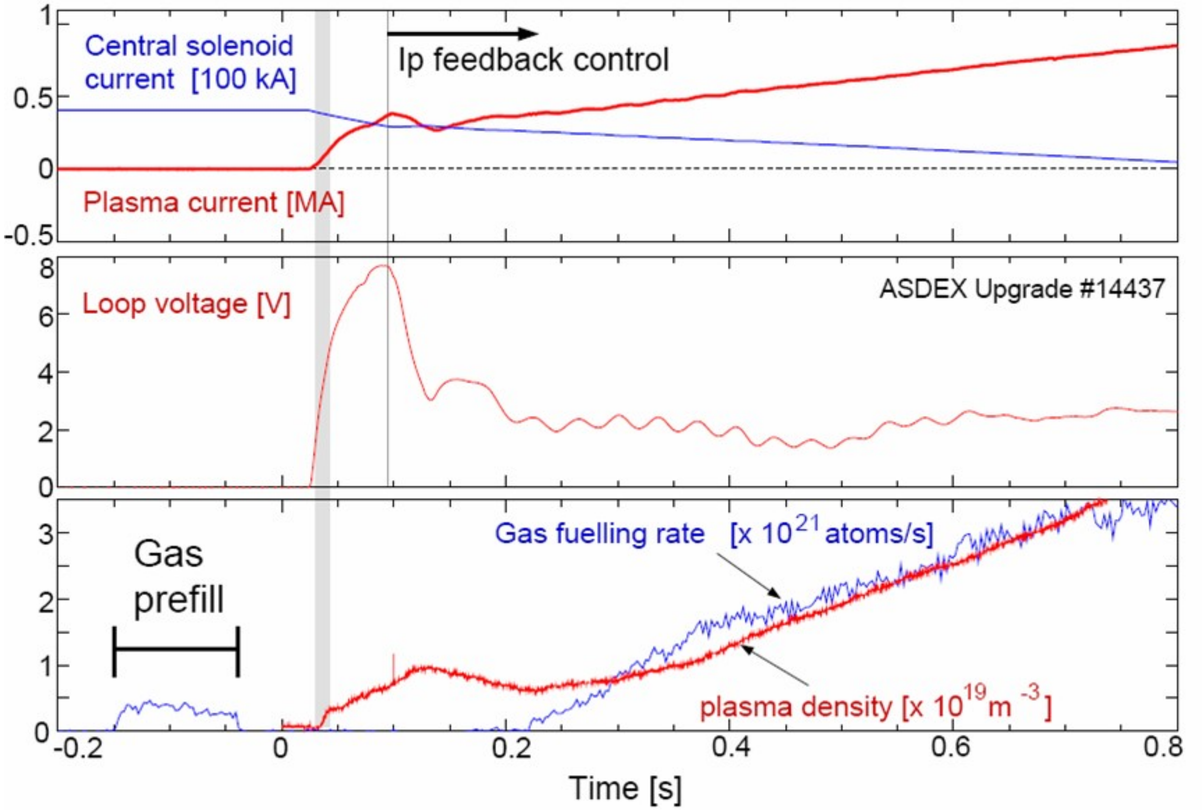
Physics of fusion power

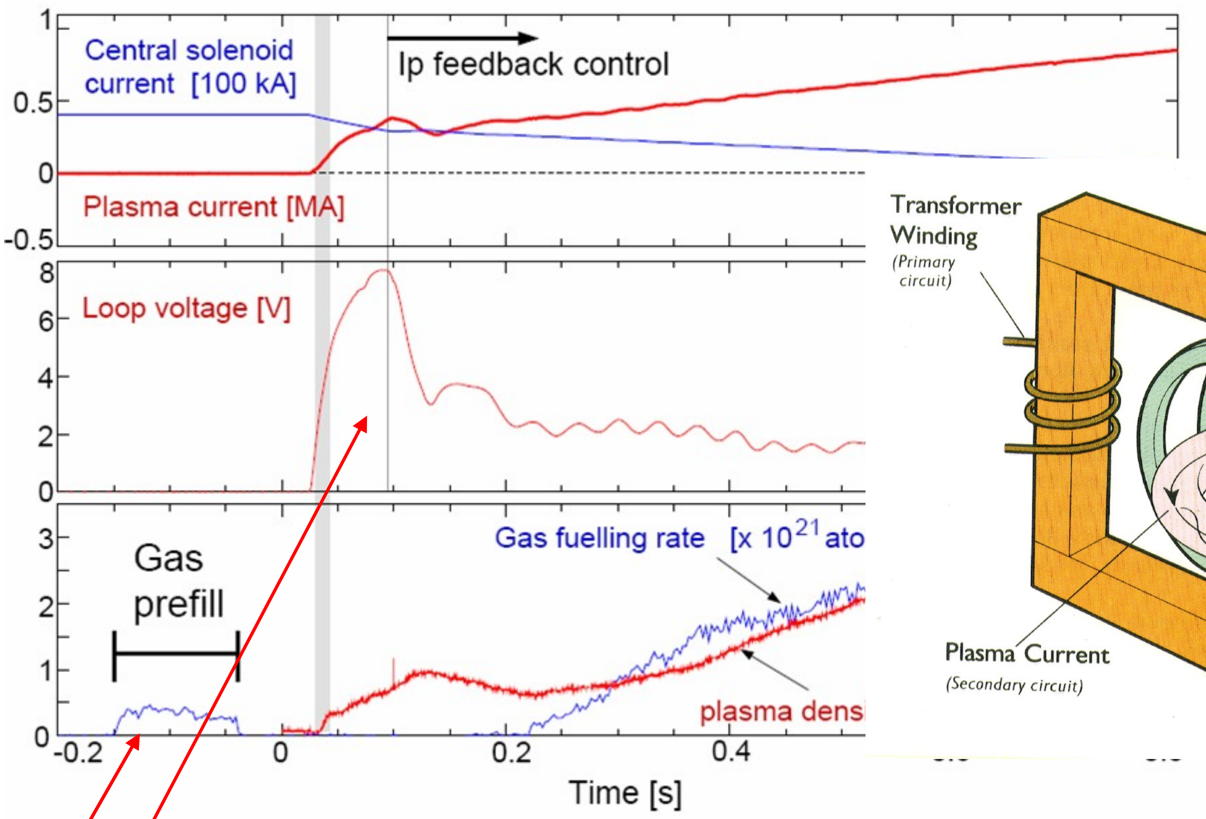
Lecture 10 : Running a discharge / diagnostics

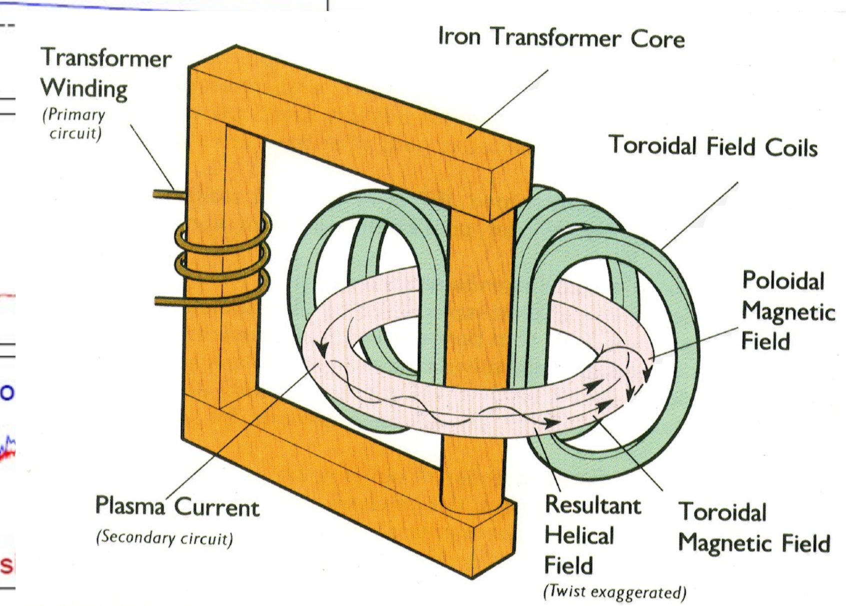
Startup



- Start with pumping the main vessel to obtain a good vacuum
- Then ramp up the toroidal field
- At the start of this picture there is a vacuum with a toroidal magnetic field

