Physics of Fusion

Lecture 1: The basics

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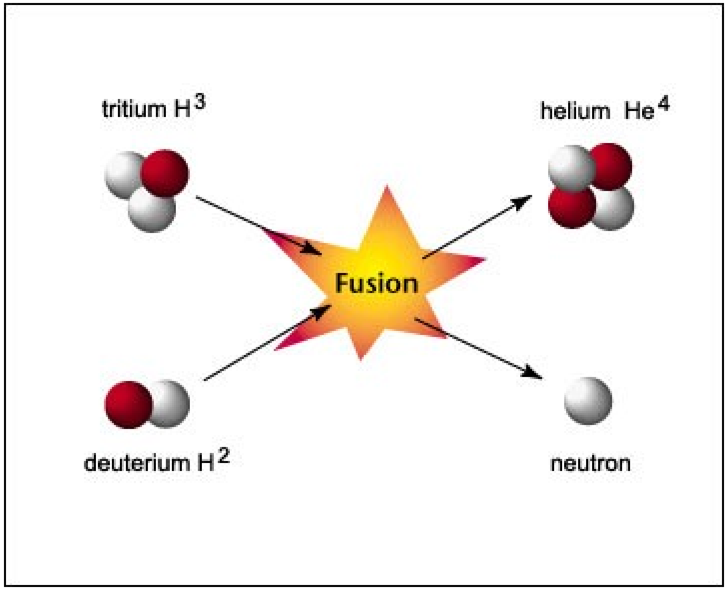
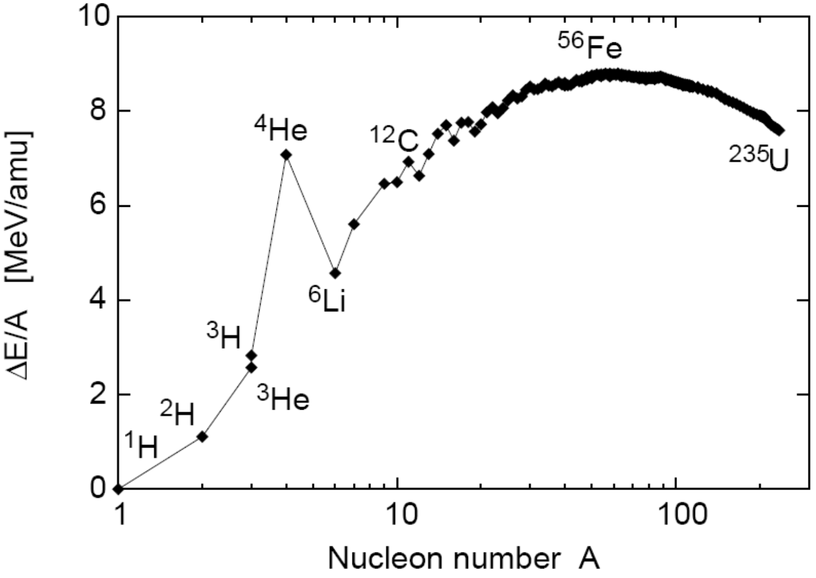
Physics of Fusion

- Fusion here refers to the controlled process in which two light atoms are fused together generating a heavier atom with the aim of generating energy



