Analysis of Dissipation in MHD Turbulence Simulations in Two and Three Dimensions

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Abstract: The present work focuses on the statistical of properties of dissipation in MHD turbulence simulations in two and three dimensions. Particular attention is paid to the problem of combined field and dissipation scaling after Komogorov's *refined similarity hypothesis*. The three dimensional data is from the simulation of Biskamp and Müller *Phys. Plasmas* **7**, 4889 (2000). The two dimensional data is from a simulation using a high order finite difference code.

1. MHD Turbulence Phenomenology

Scaling exponents (ζ_p, τ_p) give a statistical description of MHD turbulence

$$S_l^{(\pm)p} = \left\langle \partial z_l^{(\pm)p} \right\rangle = \left\langle \left| z^{(\pm)}(x+l)\cdot\frac{l}{|l|} - z^{(\pm)}(x)\cdot\frac{l}{|l|} \right|^p \right\rangle \sim l^{\zeta_p^{\pm}}$$

$$\left\langle \varepsilon_l^p \right\rangle = \left\langle \left(\frac{1}{l^D}\int_{l'} \varepsilon(x+l')dl'^3 \right)^p \right\rangle \sim l^{\tau_p}$$

Where $z^{\pm} = v \pm B/\sqrt{\rho\mu_0}$ are the Elsasser field variables, l is a differencing length, ε is the local rate of dissipation, D is the number of spatial dimensions v is the velocity field, B is the magnetic field and ρ is the fluid density.

Models:

Random eddy scrambling after Kolmogorov (K41)

Favoured model of 3D MHD turbulence

$$S_l^{(\pm)p} \sim \left\langle \varepsilon_l^{p/3} \right\rangle l^{p/3}$$

Alfven wave collisions after Iroshnikov and Kraichnan (IK)

Favoured model for 2D MHD turbulence

$$S_l^{(\pm)p} \sim \left\langle \varepsilon_l^{p/4} \right\rangle l^{p/4}$$

2. Dissipation Structure Functions

1D measure based on Elsasser field gradient

$$\left\langle \varepsilon_l^{(\pm)p} \right\rangle = \left\langle \left(\frac{1}{l}\int_0^l \left(\partial_i z_i^{(\pm)}(x+l_1,t)\right)^2 dl_1 \right)^p \right\rangle$$

3. Extended Self-Similarity (ESS)

•Numerical constraints inhibit the appearance of an extensive inertial range for direct simulations of the equations of MHD

•Use Extended Self-Similarity (ESS) which extends the scaling laws into the dissipation range such that:

$$S_l^{(\pm)p} \sim l^{\zeta_p} \qquad \left\langle \varepsilon_l^p \right\rangle \sim l^{\tau_p}$$

In the inertial range

$$S_l^{(\pm)p} \sim S_l^{(\pm)q\left(\zeta_p/\zeta_q\right)} \qquad \left\langle \varepsilon_l^p \right\rangle \sim \left\langle \varepsilon_l^q \right\rangle^{\left(\tau_p/\tau_q\right)}$$

In the inertial and dissipation range

•Models used to explain these scaling laws often use the refined similarity hypothesis (RSH). Two versions of this are shown below in ESS compatible form.

Random eddy scrambling
Kolmogorov 1941 (K41)

$$S_l^{(\pm)p} \sim \left\langle \varepsilon_l^{p/3} \right\rangle \left(S_l^3 \right)^{p/3}$$

Alfven wave collisions
Iroshnikov / Kraichnan (IK)

$$S_l^{(\pm)p} \sim \left\langle \varepsilon_l^{p/4} \right\rangle \left(S_l^4 \right)^{p/4}$$

•Directly testing these relations is an important consistency check for currently favored models of turbulence.