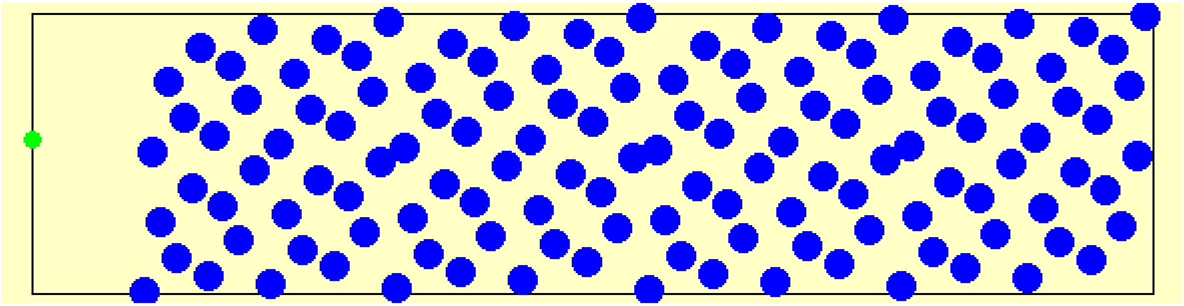
Startup





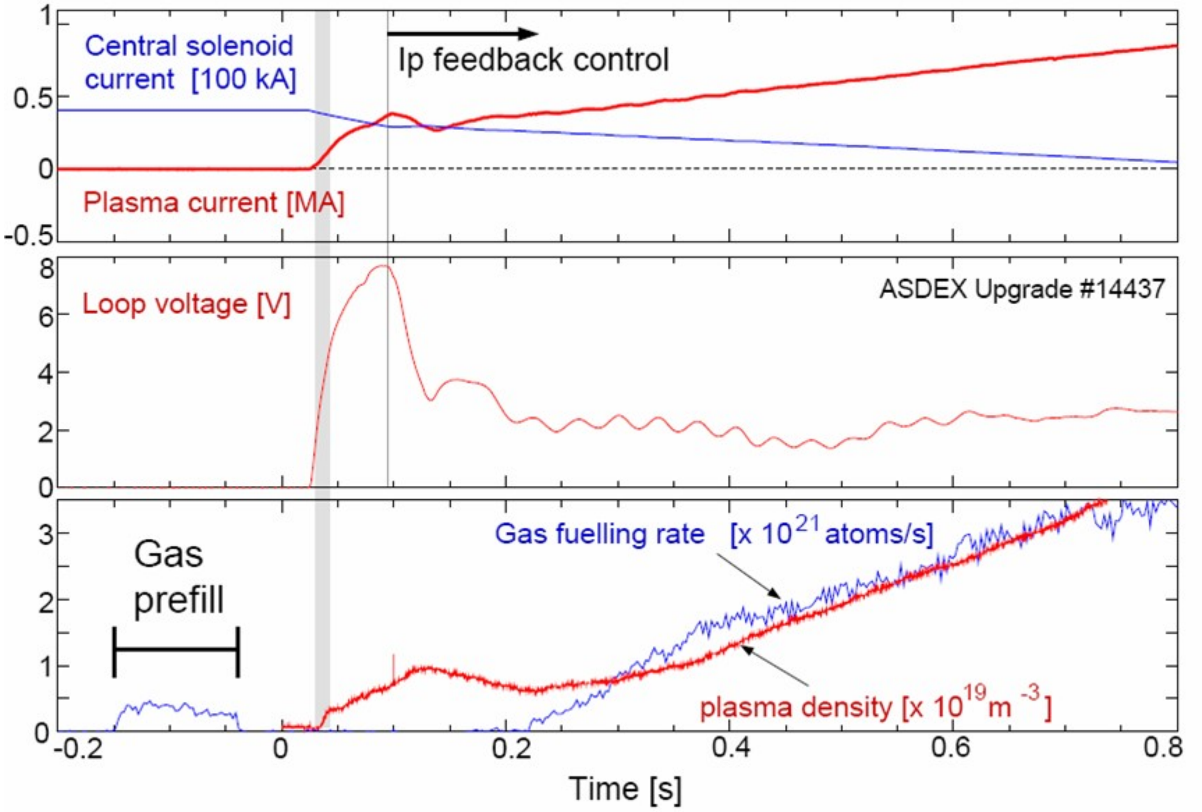
- Give a small puff of gas into the vessel (this neutral gas fills the whole vessel)
- Ramp up the flux in the transformer to obtain a high Electric field (this leads to plasma breakdown

