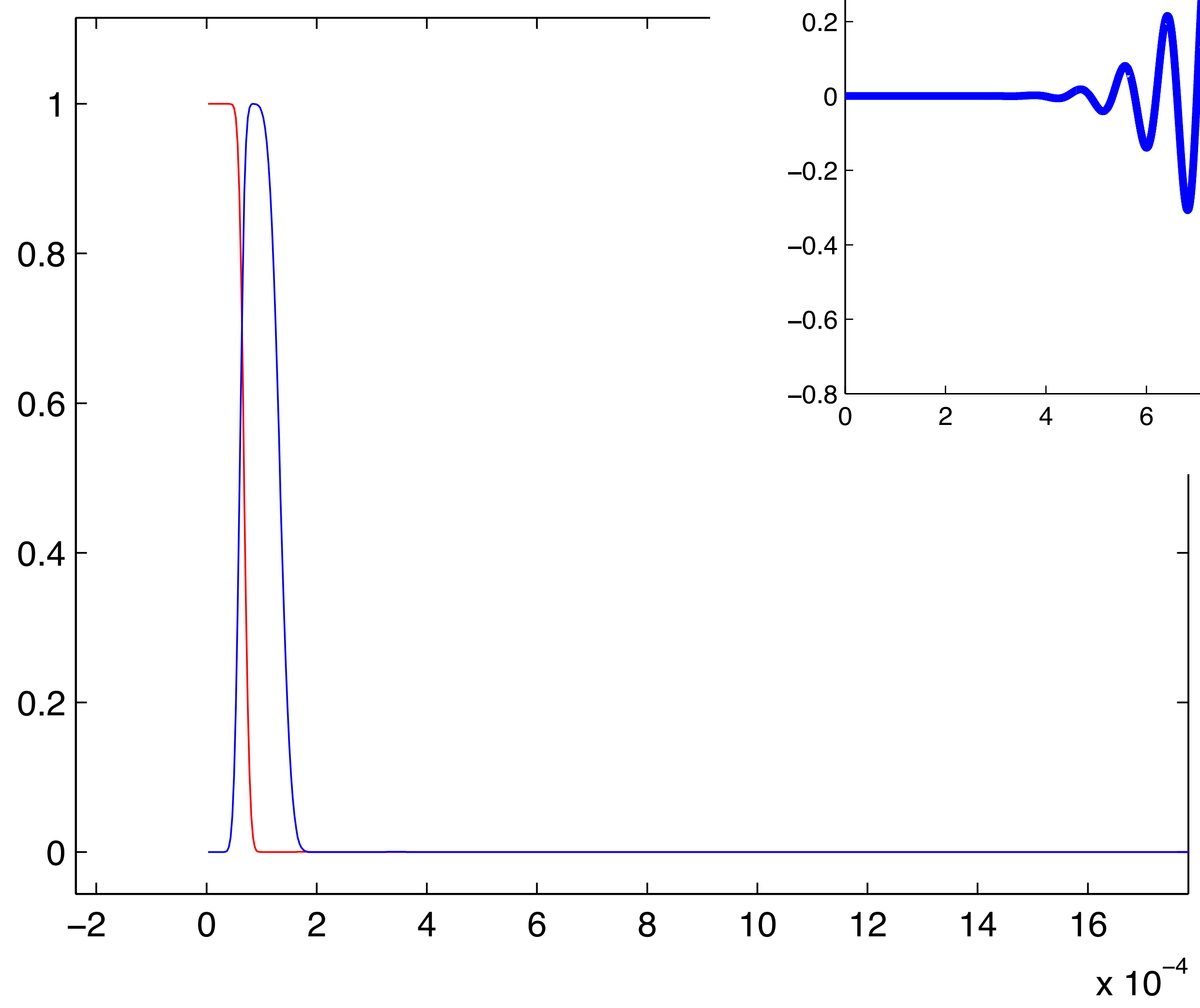
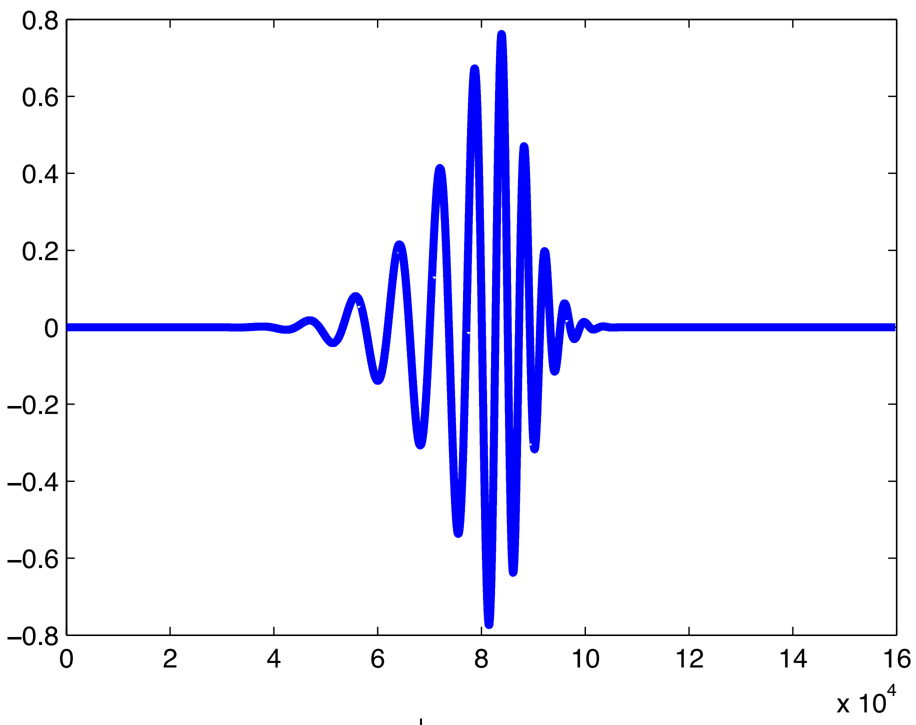