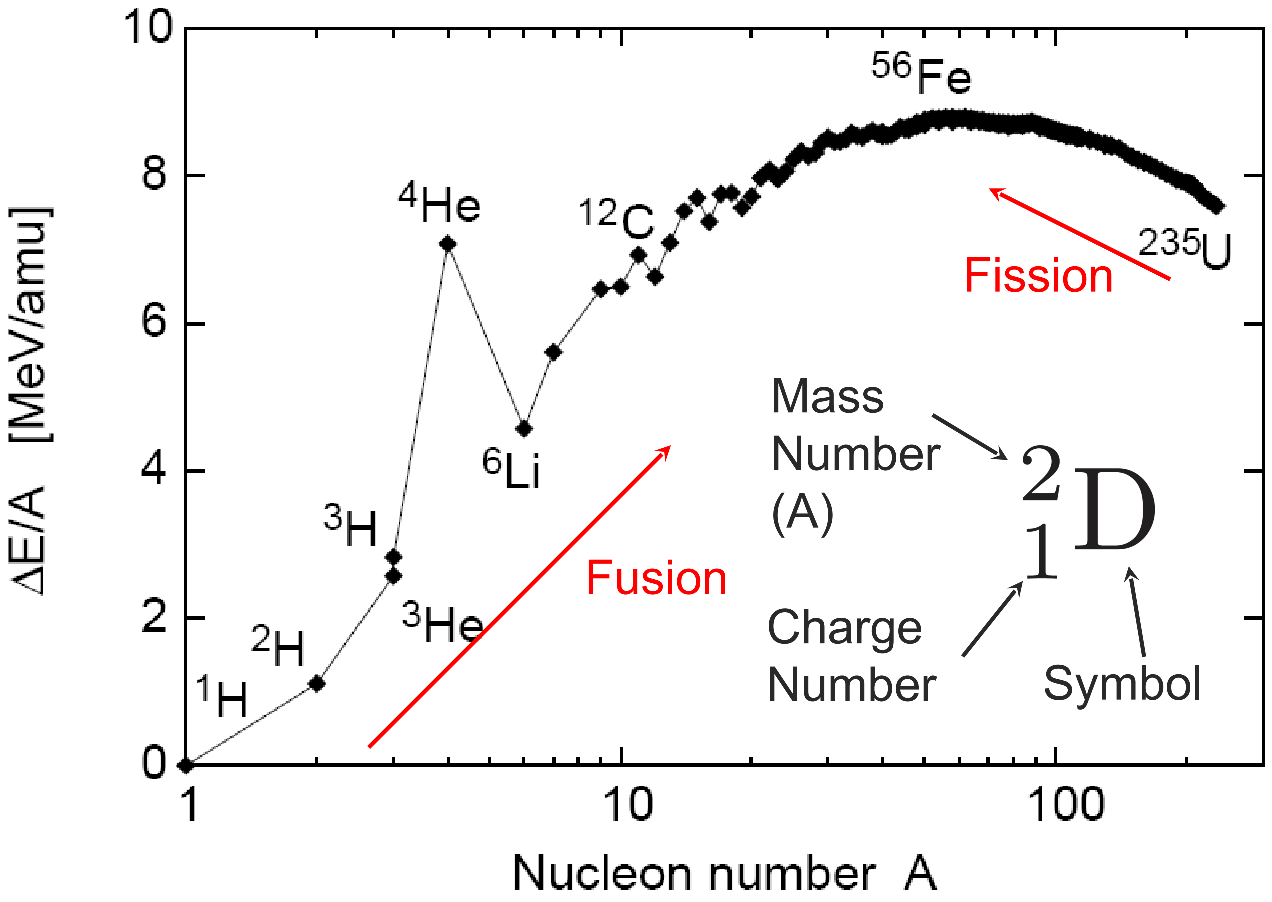
Binding Energy

- Binding energy is the energy that is released when a nucleus is created from protons and neutrons
- It is released during the formation of a nucleus
- The greater the binding energy per nucleon in the atom, the greater the atom's stability.



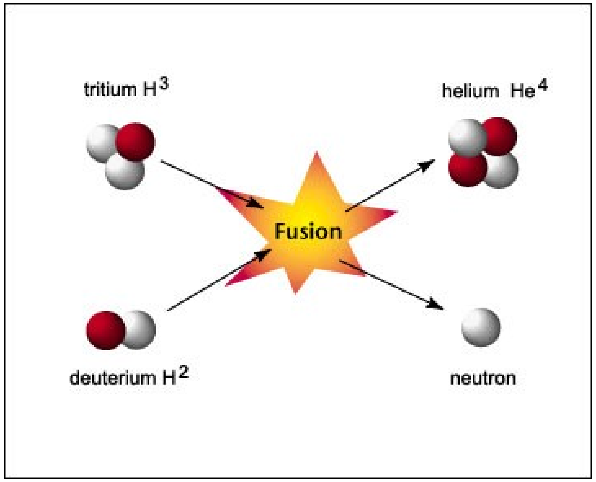
Binding energy (in MeV) per particle as a function of the mass number (A)