Plasma breakdown



- Gas mostly neutral. But always one of the electrons is free
- The electric field accelerates this electron which gains in energy
- When the fast electron hits one of the atoms it can ionize it and generate an additional electron
- The avalanche leads to the break down
- Works well for low density (long mean free path) and high electric field
- Conditions mostly empirically determined

Startup



- Short time after the plasma breakdown one starts the feed back control of the plasma current
- It is slowly ramped to a stationary value required by the discharge

Measurement of the magnetic field

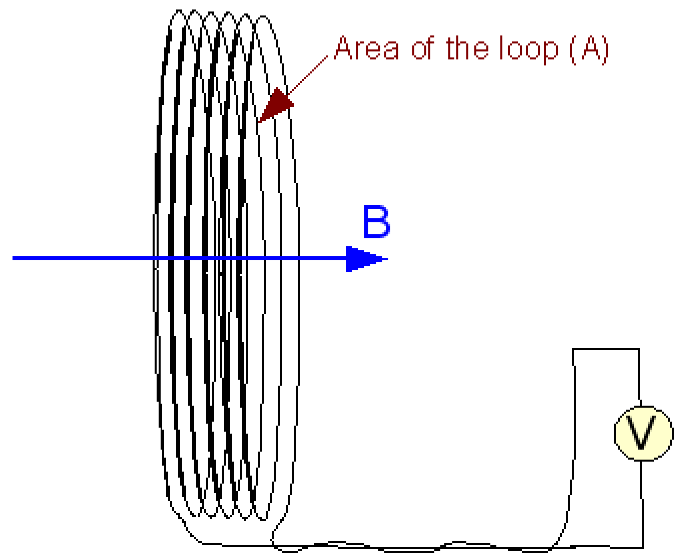
- Magnetic field is measured with small coils at many different positions

$$V = N\frac{\partial \psi}{\partial t} = NA\frac{\partial B_\perp}{\partial t}$$

Voltage — Number of windings — Area — Magnetic field

$$B_\perp = \frac{1}{NA}\int_{-\infty}^{t} V(t)\mathrm{d}t$$

- Easy to construct diagnostic. Only disadvantage related to a possible drift due to spurious voltage