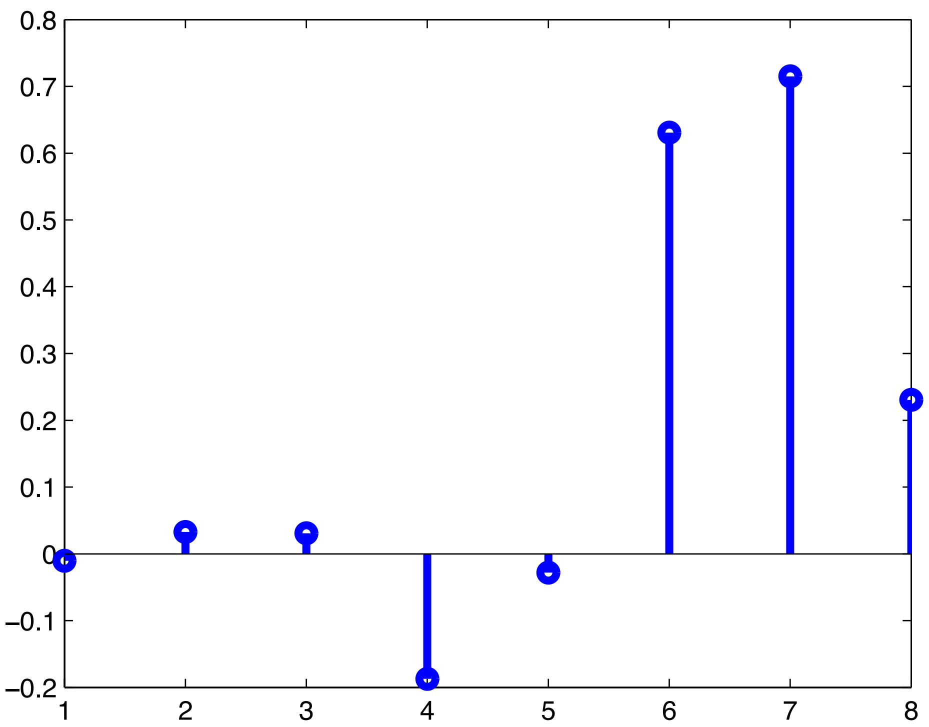
choice of wavelet transform & filters

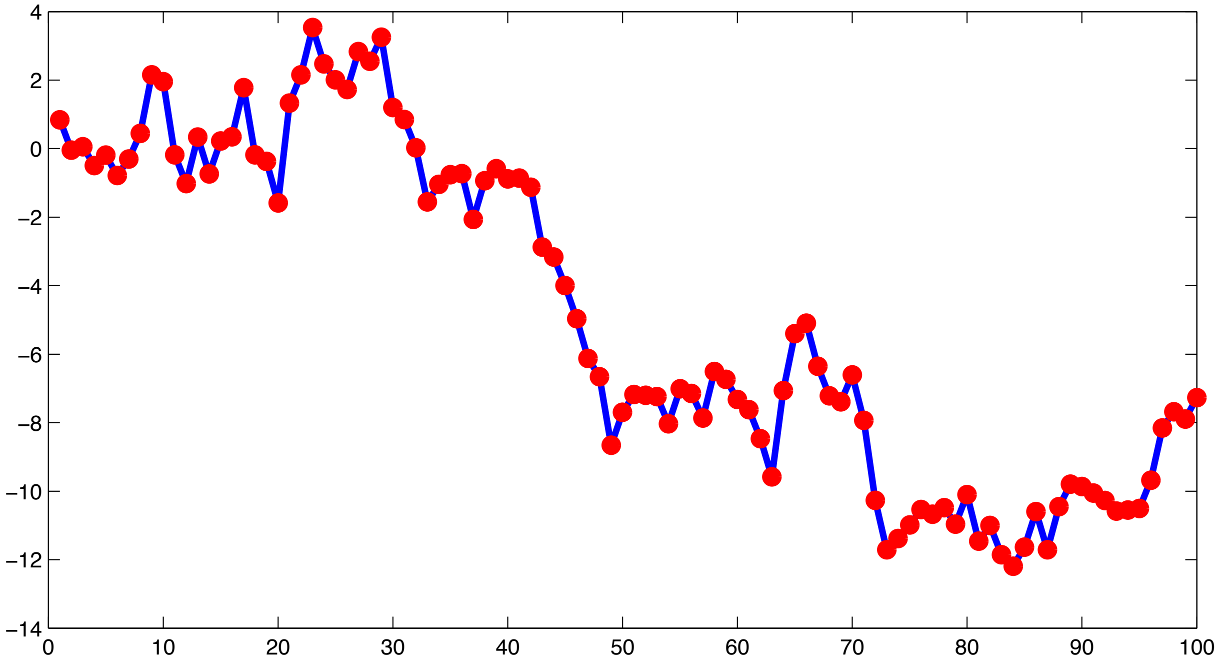
1. Self-consistent high/low pass filters [Quadrature Mirror Filters], full reconstruction -- can’t use continuous wavelets.
2. Time event preservation -- can’t use normal discrete wavelets.
3. Zero or linear phase (symmetric filters) -- for exact time localisation of events.
4. Sharp Fourier frequency resolution for studying spectra -- need higher order wavelets.
5. Time domain implementation with compact digital filters to minimise edge effects, increase time locality and reduce computational expense -- wavelet order can’t be too high; again can’t use continuous wavelets.

choice of wavelet transform & filters

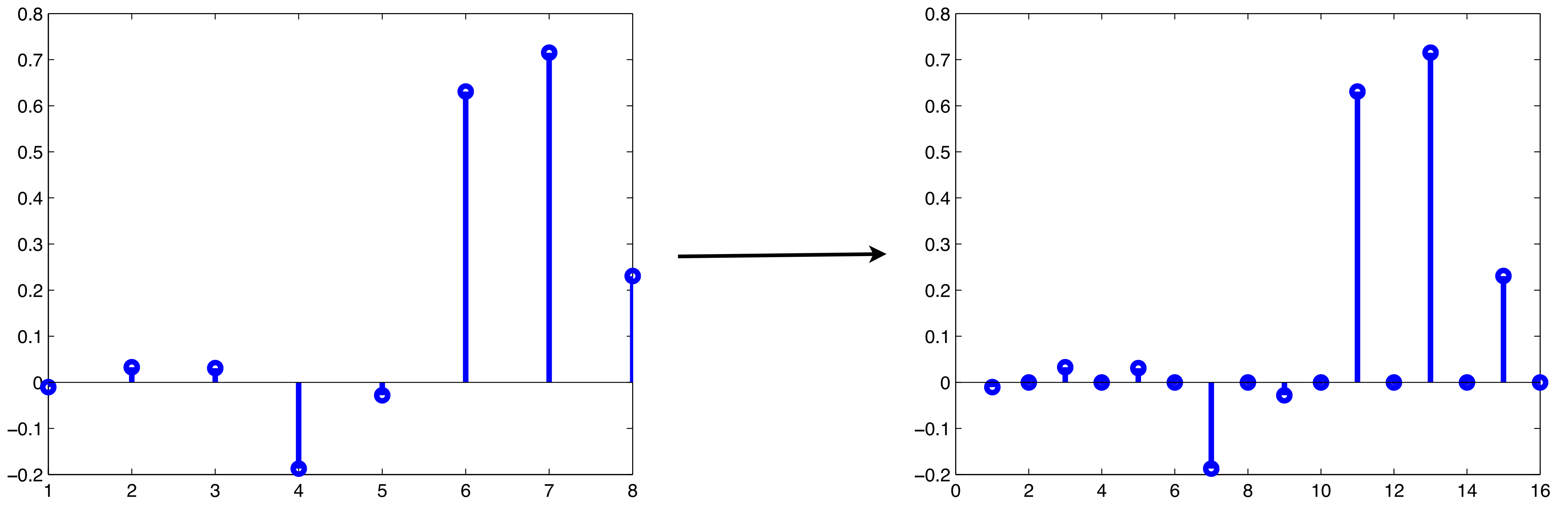
Solution: Undecimated discrete wavelet transform

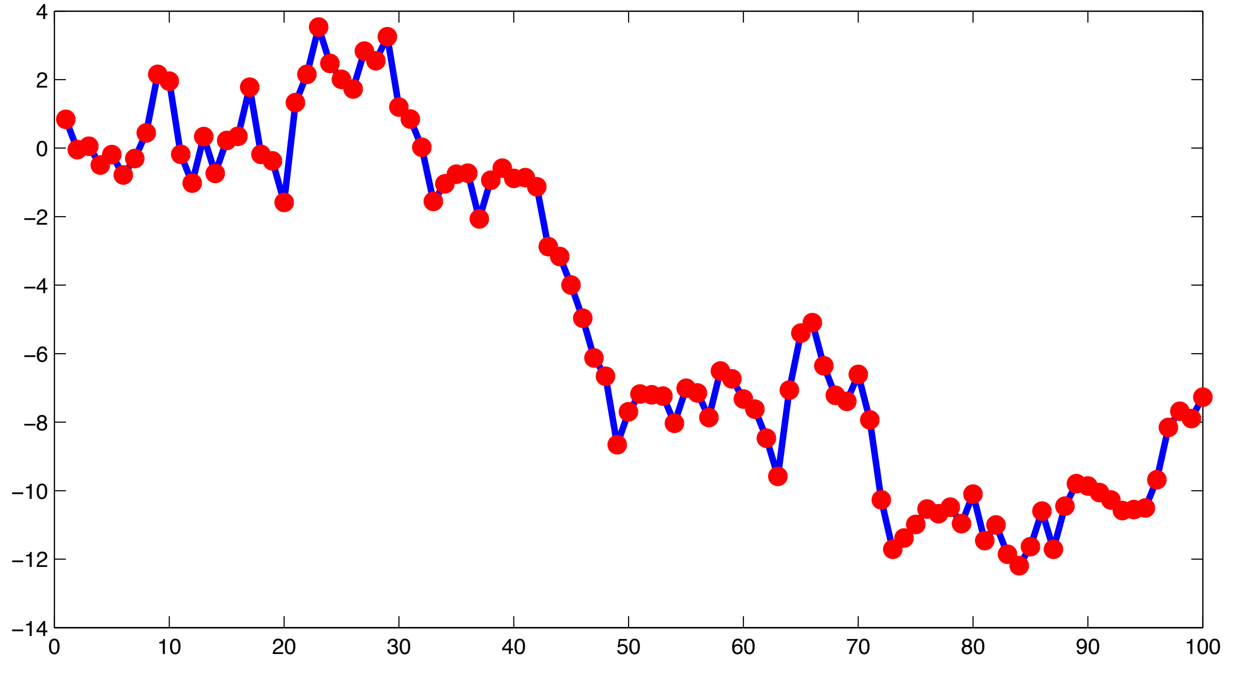
undecimated discrete wavelet transform (udwt)





undecimated discrete wavelet transform (udwt)