Fusion and Fission



Relevant fusion reactions

- Often considered fusion reactions (Note more than one reaction possible)

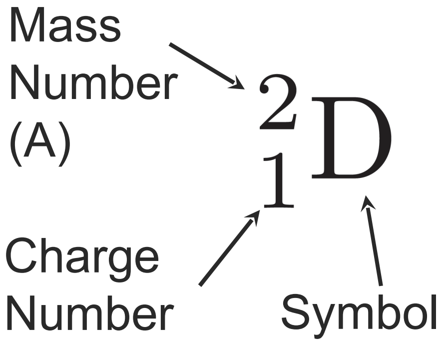


$${}^{2}_{1}\mathrm{D} + {}^{3}_{1}\mathrm{T} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + 17.6\ \mathrm{MeV}$$

$${}^{2}_{1}\mathrm{D} + {}^{2}_{1}\mathrm{D} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + 3.7\ \mathrm{MeV}$$

$${}^{2}_{1}\mathrm{D} + {}^{2}_{1}\mathrm{D} \rightarrow {}^{3}_{1}\mathrm{T} + {}^{1}_{1}\mathrm{H} + 4.03\ \mathrm{MeV}$$

$${}^{2}_{1}\mathrm{D} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H} + 18.3\ \mathrm{MeV}$$



Mass Number (A)

$${}^{2}_{1}\mathrm{D}$$

Charge Number

Symbol

Calculation of energy released

- The released energy follows from the mass deficit. Consider the reaction

$$ {}^{2}_{1}\mathrm{D} + {}^{3}_{1}\mathrm{T} \longrightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} $$

- The masses of the different products are

$$ m_D = (2 - 0.000994) m_H \qquad m_T = (3 - 0.006284) m_H $$

$$ m_{He} = (4 - 0.027404) m_H \qquad m_n = (1 + 0.001378) m_H $$

- The mass deficit (Total mass before minus total mass after) is

$$ \Delta m = 0.0187 m_H \qquad m_H = 1.6727 \cdot 10^{-27}\ \mathrm{kg} $$

Calculation of the released energy

- The mass deficit is

$$\Delta m = 0.0187 m_H \qquad m_H = 1.6727 \cdot 10^{-27} \text{ kg}$$

- The energy then follows from Einstein's formula

$$E = mc^2 = 0.0187 m_H c^2 = 2.8184 \cdot 10^{-12} \text{ J}$$

- Used unit of energy is the electron volt (eV), kilo-electron volt (1keV = 1000 eV) or Mega-electron volt (1 MeV = 10^6 eV) $1 \text{ eV} = 1.6022 \cdot 10^{-19} \text{ J}$

$$E = \frac{2.8184 \cdot 10^{-12}}{1.6022 \cdot 10^{-19}} \text{ eV} = 17.56 \text{ MeV}$$

Energies in the MeV range are far in excess of usual chemical reactions

- 1 kg of a Deuterium/Tritium mixture would allow for a number of fusion reactions N

$$N = \frac{0.5}{2.5 \cdot 1.67 \cdot 10^{-27}} = 1.2 \cdot 10^{26}$$

- This amount of reactions would generate an energy

$$E = N 2.8184 \cdot 10^{-12}\ \mathrm{J} = 3.4 \cdot 10^{14}\ \mathrm{J}$$

- This is around 4 GW for 24 hours

eV is also used as the unit of Temperature

- Temperature is always used to express an averaged energy. The unit is again eV, i.e.

