Analysis Of Incompressible 3D MHD Turbulence As Applied To The Solar Wind

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- "Extended Self-Similartiy" analysis performed on 3D MHD DNS of *Biskamp and Muller, POP, 2000*
- Application to Solar Wind

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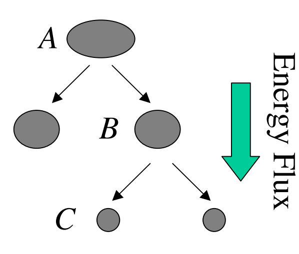
Scaling Laws

Energy Flux

- Branching Process (*Direct Cascade shown here)*
  - Large Scale eddies (pumping or driving scale)
  - *Cascade* to smaller scales unaffected by dissipation (*inertial range)*
  - Dissipate at small scales (dissipative range)

- At length scales far from the driving or dissipation scale *A*→*B* is a scaled version of *B*→*C*
- This is captured by scaling laws
  - Structure functions

$$\left\langle \partial v_l^p \right\rangle = \left\langle \left| v(x+l) - v(x) \right|^p \right\rangle \sim l^{\phi_p}$$



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Elsasser Field Structure Functions $S^p{}_l$

$$z^{\pm} = v \pm \frac{B}{\sqrt{\rho\mu_0}}$$

•Describes Left and Right travelling Alfvenic disturbances

$$\frac{\partial z^{\pm}}{\partial t} + z^{\mathrm{m}} \cdot \nabla z^{\pm} = -\frac{\nabla P}{\rho} + \mu_{+} \nabla^2 z^{\pm} + \mu_{-} \nabla^2 z^{\mathrm{m}}$$

•Can describe incompressible equations of MHD in Elsasser symmetric form see *Biskamp, MHD Turbulence, Camb. Univ. Press 2003*

$$S_l^p = \left\langle \partial z_l^{(\pm)p} \right\rangle = \left\langle \left| z^{(\pm)}(x+l) \cdot \frac{l}{|l|} - z^{(\pm)}(x) \cdot \frac{l}{|l|} \right|^p \right\rangle$$

•Construct Elsasser field structure functions ($S^p{}_l$) to describe Magneto-kinetic fluid

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Fluid Interpretation of Structure Function Scaling Laws *(Fluid Phenomenology)*

$$S_l^{(\pm)1} = \left\langle \partial z_l^{(\pm)} \right\rangle \sim \varepsilon_l^{1/\alpha} l^{1/g}$$

ε_l = local rate of dissipation averaged over ball of radius l

Kolmogorov 1941 (K41):

local in k space

nonlinear process is random eddy scrambling

g=3, α=3

Iroshnikov-Kraichnan (IK) *Kraichnon, POF, 1965* :

non-local in k space

nonlinear process is governed by Alfvenic collisions

g=4, α=4

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Universal Scaling Laws In Turbulence

- Scaling laws attractive; exponents independent of flow detail provided homogenous and isotropic

•*Universal*

$$S_l^{(\pm)p} \sim l^{\xi_p} \qquad \chi_l^{(\pm)p} = \left\langle \left[\int \frac{\left(\partial_i z_i^{(\pm)}\right)^2}{V_l} dV_l \right]^p \right\rangle \sim l^{\tau_p}$$

•$S_l^{(\pm)p}$ and χ_l^p constructed from simulation to determine ξ_p and τ_p

•χ_l acts as a 1D surrogate to ε_l for which we anticipate the same scaling see *Sreenivasan Annu. Rev. Fluid Mech. 1997*

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She-Leveque (1994) Intermittency Correction

•In practice $S_l^{(\pm)}$ does not behave as IK or K41

→ *Intermittent* eddy activity .... use theory of She-Leveque

- All parameters have a physical interpretation; C_o = codimension of most dissipative structures

$$\tau_p = -\frac{2}{g} p + C_o - C_o \left(1 - \frac{2}{gC_o}\right)^p$$

•Refined Similarity Hypothesis gives:

$$\xi_p = -\frac{2}{\alpha g} p + C_o - C_o \left(1 - \frac{2}{gC_o}\right)^{p/\alpha} + \frac{p}{g}$$