<u>UDWT steps:</u>

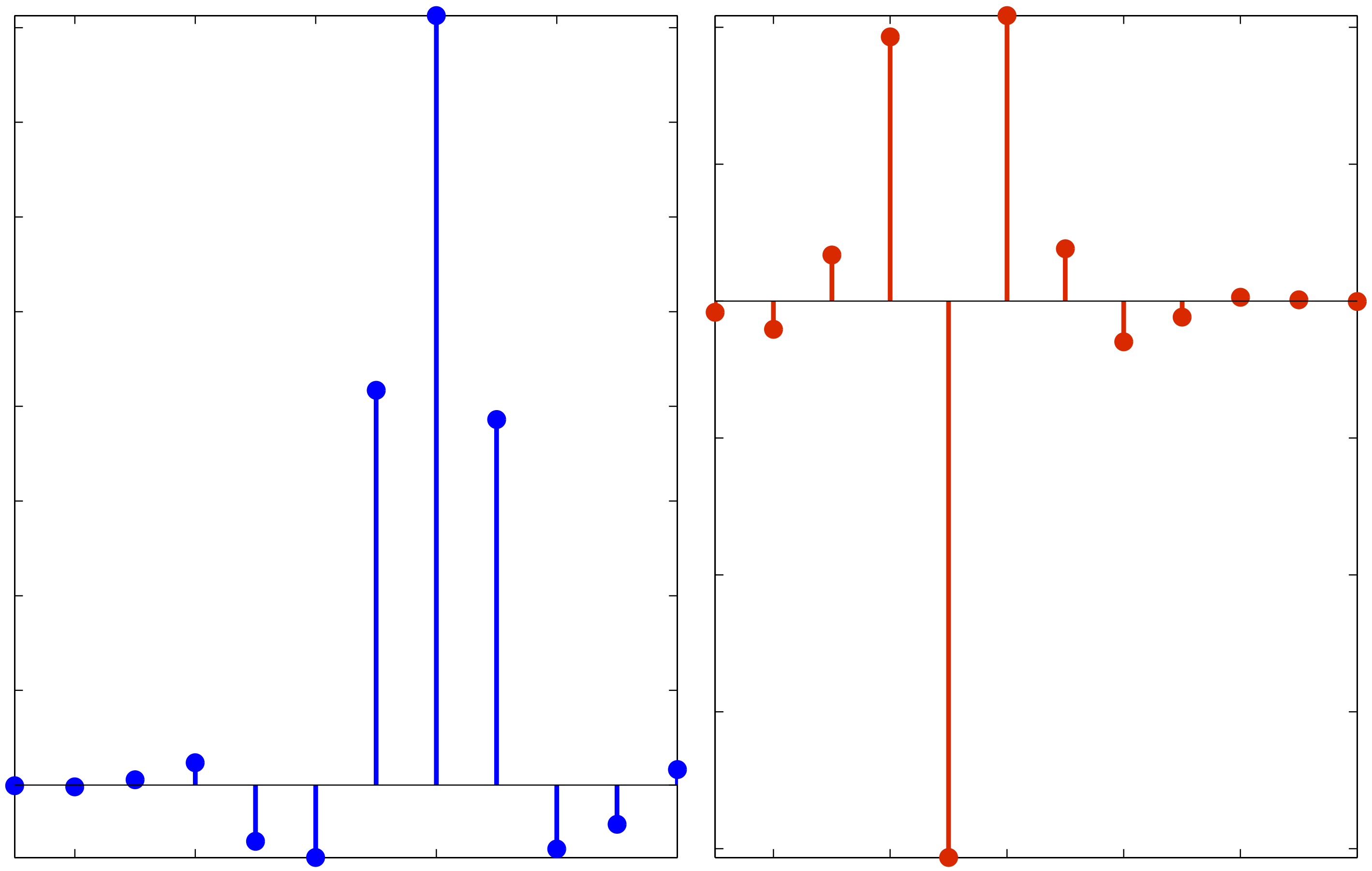
Leave signal as it is

Upsample filter by 2

choice of wavelet transform & filters

Solution: <u>Undecimated discrete wavelet transform</u>

scaling (low-pass) filter

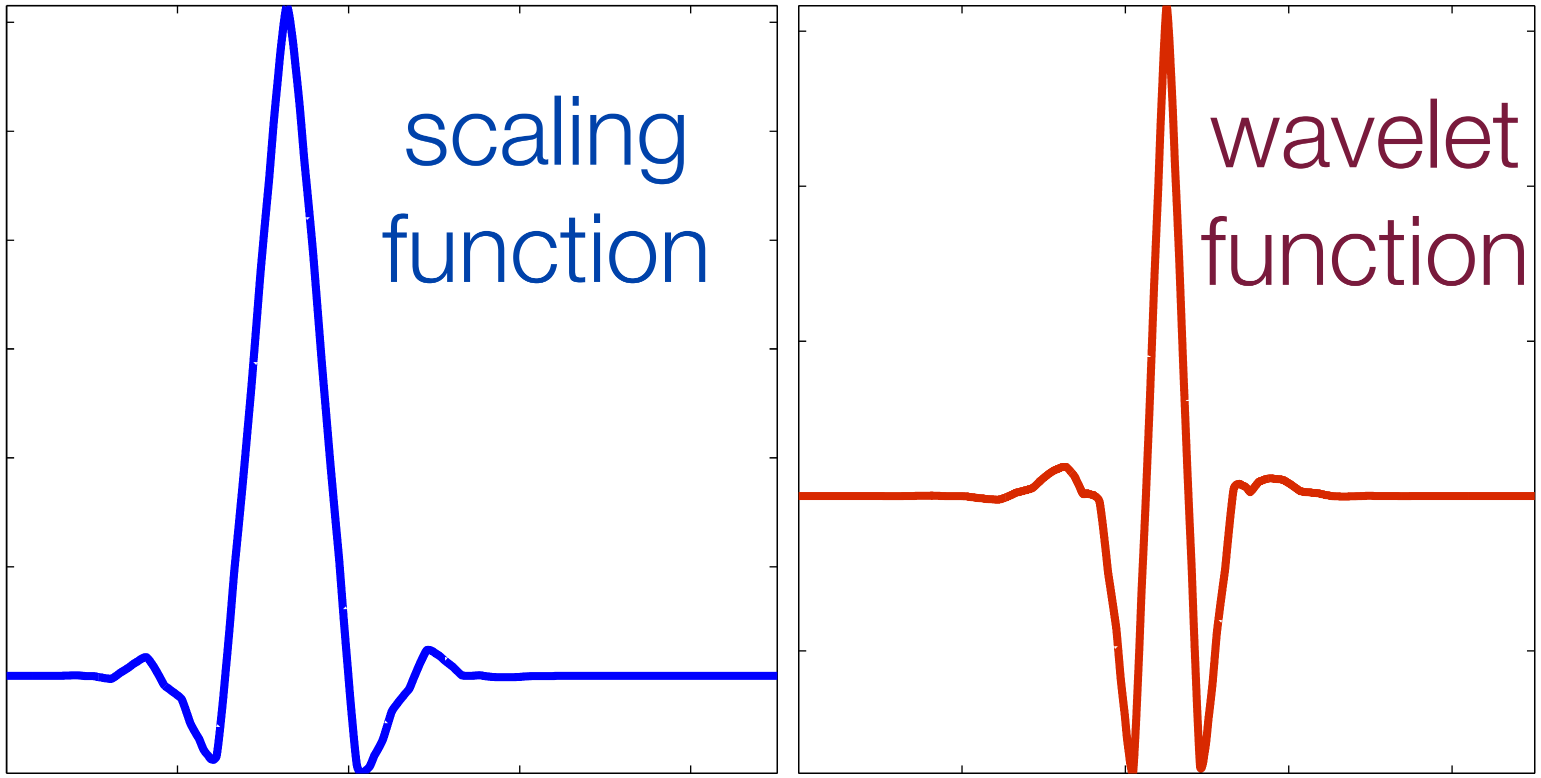


wavelet (high-pass) filter

[*Coiflet-2* wavelet and scaling (digital) time-domain filters]

choice of wavelet transform & filters

Solution: Undecimated discrete wavelet transform



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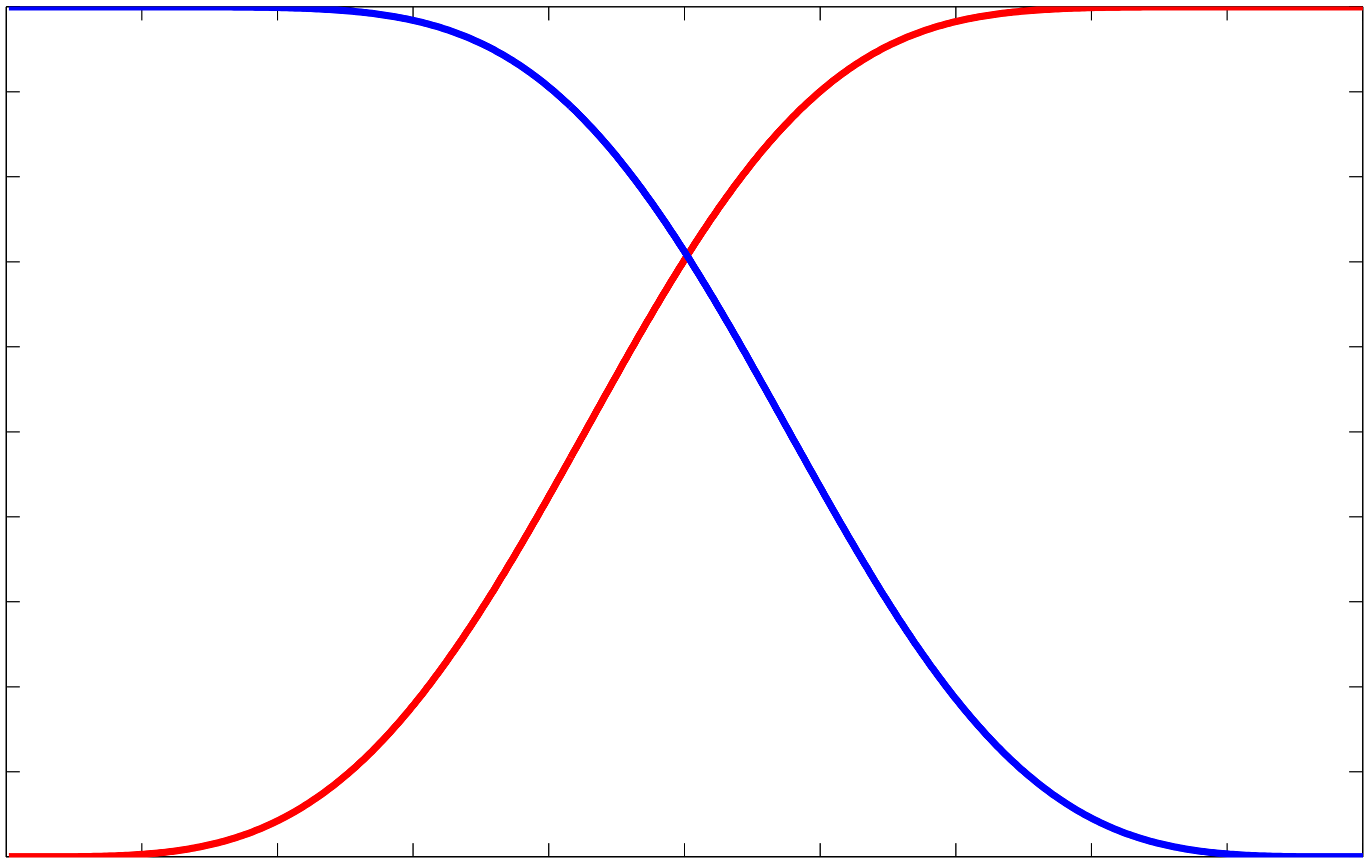

