A Krook Type Collision Operator for Electron Transport in a Laser Produced Plasma

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ABSTRACT

Existing codes for directly solving for the distribution function in laser-plasma interaction studies either assume a collisionless plasma or solve for the full Fokker-Planck collision terms. While much progress has been made in efficiently implementing the Fokker-Planck collision terms, these methods remain computationally demanding and often rely on decomposing the electron distribution function into spherical harmonics. In the laser-plasma interaction region it is desirable to use an Eulerian grid for its ability to accurately include a laser. Here we outline a Krook type collision operator and assess its accuracy by comparing it to known transport coefficients in regimes relevant to laser-produced plasmas. Following the work done by Mannheimer et. al.[1] we normalise the Krook operator to give the same thermal flux as Braginskii in the local limit and investigate its effectiveness in a full Vlasov simulation.

Introduction

Considerable progress[1][2] has been made studying transport phenomena using Vlasov-Fokker-Planck models based on the decomposition of the distribution function into spherical harmonics. However for numerical simulation of the laser-plasma interaction region it is desirable to solve Vlasov's equation directly on an Eulerian grid. A direct solve of the Vlasov equation is a problem that requires significant computational effort. While Eulerian based Vlasov-Fokker-Plank models have been developed[6], these methods incur significant computational cost.

Collisions are included in a relativistic Vlasov model through:

$$\frac{\partial f_i}{\partial t} + \frac{\boldsymbol{u}_i}{\gamma}\frac{\partial f_i}{\partial \boldsymbol{x}} + \frac{q_i}{m_i}\left(\boldsymbol{E} + \frac{\boldsymbol{u}}{\gamma} \times \boldsymbol{B}\right)\frac{\partial f_i}{\partial \boldsymbol{v}} = \left(\frac{\partial f_i}{\partial t}\right)_c$$

Where the term on the right hand side denotes the change in the distribution function due to collisions.

Krook Collisions

The Krook-type[5] collision operator assumes that collisions act to relax the distribution function to a Maxwellian at the collisional rate. Electron-electron, electron-ion and ion-ion collisions are implemented here. The Krook collision term for each of the distribution functions is then:

$$\left(\frac{\partial f_i}{\partial t}\right)_c = -\nu_{ii}(v)(f_i - f_i m) \qquad \left(\frac{\partial f_i}{\partial t}\right)_c = -\nu_{ee}(v)(f_e - f_{m1}) - \nu_{ei}(v)(f_e - f_{m2})$$

Implementation

For the Krook operator to conserve mass, momentum and energy, the advance from time n to $n+1$ must satisfy:

$$\int (f^{n+1} - f^n) v^i dv = 0 \qquad v = 0, 1, 2$$

This can be enforced numerically with a velocity dependent collision frequency if a Maxwellian f_m is chosen such that

$$\int \nu(v)(f^n - f_m) v^i dv = 0 \qquad v = 0, 1, 2$$

- Solve for parameters of Maxwellian using Newton-Rapheson, use moments of f_n as initial guesses
- Implicit time advance so for the electron update:

$$f_e^{n+1} - f_e^n = \frac{-\Delta t \nu_{ee}}{1 + \Delta t(\nu_{ee} + \nu_{ei})}(f_e^n - F_{m1}) + \frac{-\Delta t \nu_{ei}}{1 + \Delta t(\nu_{ee} + \nu_{ei})}(f_e^n - F_{m2})$$

- Collision term non-relativistic as the collision time for fast particles is longer than timescales of interest
- Can be modified to a faster method, where mass is conserved numerically but energy and momentum chosen from f using Greene's[6] analytical solutions.