$$T = kT_k/e \ (\mathrm{eV}) = 8.617 \cdot 10^{-5} T_k \ (\mathrm{eV})$$

- Where T is the temperature and T_k is the temperature in Kelvin.
- Note

1eV = 11605 K 17.56 MeV = 2 10^{11} K

Distribution of energy over the products

- The energy is released in the form of kinetic energy
- The kinetic energy is not equally distributed over the products since both energy as well as momentum need to be conserved

$$\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^2 + \frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^2 = E_{fus}$$

$$m_{\mathrm{A}} v_{\mathrm{A}} + m_{\mathrm{B}} v_{\mathrm{B}} = 0$$

- These equations can be solved to give

$$E_A = \frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^2 = \frac{m_B}{m_A + m_B} E_{\mathrm{fus}} \qquad E_B = \frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^2 = \frac{m_A}{m_A + m_B} E_{\mathrm{fus}}$$

Distribution of energy

- Momentum and energy conservation yield

$$E_A = \frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^2 = \frac{m_B}{m_A + m_B} E_{\mathrm{fus}} \qquad E_B = \frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^2 = \frac{m_A}{m_A + m_B} E_{\mathrm{fus}}$$

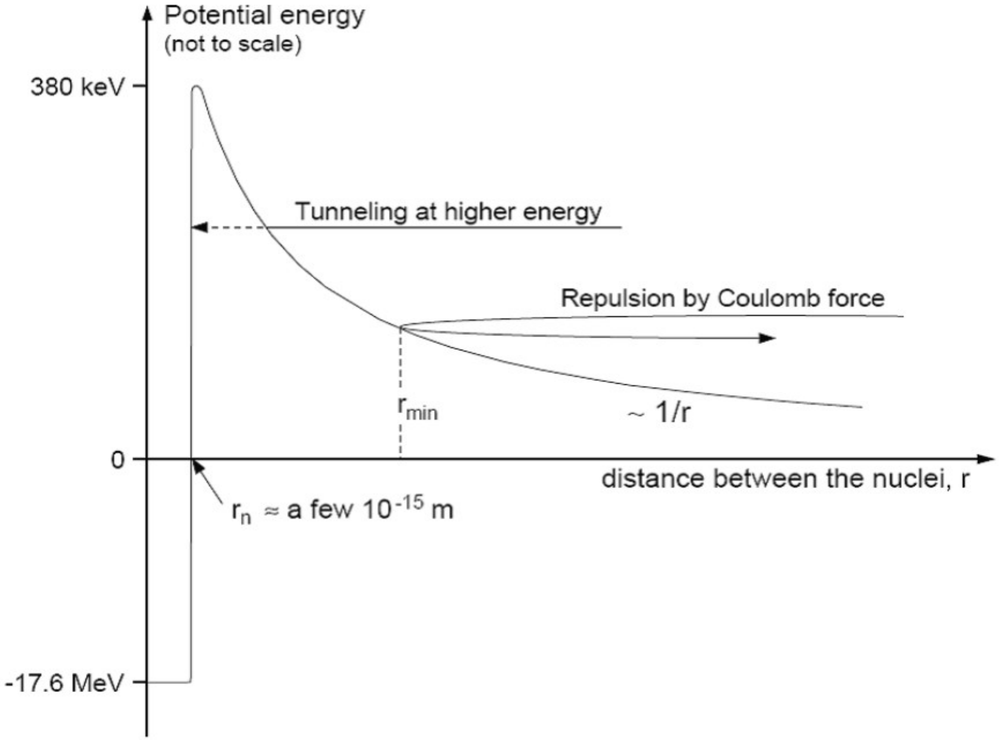
- Take the now famous reaction

$${}^{2}_{1}\mathrm{D} + {}^{3}_{1}\mathrm{T} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$$

- The Helium nuclei is roughly 4 times more heavy than the neutron and will thus acquire 20% of the energy (3.5 MeV) whereas the neutron obtains 80% (14.1 MeV)