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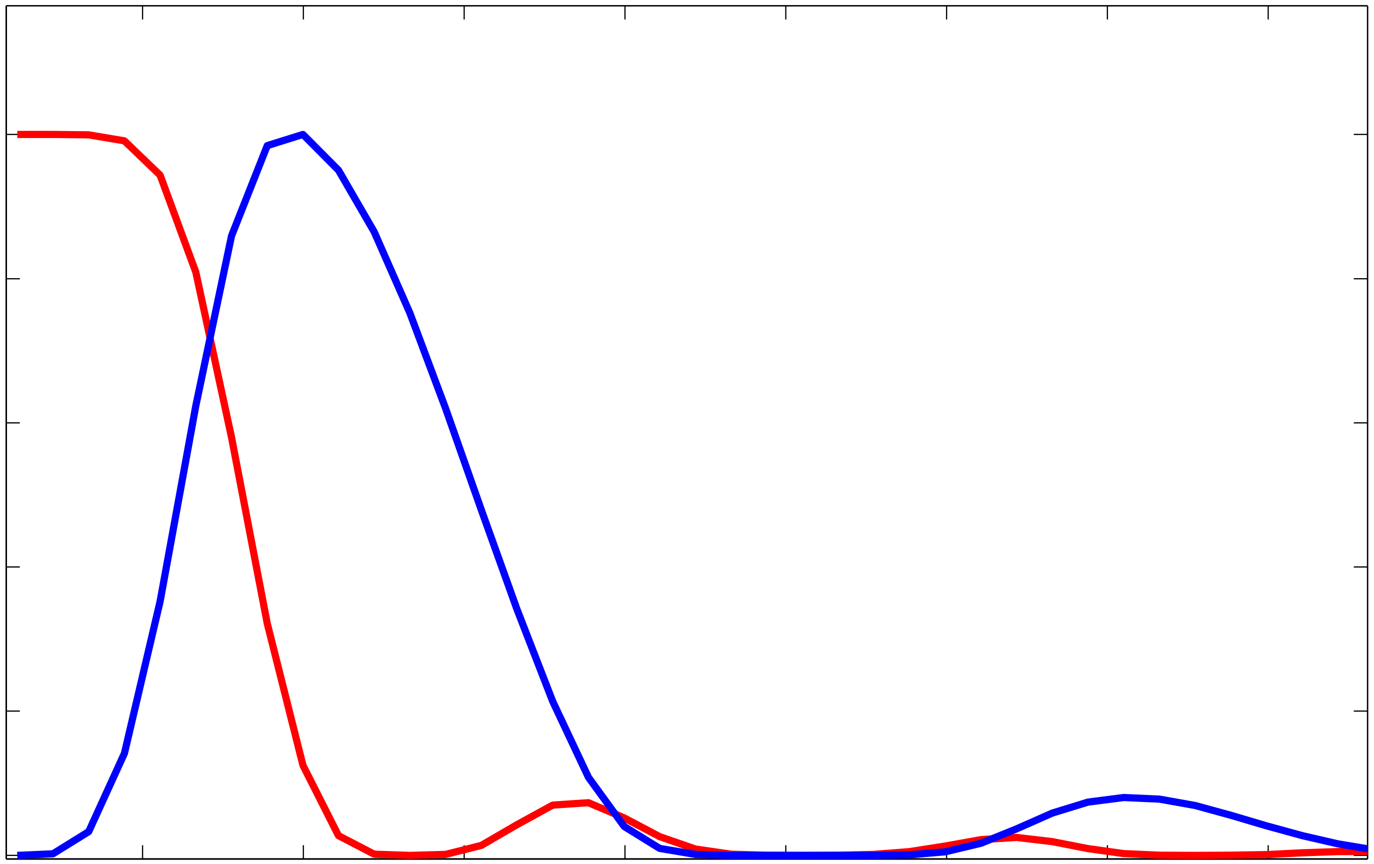

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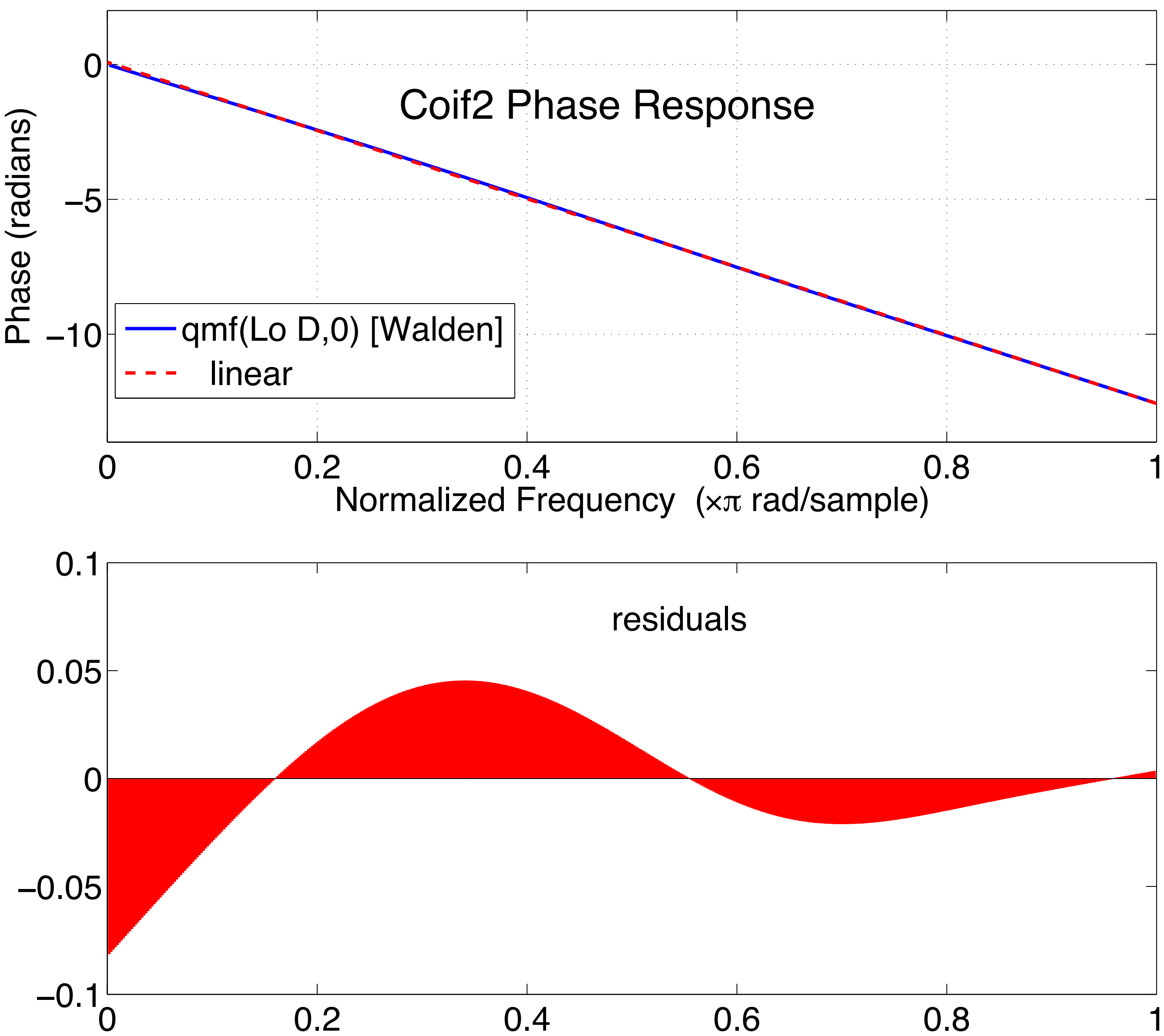
Solution: Undecimated discrete wavelet transform



choice of wavelet transform & filters

Solution: Undecimated discrete wavelet transform

Rice wavelet toolbox for Matlab (C mex files)

