Numerical study of the small scale dynamics of two-dimensional magnetohydrodynamic turbulence

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Recent improvements in the scale and accuracy of direct numerical simulations of isotropic magnetohydrodynamic turbulence enable many of its fundamental properties to be investigated anew. Here we report progress on questions regarding the small scale dynamics of compressible two-dimensional magnetohydrodynamic turbulence as well as issues concerning its accurate simulation.

In many astrophysical and laboratory plasmas, the magnetic Reynolds number is large and the magnetohydrodynamic (MHD) approximation is valid. It is in these cases plasmas exhibit MHD turbulence. This paper presents results from numerical studies of 2D MHD turbulence. Contrary to the case of hydrodynamic turbulence, absolute equilibrium theory shows that MHD turbulence is expected to display similar cascade properties in two and three dimensions, such as a direct cascade in total energy. However, recent numerical studies of the time evolution of MHD turbulence suggest that the cascade may be governed by processes in 2D and 3D that are different, namely, the Alfven effect discussed by Irishnokov and Kraichnan in the former case and random eddy scrambling in the latter case. See for example Biskamp and Schwarz (2001) and Biskamp and Müller (2000) for 2D and 3D results respectively. Furthermore, direct numerical simulations show, see Müller *et. al* (2003), that the the small scale turbulent dynamics of 3D turbulence with an applied mean magnetic field crosses over to that of 2D turbulence as the strength of the magnetic field increases. As such, the 2D equations of MHD can be used to model plasma turbulence where the perturbed magnetic field is small when compared to the mean magnetic field and is essentially perpendicular to it, with particular relevance to solar system and astrophysical applications

We have developed high order finite difference simulations of the two dimensional isothermal equations of MHD. We compare third and fourth order Runge-Kutta time iteration methods for turbulence simulations. Spatial derivatives are estimated to sixth order. These solvers are less computationally intensive than their spectral equivalents and posses near spectral accuracy, as discussed by Brandenburg and Dobler (2002). This computational saving allows for higher resolution simulations for a given amount of computational resources.

The results presented in this paper study the effect of compressibility on the small scale turbulent dynamics of 2D MHD turbulence. Particular attention is paid to the importance of the development of shock like structures in which energy can be strongly dissipated. These structures can be thought to be particularly important due to their relevance to the She-Leveque intermittency correction, which seems to be the currently favored model for MHD turbulence. The She-Leveque intermittency correction and its extension to MHD turbulence is discussed in Politano and Pouquet (1995).

$$\zeta_p = p/g - xp/a + C_0[1 - (1 - x/C_0)^{p/g}] \qquad (1)$$

Here ζ_p are the scaling exponents describing the statistical self-similarity in the Elsässer field structure functions $\langle \delta z_l^{(\pm)} \rangle \sim l^{\zeta_p}$ where l is a differencing length and the Elsässer field variables are defined as $\mathbf{z}^{(\pm)} = \mathbf{v} \pm \mathbf{B}(\mu_0 \rho)^{-1/2}$ in which $\mathbf{B}$, v, ρ and μ_0 are the fluid velocity, the magnetic field, the fluid density and magnetic diffusivity respectively. The parameters in (1): a, g, and x relate to the phenomenology used such that the field difference $\langle \delta z_l^{(\pm)} \rangle \sim \langle \epsilon_l^{1/g} \rangle l^{1/a}$, where ϵ_l is the local rate of dissipation averaged over a ball of radius l, and the cascade time $t_l \sim l^x$. Phenomenological arguments are used to argue that the values of these numbers only linearly relate ζ_p to p so any non-linear dependence, or multifractal scaling, must come from other reasoning. She and Leveque argued that this non-linear dependence can be determined by the space filling nature of the most dissipative structures in the flow. This is described by the parameter C_0 in (1) which is the codimension of these structures. The importance of dissipation by shock like structures in compressible MHD turbulence now becomes apparent because of the affect this will have on the value of C_0

Investigations are performed both for freely decaying and driven turbulence and differences in the resulting scaling laws are noted.

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