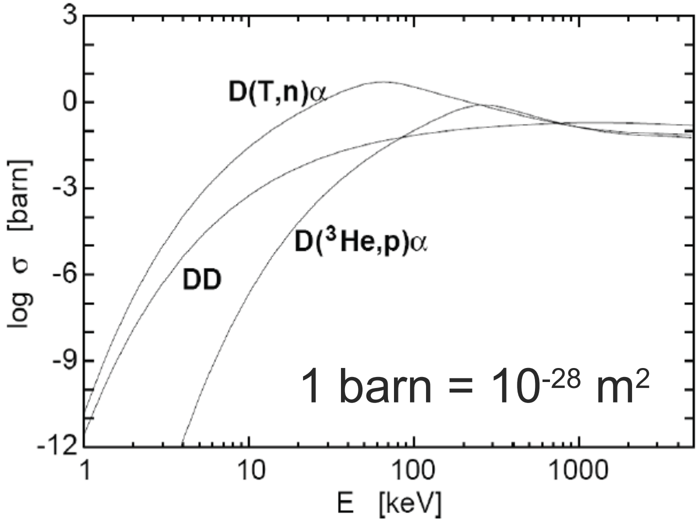
Key problem of fusion

- …. Is the Coulomb barrier



Cross section

- The cross section is the effective area connected with the occurrence of a reaction
- For snooker balls the cross section is πr^2 (with r the radius of the ball)



The cross section of various fusion reactions as a function of the energy. (Note logarithmic scale)

Averaged reaction rate

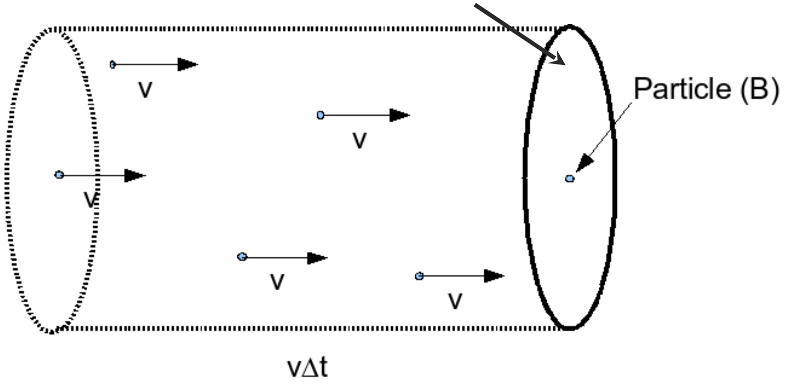
- One particle (B) colliding with many particles (A)
- Number of reactions in Δt is

$$\Delta N = n_A \sigma v \Delta t \quad \rightarrow$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = n_A \langle \sigma v \rangle$$

- Both σ as well as v depend on the energy which is not the same for all particles. One builds the average

The cross section σ

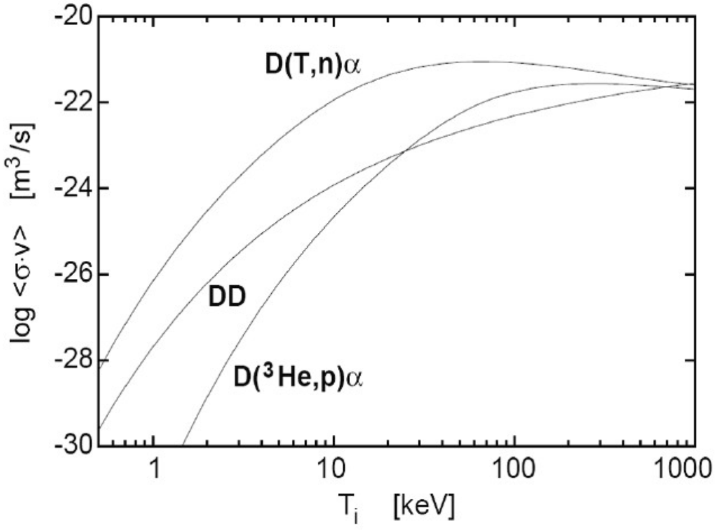


Schematic picture of the number of reactions in a time interval Δt

Averaged reaction rate …..