https://github.com/ricedsp/rwt

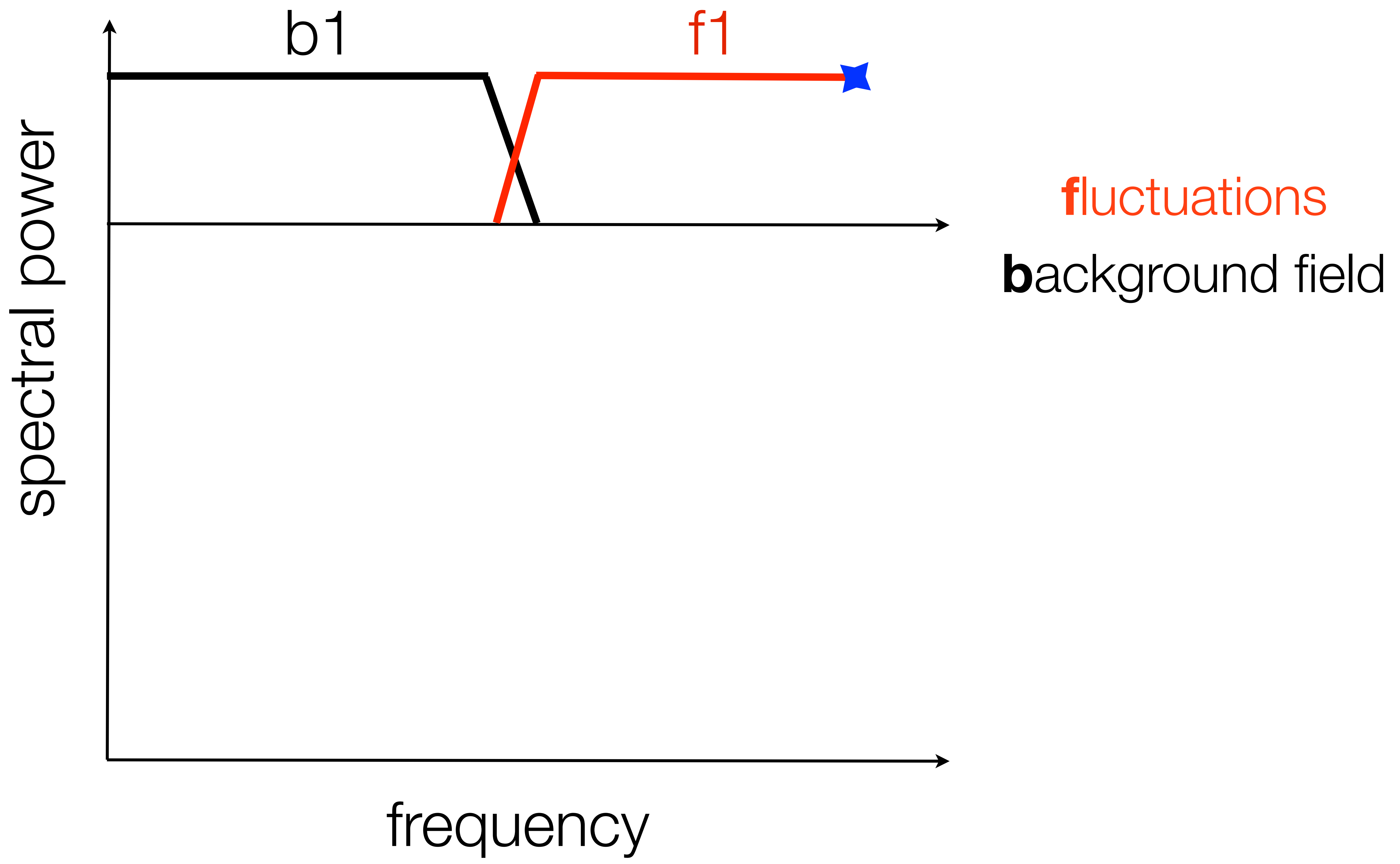
http://dsp.rice.edu/software/rice-wavelet-toolbox

PyWavelets (Cython)

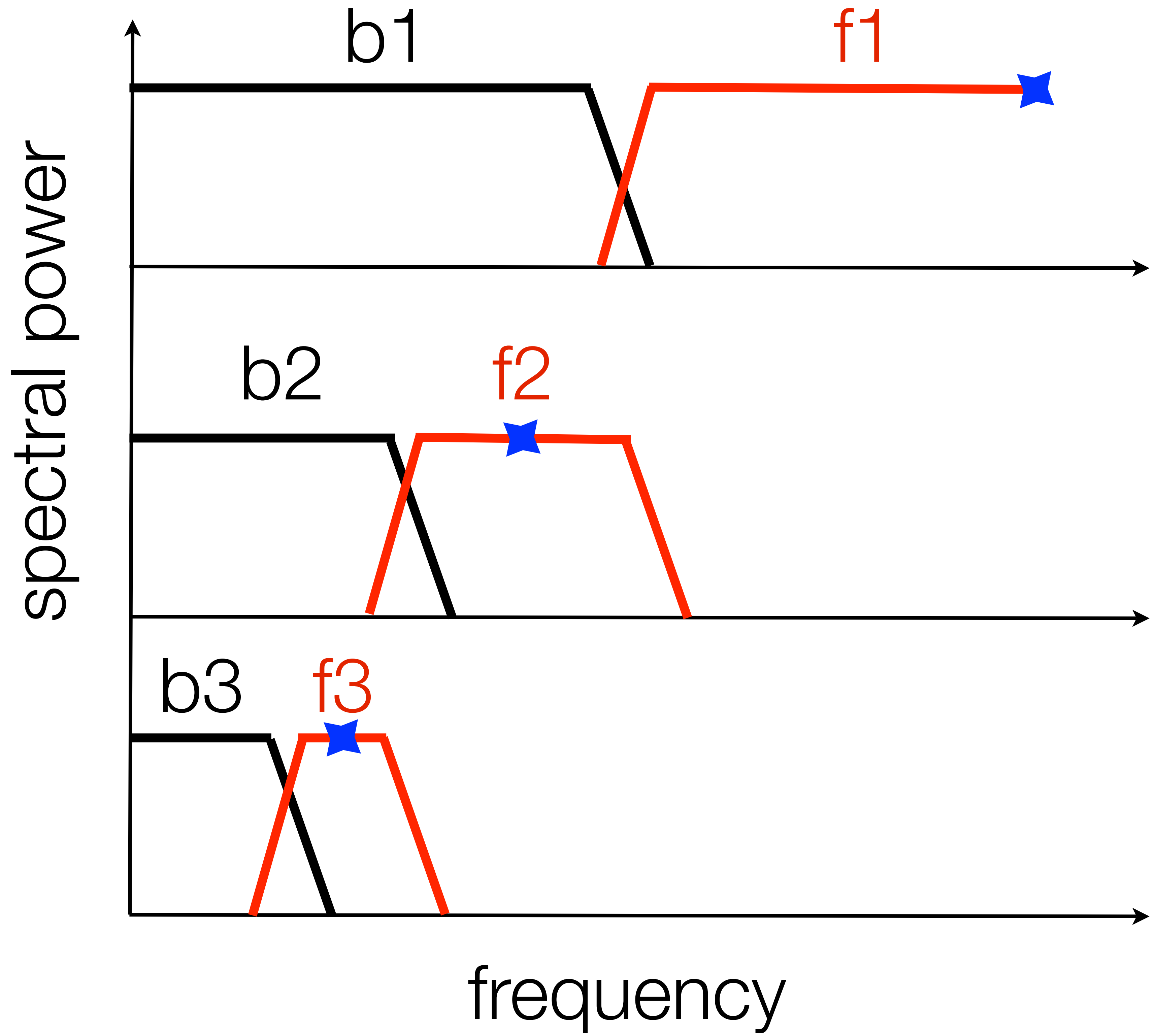
http://www.pybytes.com/pywavelets/

Anisotropic plasma turbulence applications

local scale-dependent fluctuations *and* background field using low/high pass filters



local scale-dependent fluctuations *and* background field using low/high pass filters