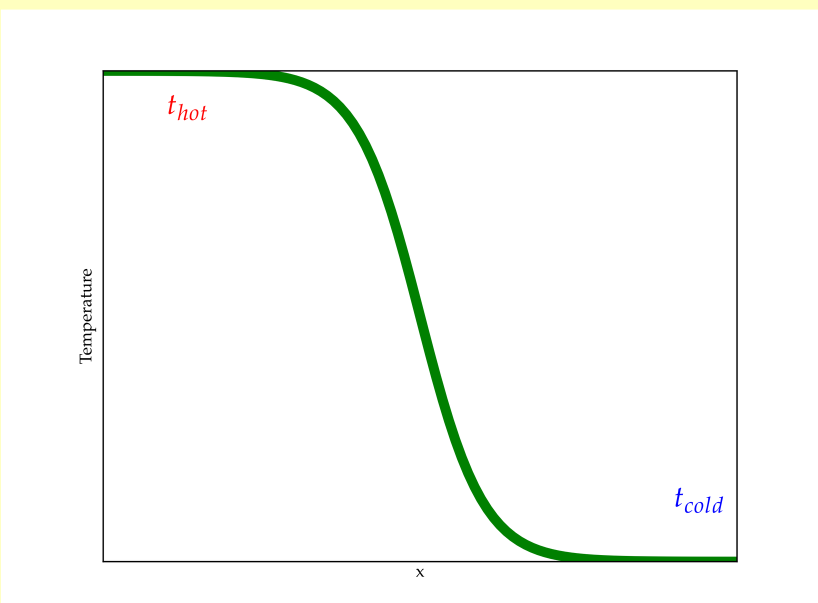
Fokker-Planck Heat Flux Normalisation

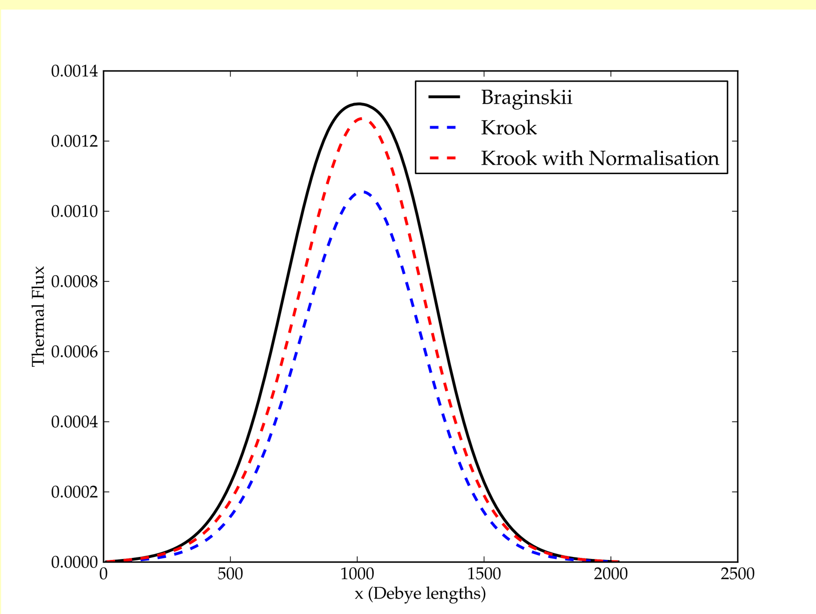
In the local limit Mannheimer et al[3]. found that the heat flux calculated using a Krook operator differed from that produced using the full Fokker-Planck collision term by a factor $\zeta(z)$.
Rather than normalise the entire collision operator with this factor we instead normalise our slow limit electron-electron collision frequency using this factor to improve thermal transport performance whilst retaining the accuracy of the electrical transport.
The electron-electron collision frequency becomes

$$\nu_{ee} = \frac{\zeta(z)\nu_{slow}}{1 + \frac{\zeta(z)\nu_{slow}}{\nu_{fast}}}$$

Heat Flow in the Local Limit

The initial temperature profile is shown(left) with $t_{hot} = 10eV$ and $t_{cold} = 9eV$. The results here are for a fully ionised hydrogen plasma. The ions are mobile and are treated with the quick Krook operator while the conservative operator is used on the electrons.





Snapshots were taken after 1400 plasma periods.