- The cross section must be averaged over the energies of the particles. Assume a Maxwell

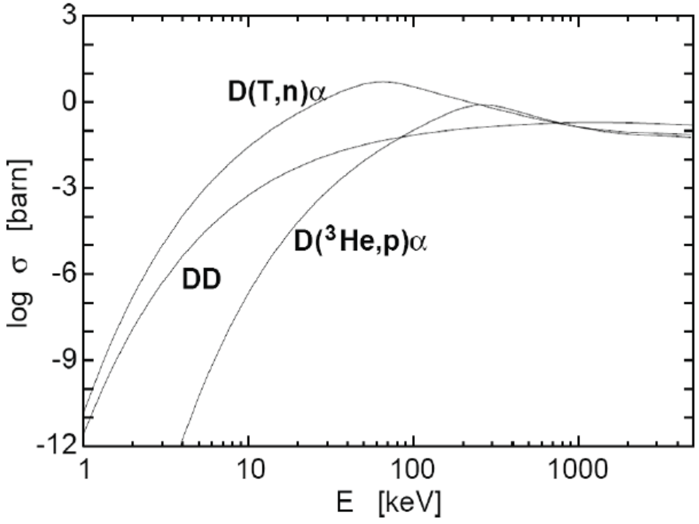
$$F_M(v) = \frac{n}{(2\pi T/m)^{3/2}} \exp\left[-\frac{mv^2}{2T}\right]$$



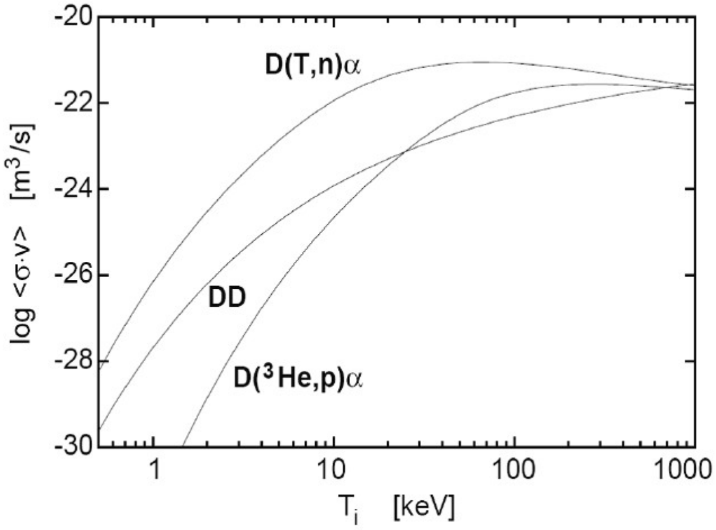
Averaged reaction rates for various fusion reactions as a function of the temperature (in keV)

Compare the two

The averaged reaction rate does not fall of as strongly when going to lower energies



Cross section as a function of energy



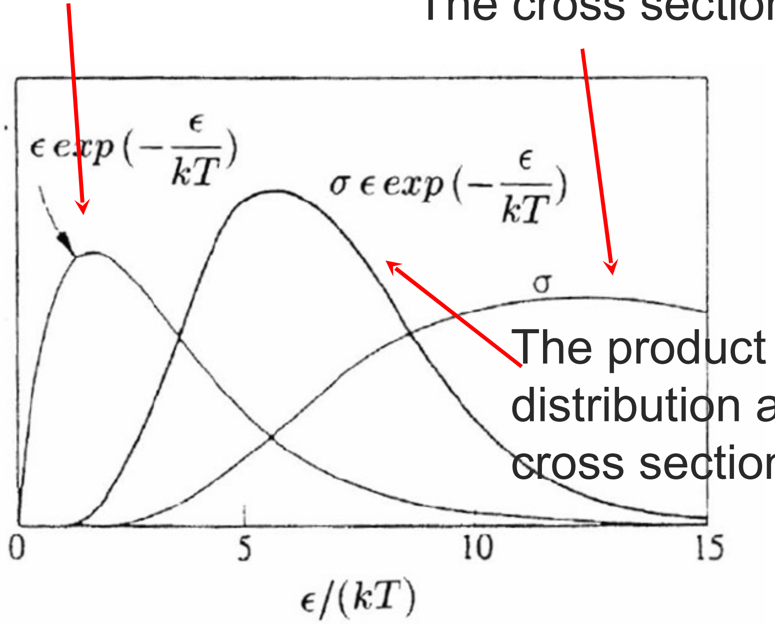
Averaged reaction rate as a function of Temperature

The reason…..