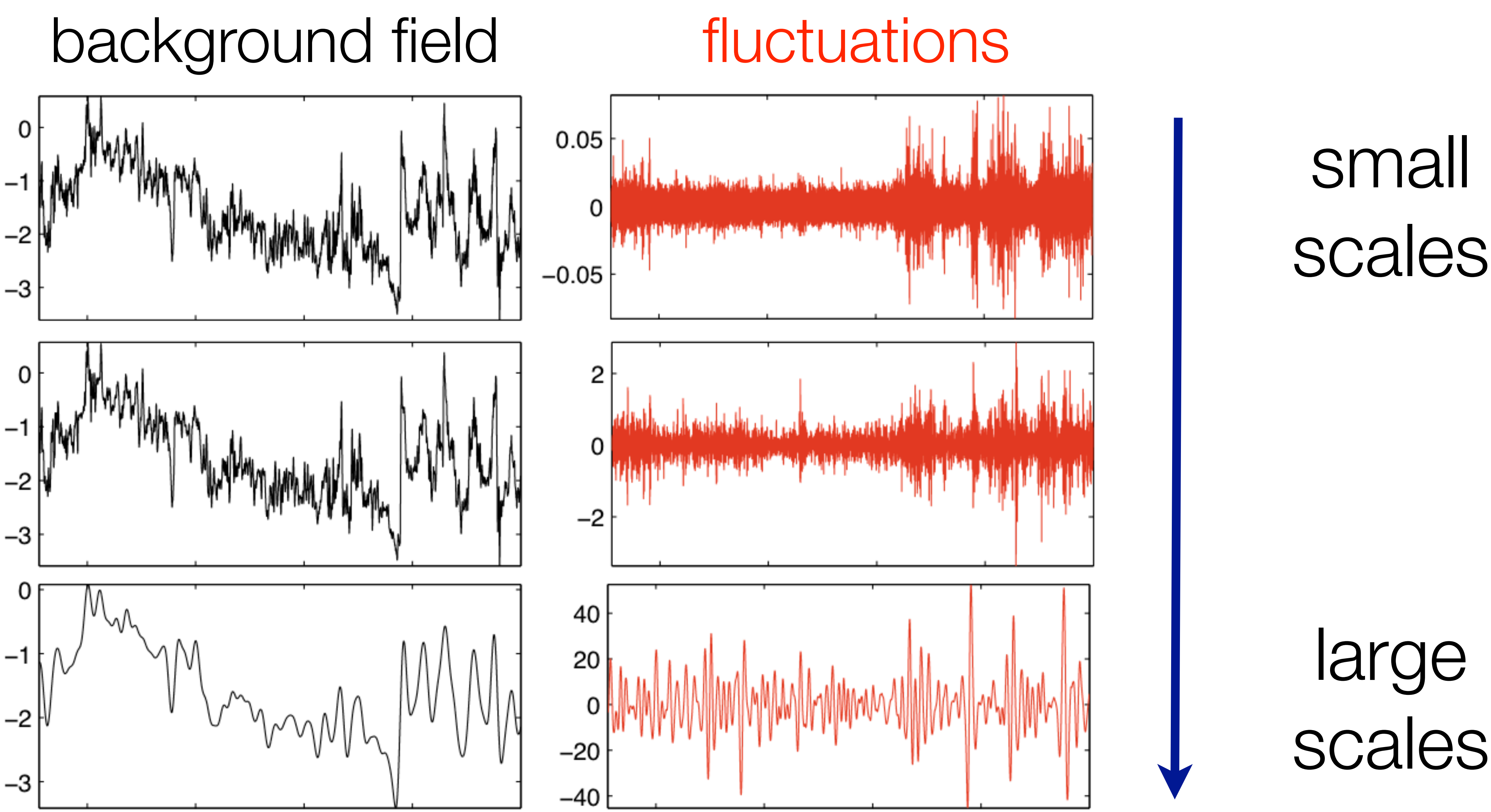


fluctuations

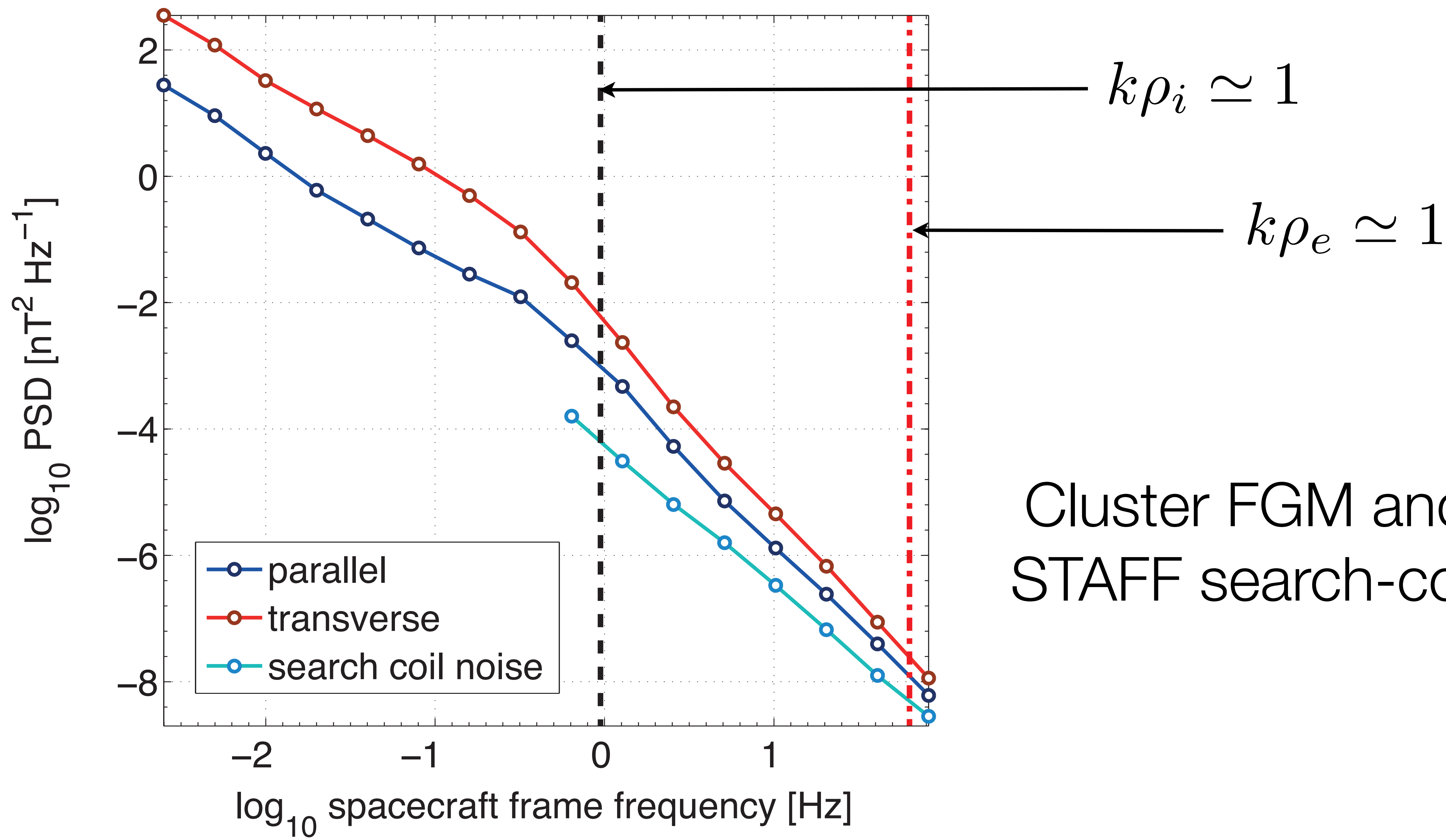
background field

filter bank implementation using quadrature mirror filters

adaptive local scale-dependent fluctuations **and** background guide field



power spectral density



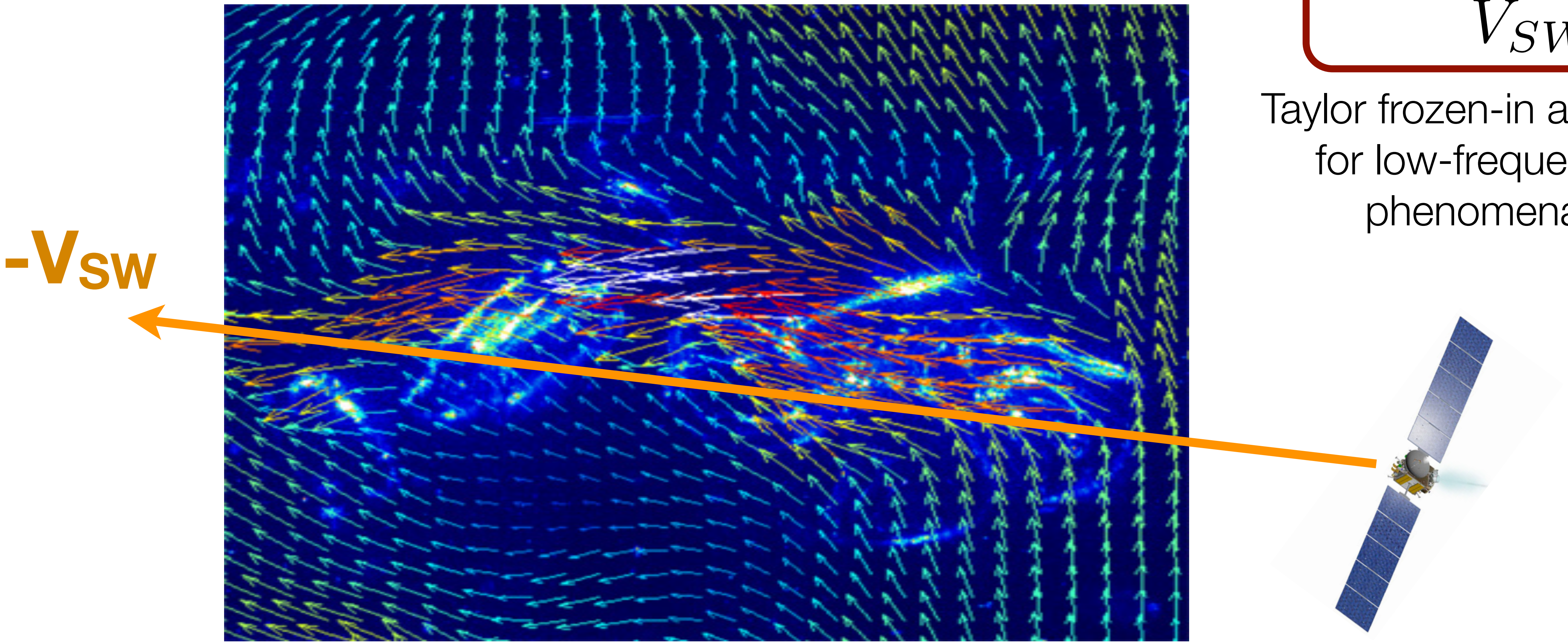
Cluster FGM and STAFF search-coil

Kiyani et al. ApJ (2013)