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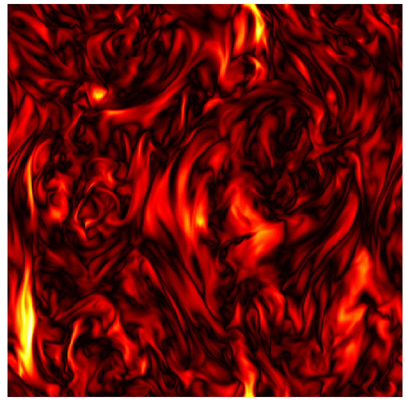
The DNS *Biskamp and Muller, POP, 2000*

- Pseudo-spectral incompressible MHD solver
- 512^3 Fourier modes
  - 3 dimensional
- Turbulence is decaying:
  - Smooth time evolution so relatively few large eddy turn over times give good statistics
  - Results independent of any driving scheme
- Micro-scale Re = 94
- Kinematic Viscosity = Magnetic Diffusivity



2D slice of velocity magnitudes from 3D simulation

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Normalisation and Extended Self-Similarity

- Since turbulence decaying, need normalisation before time averaging
  - Elsasser fields normalised to total energy (***E***)
  - χ_l normalised to average local rate of dissipation ($<\varepsilon>$)

    $S'^p_l = S^p_l / E^{p/2}$ and $\chi'^p_l = \chi^p_l / <\varepsilon>^p$
- Find scaling laws from DNS via Extended Self Similarity (ESS), see *Benzi et al., PRE, 1993*

$$S_l^p \sim S_l^{q\left(\xi_p/\xi_q\right)} \qquad \chi_l^p \sim \chi_l^{q\left(\tau_p/\tau_q\right)}$$

- these hold in the inertial and dissipative range (above the dissipative length scale)
- Scaling law is *extended* into the dissipative range.

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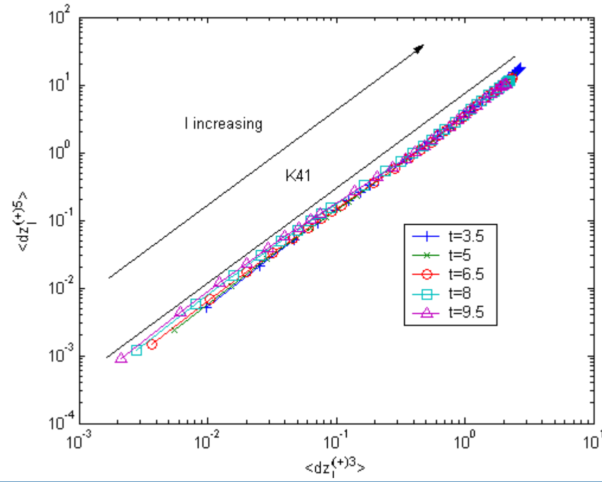
Known: ESS in Elsasser fields (as investigated in *Biskamp and Muller, POP, 2000)*



$$S'^{p}_{l} \sim S'^{q\left(\xi_p/\xi_q\right)}_{l}$$

- Structure functions time averaged
- Values for ξ_p/ξ_3 measured

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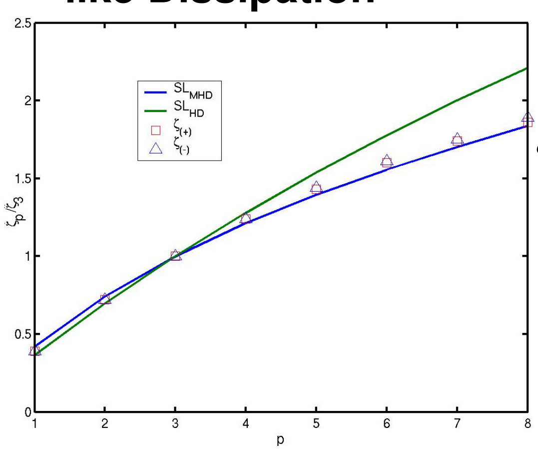
Known: Scaling Exponents for Sheet-like Dissipation