Schematic drawing of the coil with which the magnetic field is measured

Measurement of the current

- Plasma current is measured by a Rogowski coil

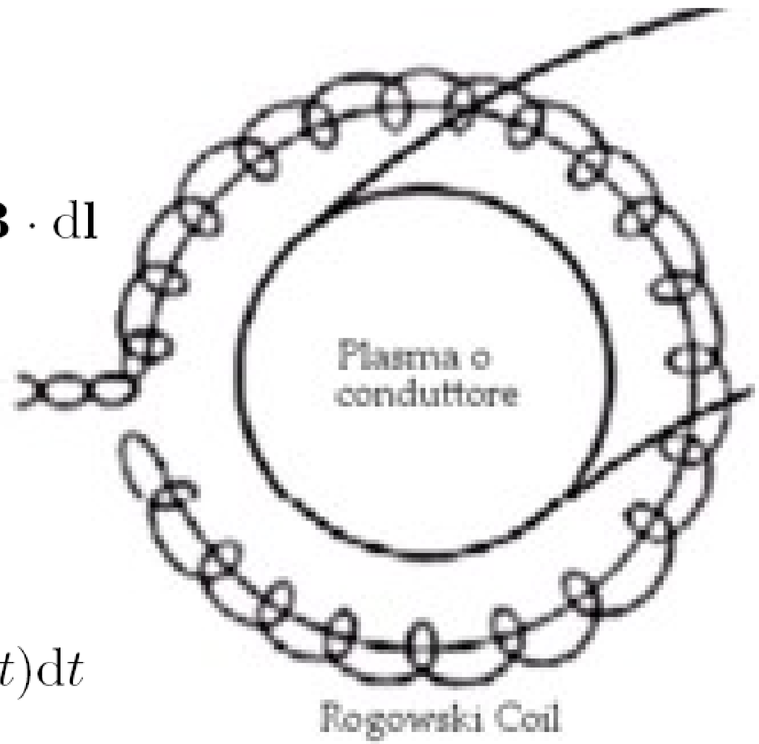
$$V = A\sum_i \frac{\partial B_{\perp i}}{\partial t} \approx \frac{A}{\Delta l}\frac{\partial}{\partial t}\oint \mathbf{B}\cdot \mathrm{d}\mathbf{l}$$