angles of measurement w.r.t. B

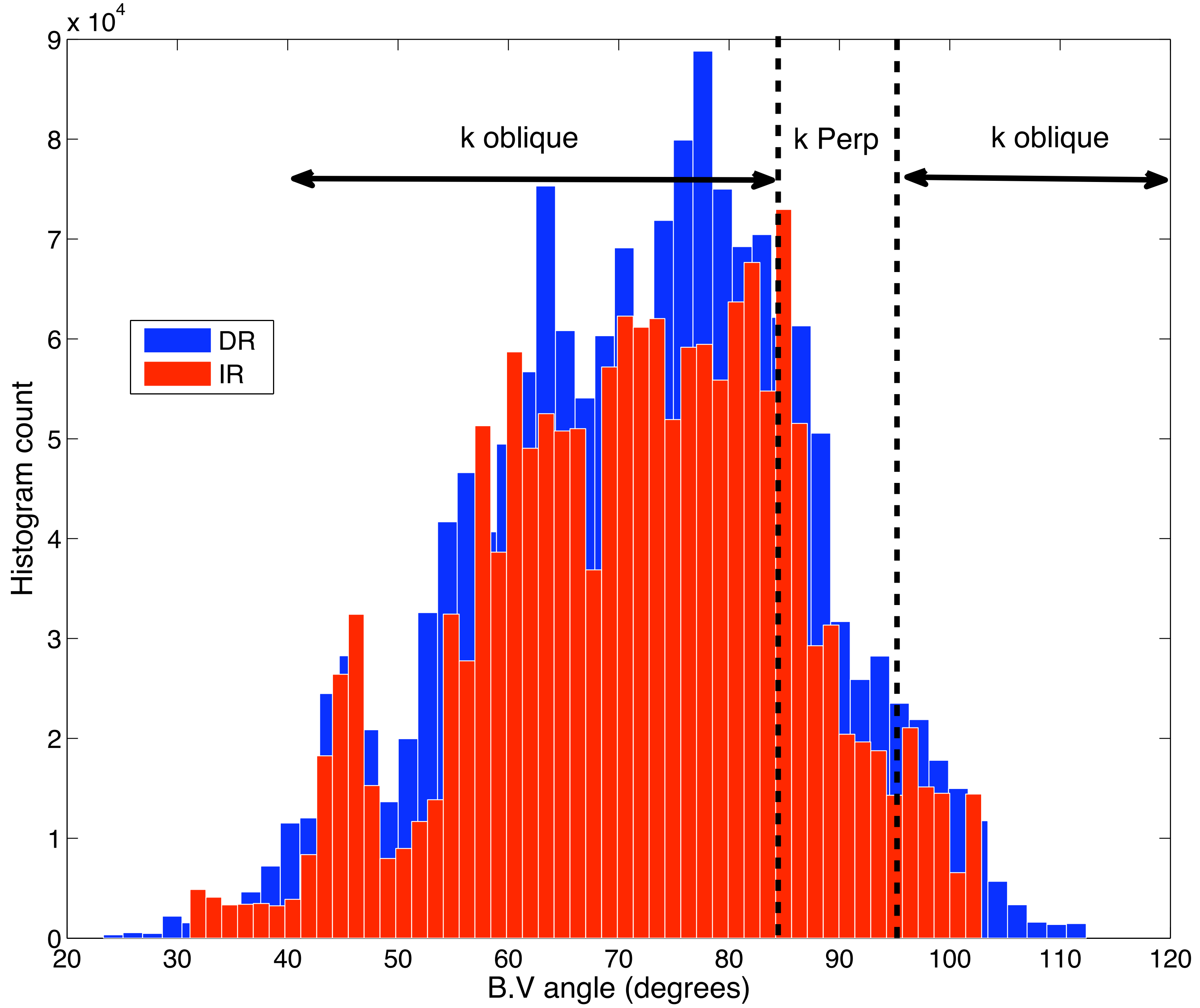
$$k = \frac{2\pi f_{sc}}{V_{SW}}$$

Taylor frozen-in approx, for low-frequency phenomena

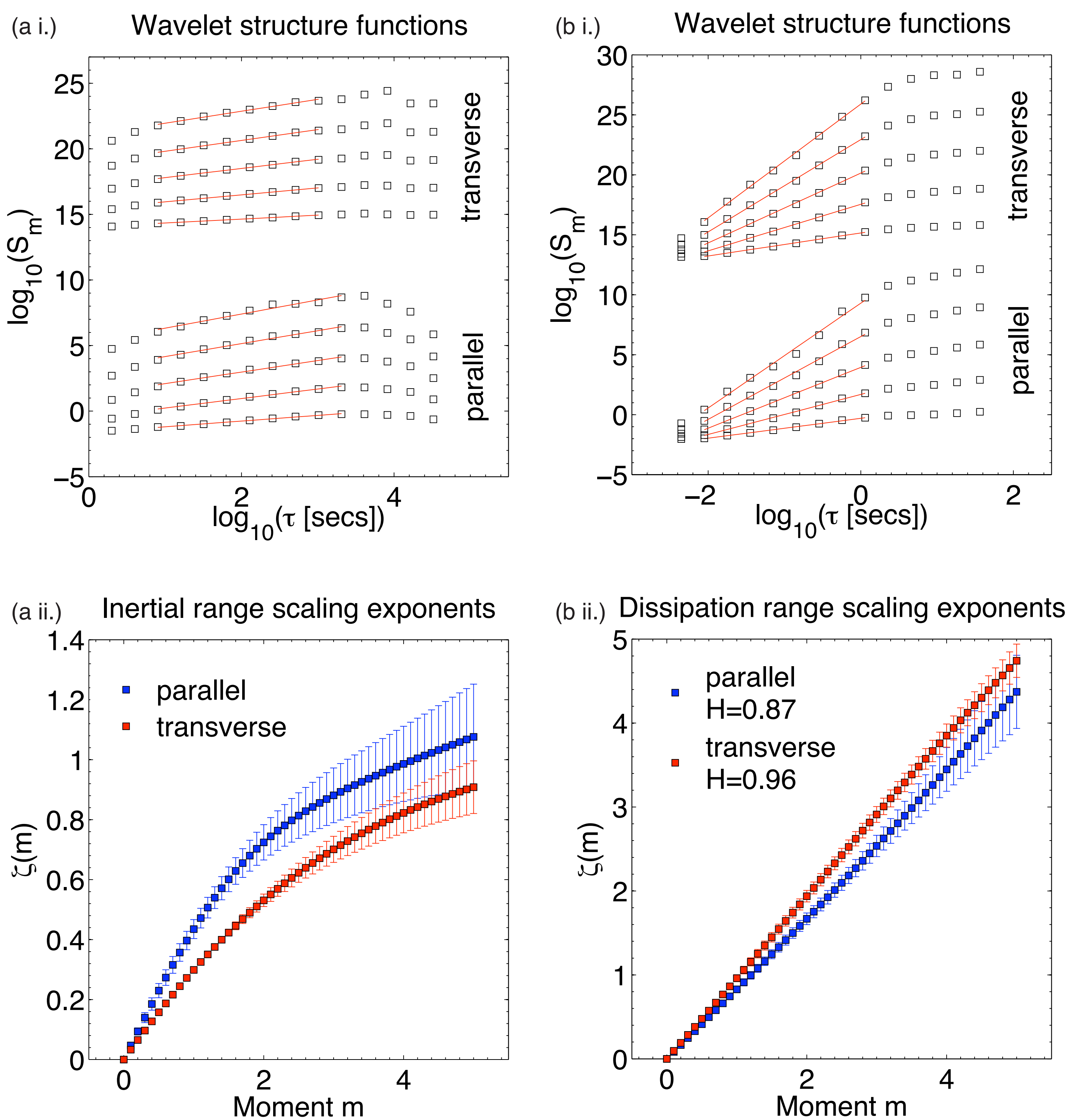


MHD $\frac{V_A}{V_{SW}} < 1$ else $\frac{V_\phi}{V_{SW}} < 1$

Anisotropy (angle between B and V)



higher order scaling



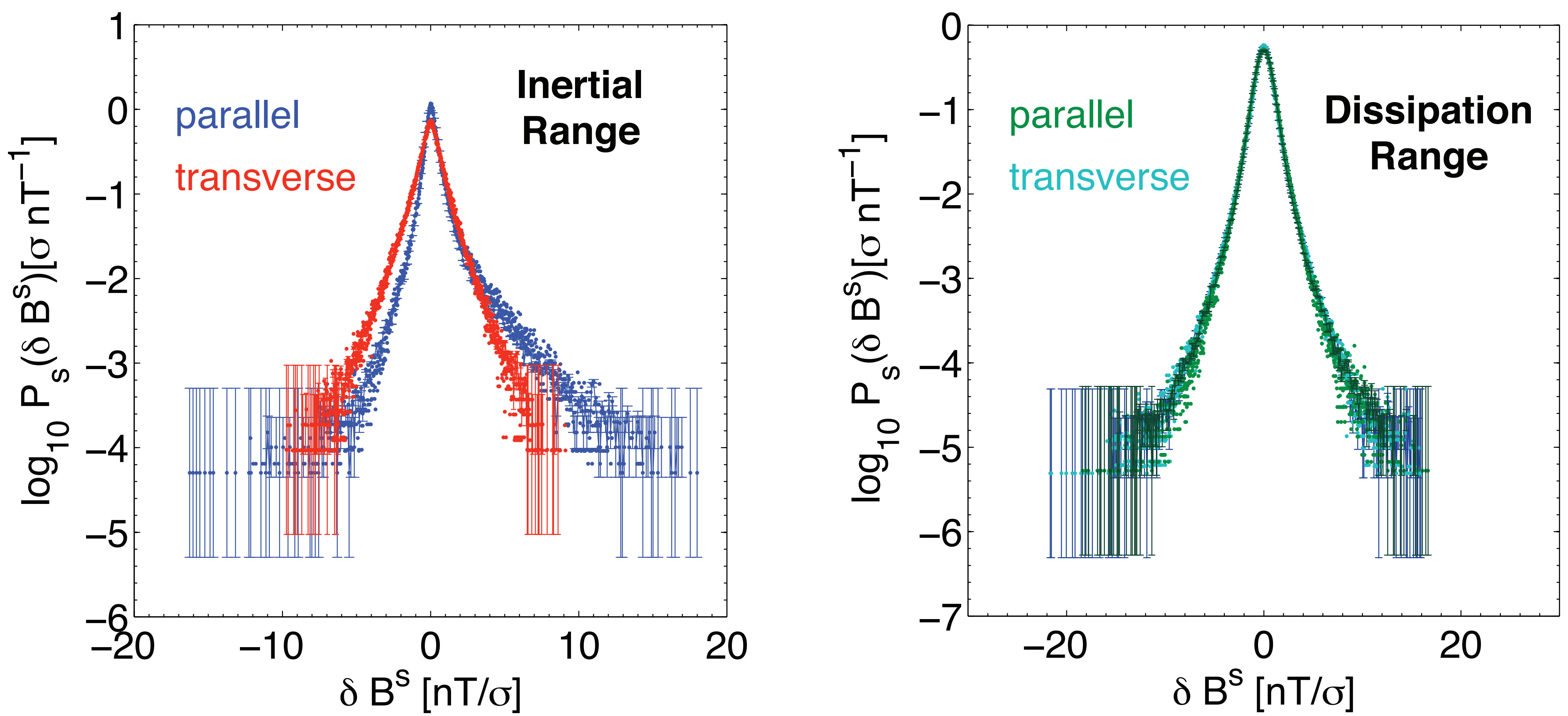
Structure Functions

$$S^m_{\parallel(\perp)}(\tau) = \frac{1}{N}\sum_{j=1}^{N}\left|\frac{\delta B_{\parallel(\perp)}(t_j,\tau)}{\sqrt{\tau}}\right|^m$$