- Even for temperatures below the energy at which the cross section reaches its maximum, there is a sufficient amount of fusion reactions due to the number of particles in the tail of the Maxwell distribution

The Maxwell (multiplied with the velocity)

The cross section

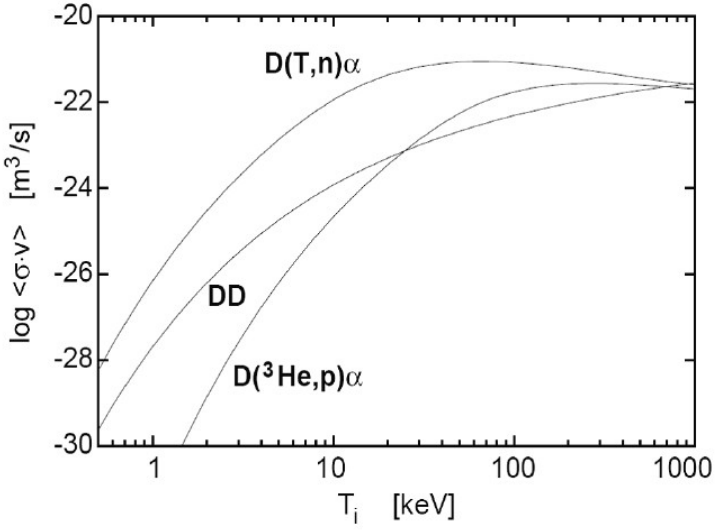


The product of distribution and cross section

Schematic picture of the calculation of the averaged reaction rate (Integrand as a function of energy)

Current fusion reactor concepts

- are designed to operate at around 10 keV (note this is still 100 million Kelvin, matter is fully ionized or in the plasma state)
- Are based on a mixture of Deuterium and Tritium
- Both are related to the cross section



Averaged reaction rates for various fusion reactions as a function of the temperature (in keV)

Availability of the fuel

- The natural abundance of Deuterium is one in 6700. There is enough water in the ocean to provide energy for 3 10^{11} years at the current rate of energy consumption (larger than the age of the universe)
- Deuterium is also very cheaply obtainable. Calculating the price of electricity solely on the basis of the cost of Deuterium, would lead to a drop of 10^{3} in your electricity bill
- Tritium is unstable with a half age of 12.3 years. There is virtually no naturally resource of Tritium

Availability of the fuel

- Tritium however can be bred from Lithium

$$ {}^{6}_{3}\mathrm{Li} + {}^{1}_{0}n \rightarrow {}^{4}_{2}\mathrm{He} + {}^{3}_{1}\mathrm{T} + 4.8\ \mathrm{MeV} $$

$$ {}^{7}_{3}\mathrm{Li} + {}^{1}_{0}n \rightarrow {}^{4}_{2}\mathrm{He} + {}^{3}_{1}\mathrm{T} + {}^{1}_{0}n - 2.5\ \mathrm{MeV} $$

- Note that the neutron released in the fusion reaction can be used for this purpose
- The availability of Lithium on land is sufficient for at least 1000 if not 30000 years, and the cost per kWh would be even smaller than that of Deuterium.
- If the oceans is included it is estimated that there is enough fuel for 3 10^7 years.

Why fusion ….