Blue curve is overlay of ξ_p/ξ_3 with

$$\xi_p = \frac{p}{g}\left(1-\frac{2}{\alpha}\right)+C_o-C_o\left(1-\frac{2}{gC_o}\right)^{p/\alpha}$$

g = 3 α = 3 (K41)

C_o = 1 (most intensely dissipating structures are sheet-like)

see Biskamp and Muller, POP, 2000

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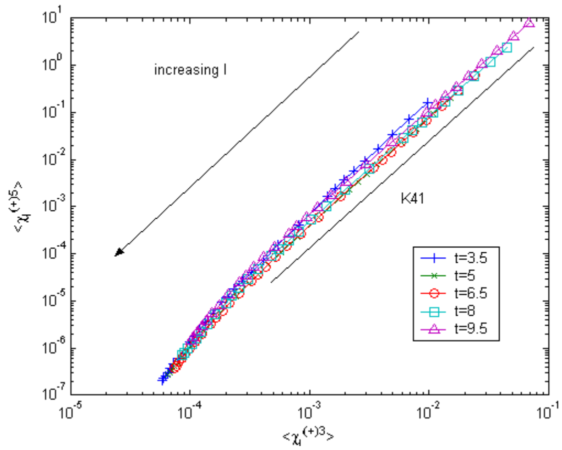
New: ESS in 1D Surrogate of Dissipation



$$\chi'^{\,p}_{l} \sim \chi'^{\,q\left(\tau_p/\tau_q\right)}_{l}$$

SL scaling of *Biskamp and Muller, POP, 2000* requires this self similarity in χ_l^p

Scaling recovered relies mostly on small ***l*** measurements → dissipative range

•Break of scaling at large ***l*** could be finite size effect

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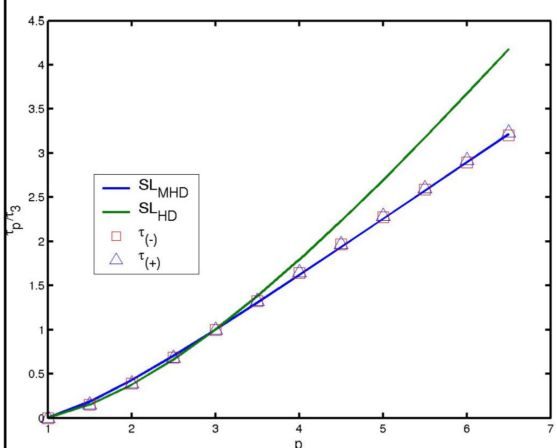
Recover Scaling Exponents



Blue curve is an overlay of τ_p/τ_3

$$\tau_p = -\frac{2}{g}p + C_o - C_o\left(1 - \frac{2}{gC_o}\right)^p$$

g = 3 α = 3 (K41)

Co = 1 (sheet-like most intensely dissipating structures)

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Application to Solar Wind despite mean $\overline{B}$ anisotropy

- Has been applied to ion thermal velocities measured from ACE see *A. Bershadskii, POP, 2003*
- Effect of mean field on exponents is quite weak for fluctuations $\perp$ to $\overline{B}$
- Smooth crossover to 2D behaviour seen for large $\overline{B}$

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Conclusions

- New: Self-similarity has been demonstrated in the local rate of dissipation using ESS
  - This is an expected corollary of the SL interpretation given in *Biskamp and Muller, POP, 2000*
- Ratios of measured scaling exponents support the findings of *Biskamp and Muller, POP, 2000*
  - Non-linear transfer by random eddy scrambling
  - Most strongly dissipating structures are sheet-like

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