Sum over the windings

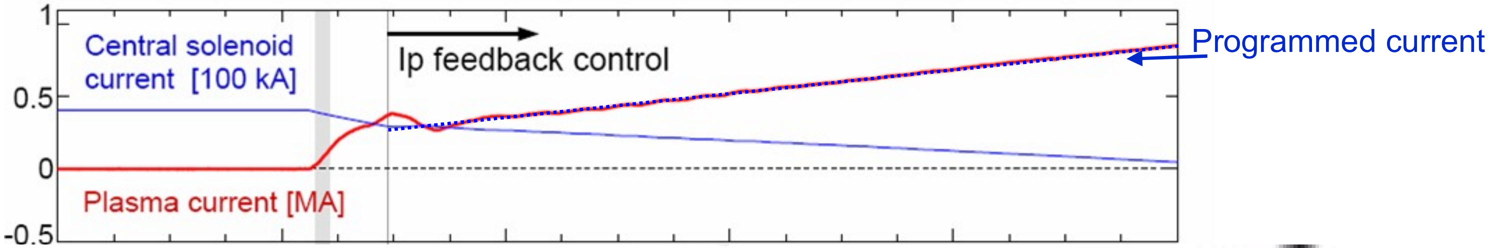
Distance between the windings

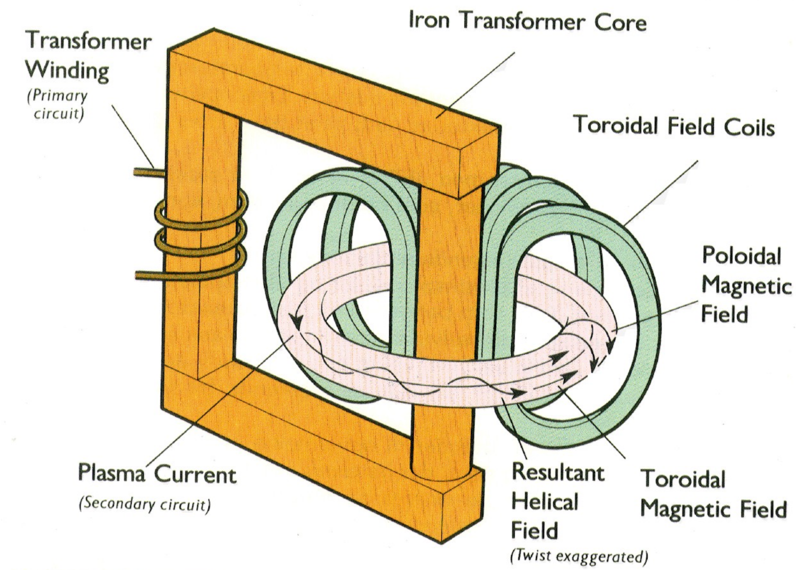
- Enclose current directly follows from

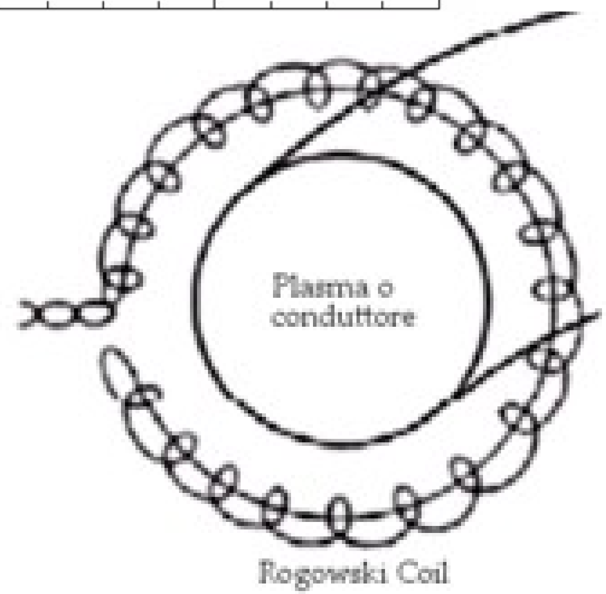
$$\mu_0 I = \oint \mathbf{B}\cdot \mathrm{d}\mathbf{l} = \frac{\Delta l}{A}\int_{-\infty}^{t} V(t)\mathrm{d}t$$



Startup

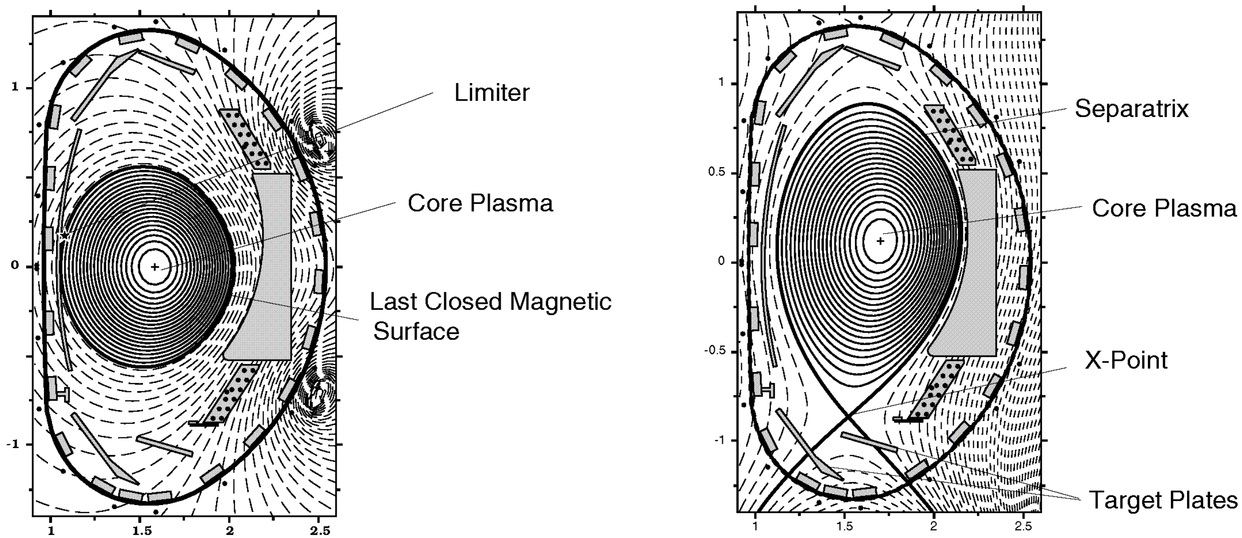






- Plasma current is measured by the Rogowski coil
- If the value is lower than desired one ramps the current in the solenoid a little faster