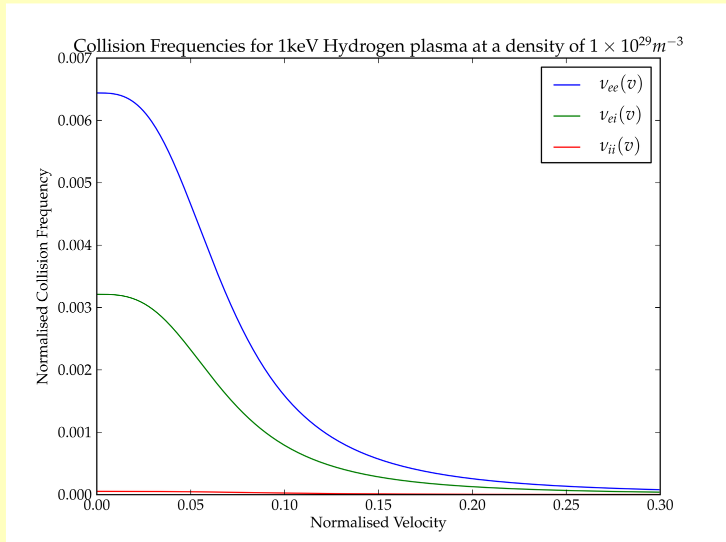
Velocity Dependent Collision Frequencies

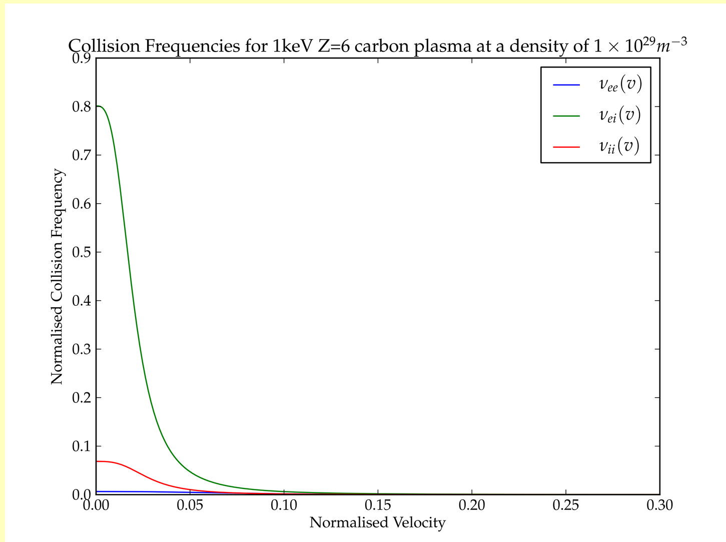
Velocity dependent collision frequencies are vital for studying hot distribution functions where the collision time of the faster particles could be orders of magnitude longer than that of the slow particles. For ease of computation we take the slow and fast limit collision frequencies from the NRL for a test particle α colliding with a field particle β.
The fast and slow limits are defined when:

$$\frac{m_\beta v_\alpha}{2kT_\beta} \ll 1 \qquad \frac{m_\beta v_\alpha}{2kT_\beta} \gg 1$$

Then following Mannheimer et al[3], make an analytic connection between the two:

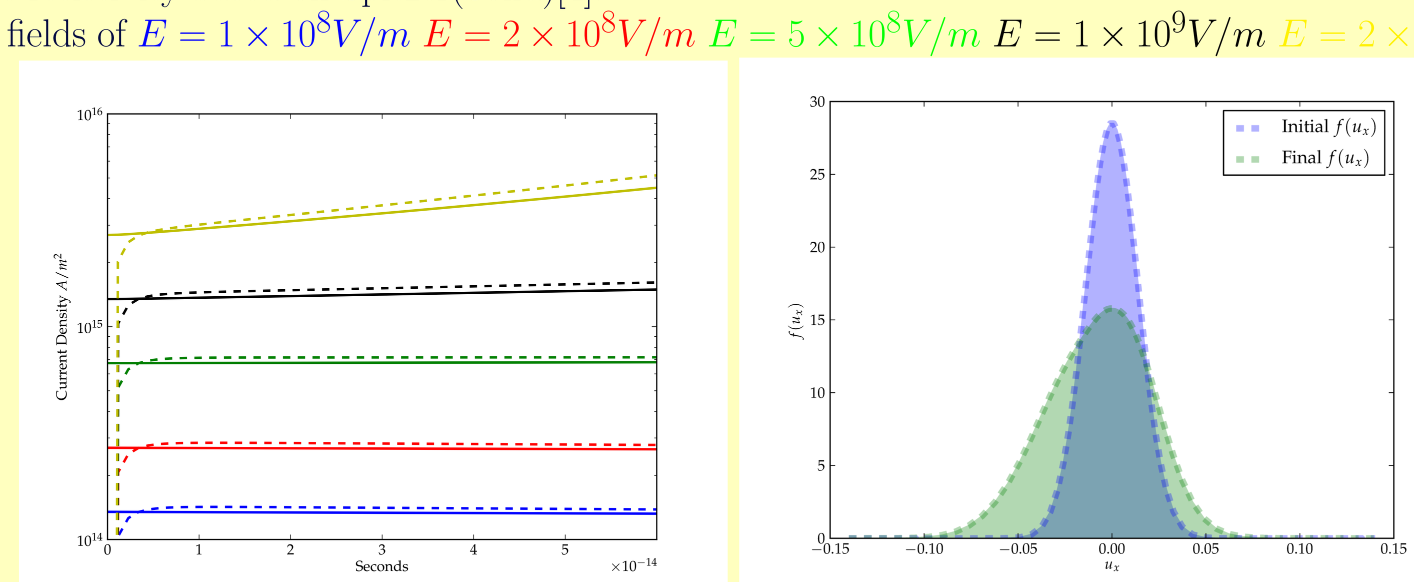
$$\nu = \frac{\nu_{slow}}{1 + \frac{\nu_{slow}}{\nu_{fast}}}$$





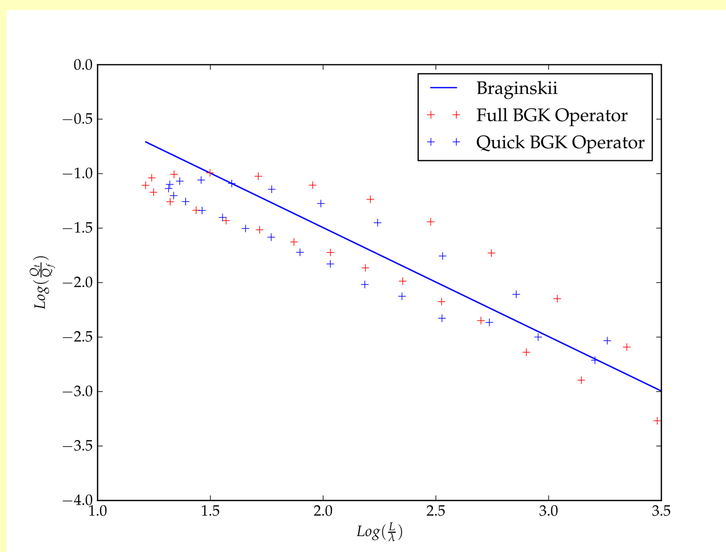
Spitzer Electrical Conductivity

To assess the accuracy of the Krook operator an electric field is applied across a 1d block of plasma. The resulting current(Dashed) is then plotted as a function of time. This is compared against the value obtained using the resistivity based on Spitzer(Solid)[4].
Applied fields of $E = 1 \times 10^8 V/m$ $E = 2 \times 10^8 V/m$ $E = 5 \times 10^8 V/m$ $E = 1 \times 10^9 V/m$ $E = 2 \times 10^9 V/m$.



Non-Local Transport

The steep temperature gradients produced in laser-plasma interactions moves heat flow into the non-local regime. The mean free paths of the electrons in the hot distribution are often orders of magnitude larger than the typical temperature scale length. In shock ignition scenarios these electrons can heat plasma ahead of the incoming shockwave, decreasing the efficiency of the compression.
The tests here are identical to those in the local limit, however the temperature gradient is now much steeper, from $t_{hot} = 400eV$ to $t_{cold} = 100eV$



The calculated heat flux exceeds Braginskii in the cool region due to contribution from hot electrons from the hotter region.

References

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