•ESS in the local rate of dissipation (as shown above) is also implied by these relations

4. Previous Work

•Analysis performed on 3D decaying turbulence simulation of Biskamp and Müller [1]
•ESS in the local rate of dissipation was recovered [2]
•This was found to be consistent with the K41 based model proposed in [1]
•The K41 RSH shown in section 3 was not verified to high order

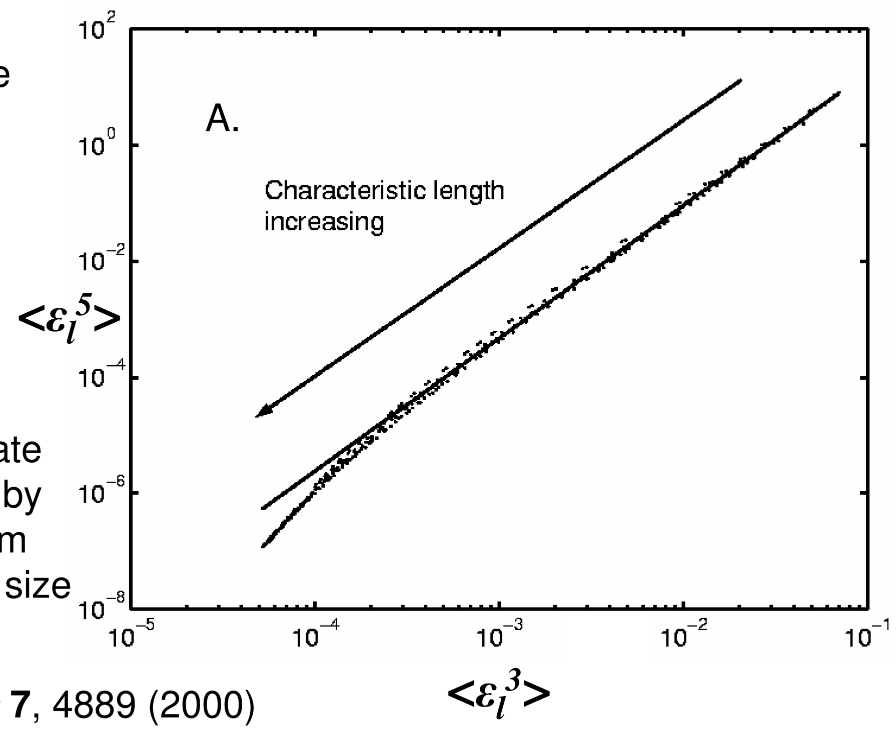


Figure A: Extended self-similarity in the local rate of dissipation (see section 3) as approximated by the 1D measure in section 2. The deviation from power law scaling at large l is probably a finite size effect

[1] D. Biskamp and W.-C. Müller *Phys. Plasmas* **7**, 4889 (2000)
[2] J. A. Merrifield *et.al. Phys. Plasmas* **12**, 022301 (2005)

5. Numerical Scheme

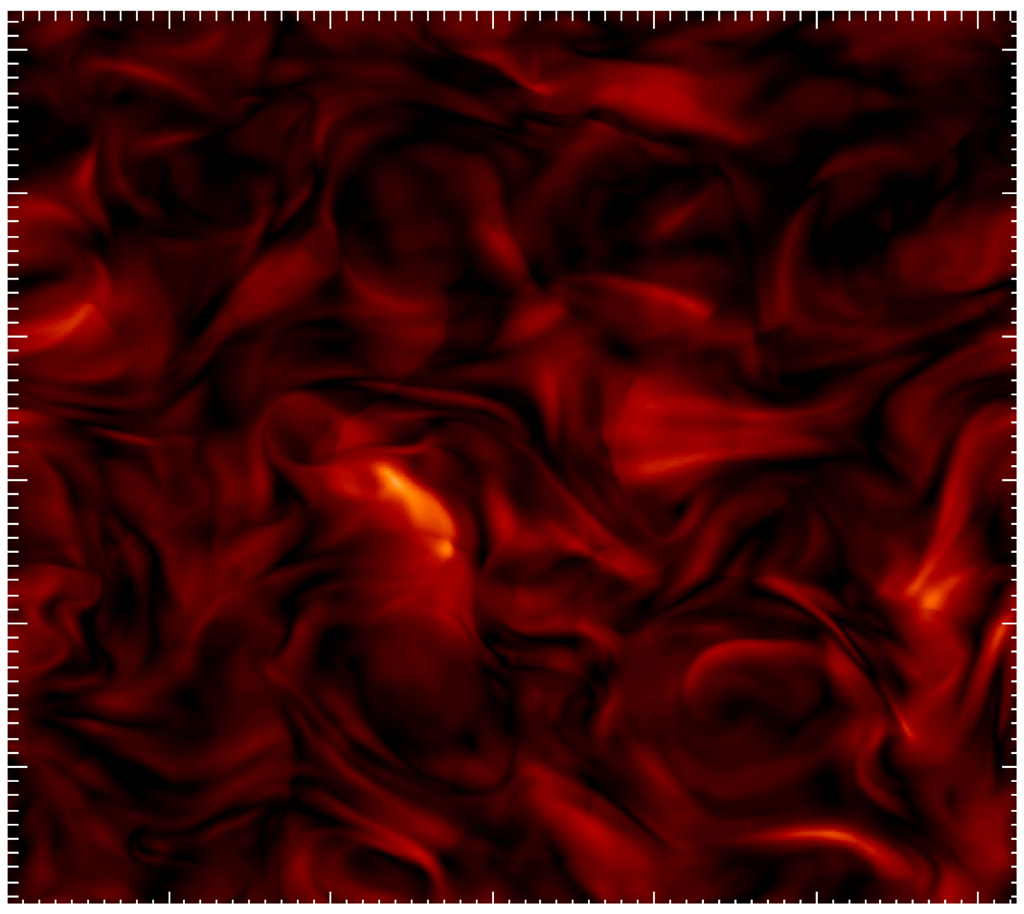
•We have developed high order finite difference simulations of the two dimensional isothermal equations of MHD on a 1024 square grid with periodic boundaries