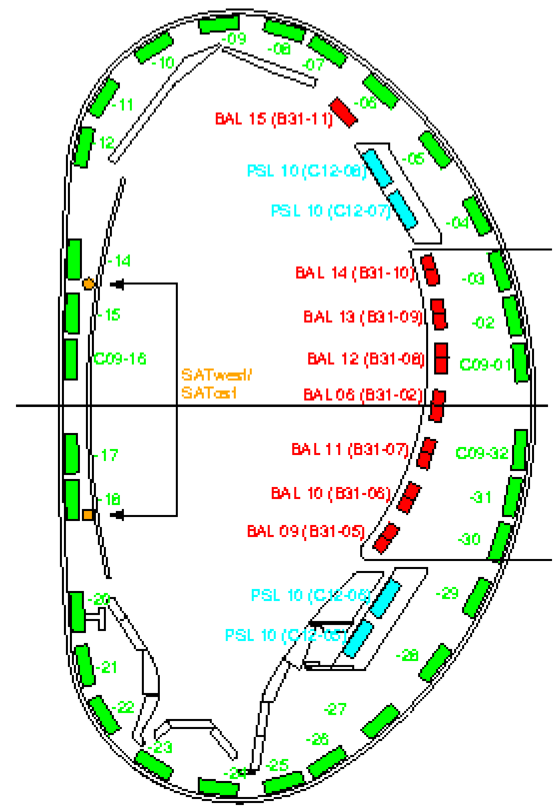
Breakdown also at non controlled position



- Left possible position of the plasma at breakdown
- Right what one wants to achieve

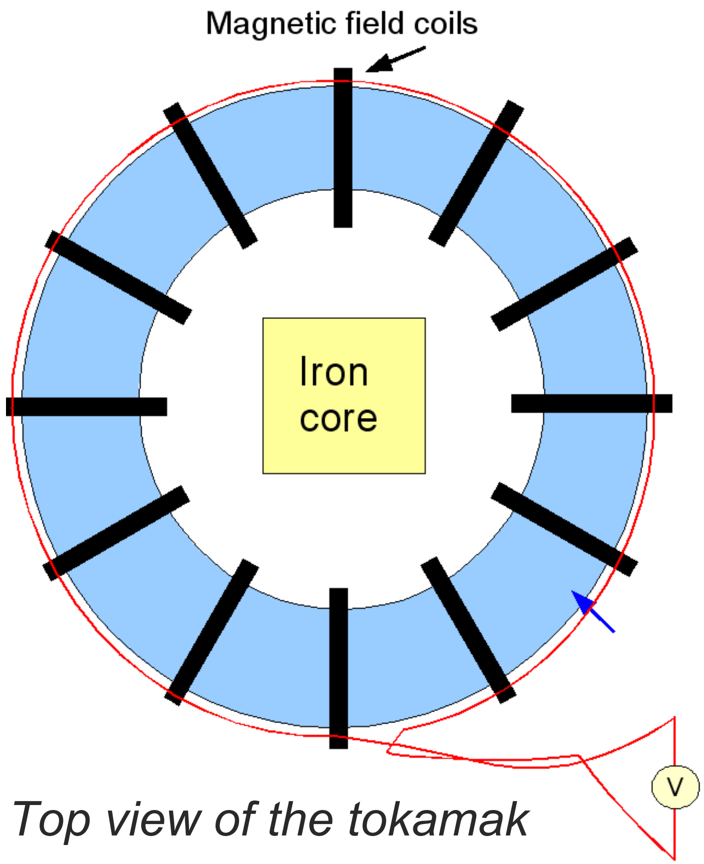
Set of coils measure the magnetic field

- Magnetic field is measured at the boundary
- Green – Poloidal field
- Red – Radial field
- Blue – poloidal flux
- The plasma position and shape can be reconstructed from these measurements
- Control system then changes the current in the vertical field coils to shape the plasma



Loop voltage

- The loop voltage can be measured straight forwardly by winding a coil in the toroidal direction