note: Inertial range data is from ACE

Also see:
K. H. Kiyani et al.
Phys. Rev. Lett.
103, 075006 (2009)

standardised probability densities

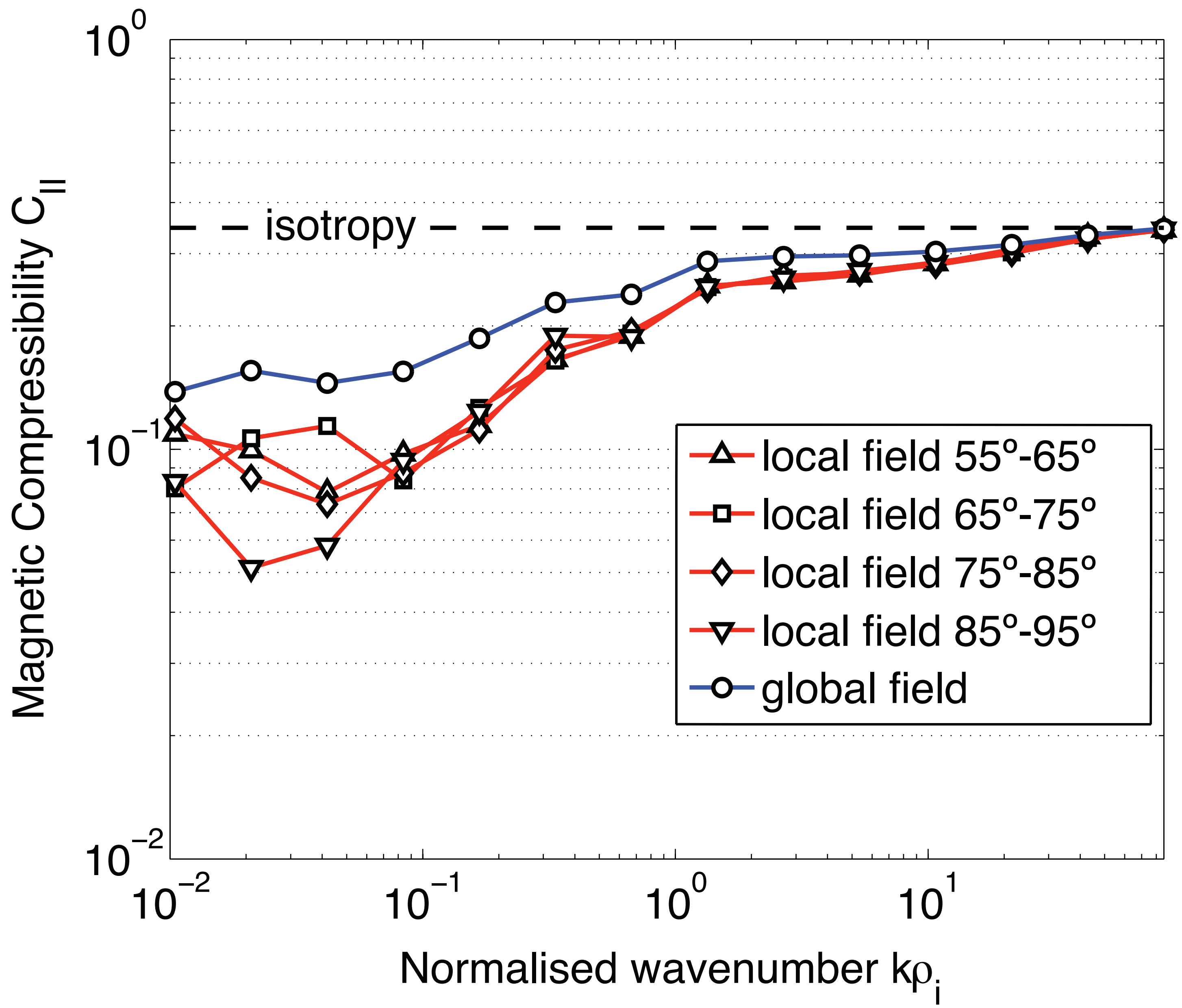


isotropy

Kiyani et al. ApJ (2013)

magnetic compressibility

$$C_{\parallel}(f) = \frac{1}{N}\sum_{j=1}^{N}\frac{\delta B_{\parallel}^{2}(t_j, f)}{\delta B_{\parallel}^{2}(t_j, f) + \delta B_{\perp}^{2}(t_j, f)}$$



Kiyani et al. ApJ (2013)

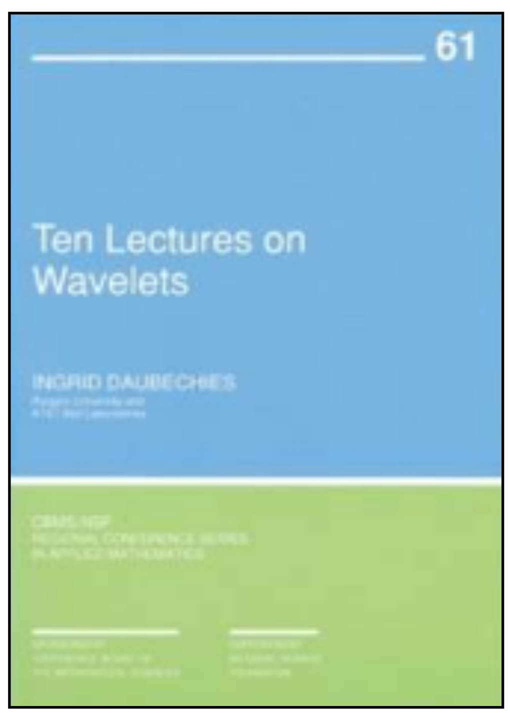
take home message

- Wavelets are excellent tools for studying turbulence and anisotropy
- They are especially useful for both simultaneous event localisation as well as frequency localisation
- Tried extending this to m-band wavelet packets to get better frequency resolution; but generally this is not possible. Recent work on over/undersampling is more promising.

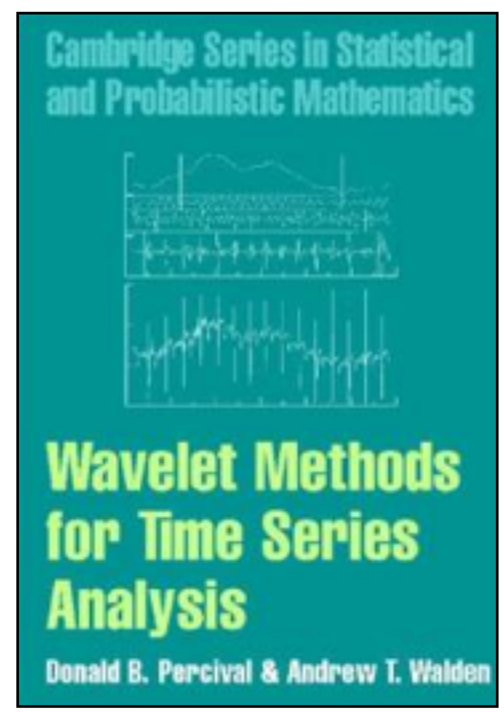
guides and literature

- M. Farge et al., *Wavelets and Turbulence*, Proc. IEEE, **84**(4), 639, (1996)
- A. Walden & A. Cristan, Proc. R. Soc. Lond. A, **454**, 2243 (1998)
- K. Kiyani et al, ApJ, (2013)

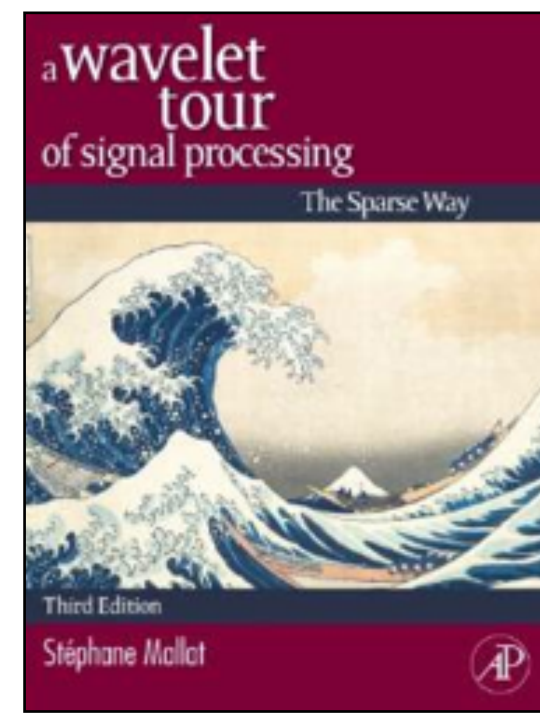
Essential Wavelets for Statistical Applications and Data Analysis
T. Ogden



Ten Lectures on Wavelets
I. Daubechies



Wavelet Methods for Time Series Analaysis
D. Percival & A. Walden



A Wavelet Tour of Signal Processing
S. Mallat