•Spatial derivatives are calculated to sixth order and time is iterated by a fourth order Runge-Kutta method

•The simulation is driven by forcing v and B in the wave number range 4±0.5 with random phases. The driver is divergence free

•A steady state is reached with a Mach number of ≈0.25

Shown right is a contour plot of velocity magnitude from a time when the turbulence is fully developed



6. 2D Results

•The driving scheme leads to a small alignment of the velocity and magnetic fields (Figure B.)

•Extended self-similarity is recovered from Elsasser field variables as expected (Figure C.)

•Statistics from 2D runs enable us to address the combined field and dissipations scaling laws (RSH see section 3). Results are tabulated in Table 1.

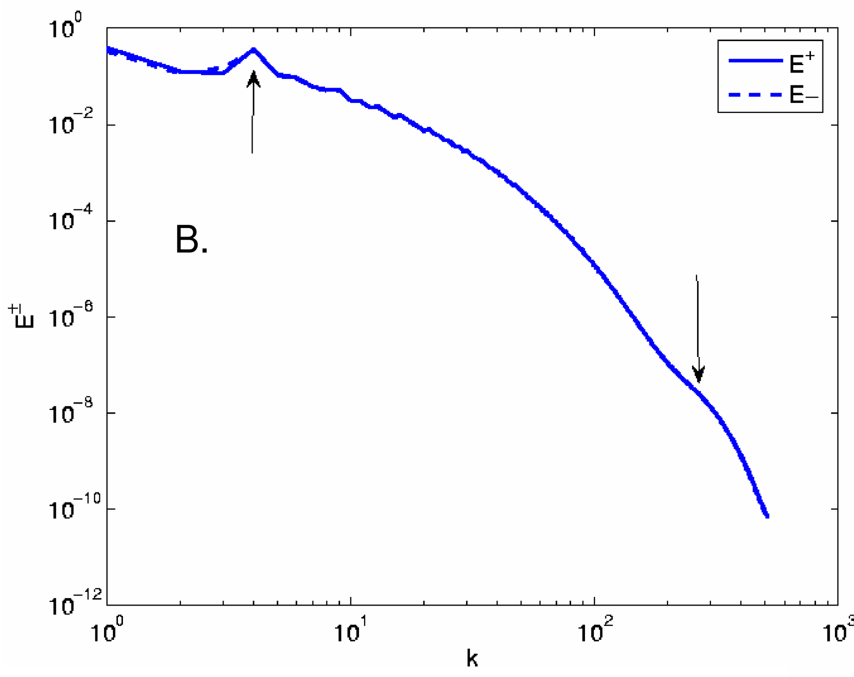


Figure B: Power spectra of the z^+ and z^- Elsasser field variables (E^+ and E^- respectively). Marked are the driving shell (k=4) showing enhanced power and the Kolmogorov dissipation scale where energy falls off rapidly. Both spectra overlie perfectly suggesting the overall alignment of v with B is small. A high degree of field alignment kills the turbulent dynamics.