- There is a large amount of fuel available, at a very low price.
- Fusion is CO_2 neutral.
- It would yield only a small quantity of high level radio active waste.
- There is no risk of uncontrolled energy release.
- The fuel is available in all locations of the earth. Fusion is of interest especially for those regions that do not have access to other natural resources.
- There is only a small threat to non-proliferation of weapon material

But ……

- A working concept is yet to be demonstrated. The operation of a fusion reactor is hindered by several, in itself rather interesting, physics phenomena
- The cost argument isn’t all that clear, since the cost of the energy will be largely determined by the cost of the reactor.

Limitations due to the high temperature

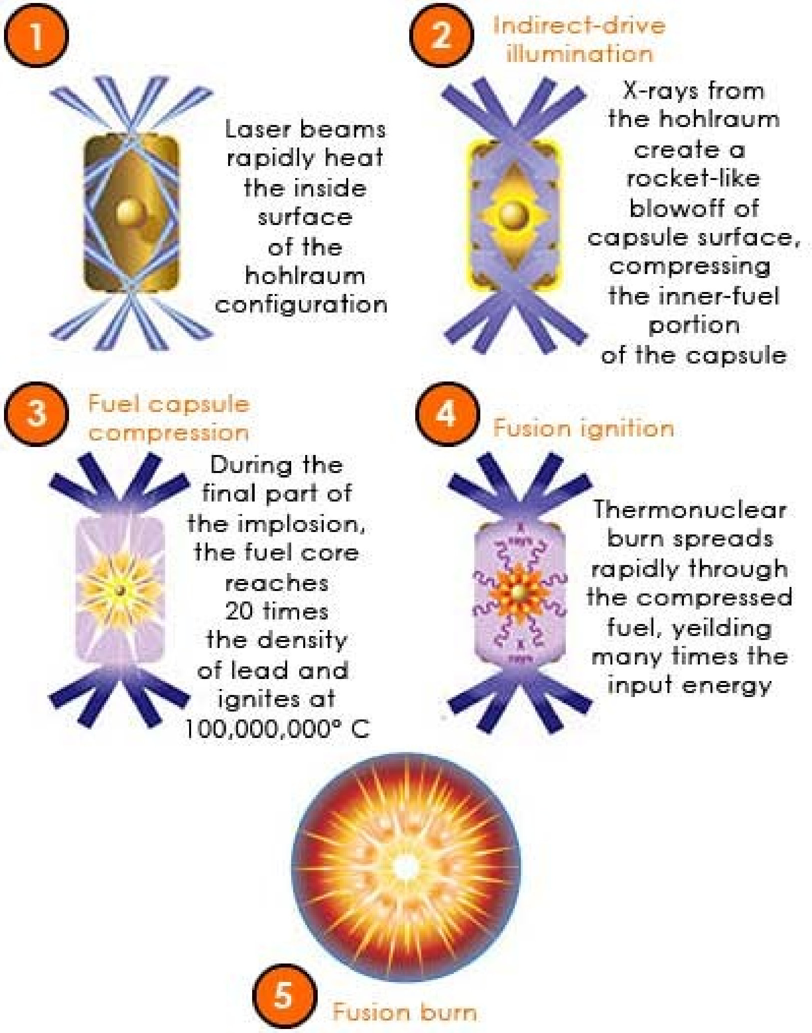
- 10 keV is still 100 million Kelvin (matter is fully ionized, i.e. in the plasma state)
- Some time scales can be estimated using the thermal velocity

$$v_{th} = \sqrt{2T/m}$$

- This is 10^6 m/s for Deuterium and 6 10^7 m/s for the electrons
- In a reactor of 10 m size the particles would be lost in 10 μs.

Two approaches to fusion

- One is based on the rapid compression, and heating of a solid fuel pellet through the use of laser or particle beams. In this approach one tries to obtain a sufficient amount of fusion reactions before the material flies apart, hence the name, inertial confinement fusion (ICF).



Magnetic confinement ..

- The Lorentz force connected with a magnetic field makes that the charged particles can not move over large distances across the magnetic field
- They gyrate around the field lines with a typical radius

$$\rho = \frac{m v_{th}}{ZeB}$$

At 10 keV and 5 Tesla this radius of 4 mm for Deuterium and 0.07 mm for the electrons