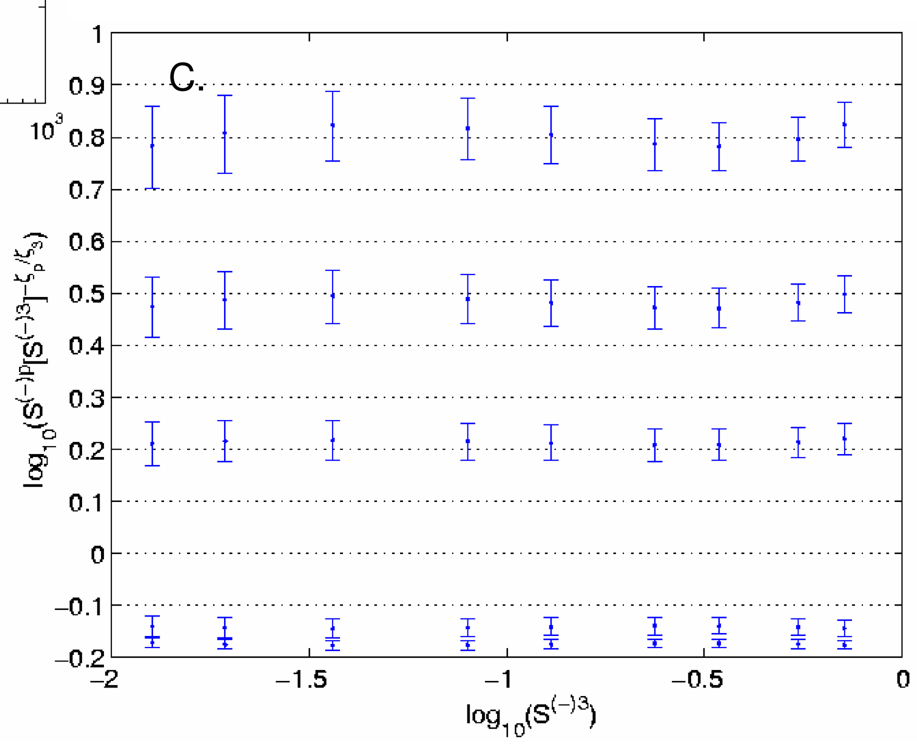


Figure C: ESS obtained from the z^- Elsasser field. Orders p=6 to p=1 top to bottom (excluding p=3 since this necessarily gives perfect scaling). The y axis has been normalised so that perfect scaling appears as a horizontal line. Error bars show the standard error present in the time averaging process.

Table 1: The scaling relations found when the RSH relations of section 3 are tested. Values entered are the gradients found from plotting these hypotheses on a log-log plot. A value of 1 represents perfect agreement. IK or K41 refers to the version of the hypothesis and the symbols + or – represent whether the scaling is derived from z^+ or z^- fields. Neither hypothesis fits our data perfectly.

--	p=1	p=2	p=3	p=4	p=5
K41 +	1.02	1.01	---	0.97	0.93
K41 -	1.02	1.02	---	0.96	0.91
IK	1.12	1.10	1.06	---	0.94
IK	1.14	1.11	1.07	---	0.93

7. Conclusions

•The problem of combined field and dissipation scaling has been addressed

•Neither IK or K41 versions of the ESS compatible RSH agree with our results perfectly

•Low order measures, for which statistical confidence is higher, are better approximated by K41

•Some previous high resolution simulations suggest that the IK phenomenology is most appropriate for 2D MHD turbulence [3,4]

•A satisfactory model for the Elsasser structure functions see (section 1) remains elusive

•The theoretical divergence uncovered here may contribute to this

[3] D. Biskamp and E. Schwartz *Phys. Plasmas* **8**, 3282 (2001)
[4]H. Politano *et.al.* Europhys. Lett **43**, 516 (1998)

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