Top view of the tokamak

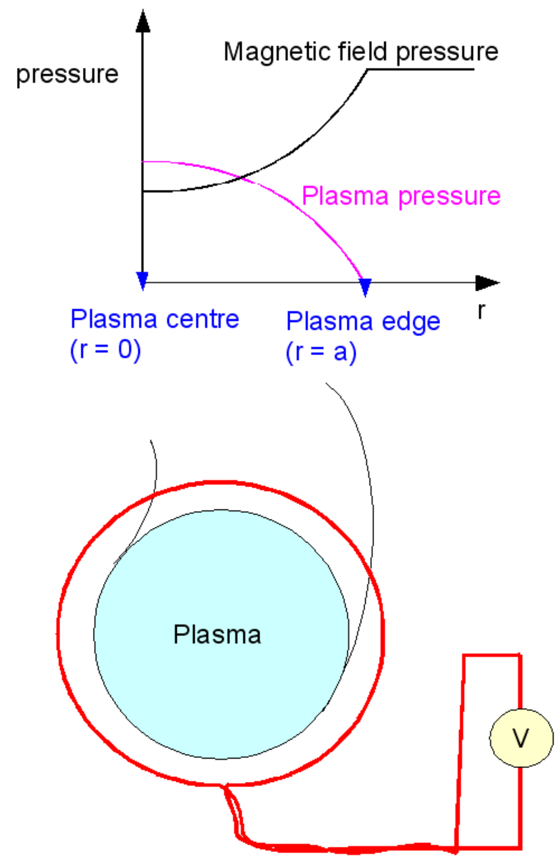
Finally the toroidal flux

- Using a poloidal coil one can measure the toroidal flux
- This flux is changed by the pressure in the plasma

$$B_t^2(r) = B_t^2(a) - 2\mu_0 p(r)$$

- This links the flux to the pressure

$$\int \mathrm{d}^2 A\, B_t = \int \mathrm{d}^2 A\, B_t(a) \sqrt{1 - \frac{2\mu_0 p(r)}{B_t^2(a)}}$$



Toroidal flux and stored energy are related

$$\int \mathrm{d}^2A\,B_t = \int \mathrm{d}^2A\,B_t(a)\sqrt{1-\frac{2\mu_0 p(r)}{B_t^2(a)}}$$

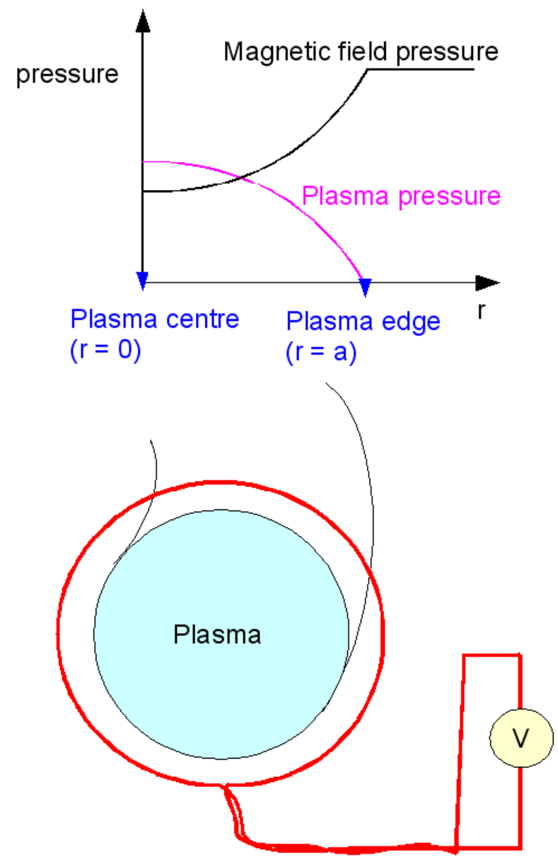
$$\int \mathrm{d}^2A\,B_t \approx B_t(a)\int \mathrm{d}^2A\left[1-\frac{p(r)}{B_t^2(a)}\right]$$

$$\int \mathrm{d}^2A\,B_t \approx C - \frac{\mu_0}{B_t(a)}\int \mathrm{d}^2A\,p(r)$$

Toroidal flux

$$W = \int \mathrm{d}^3V\,3p = 2\pi\int \mathrm{d}^2A\,p(r)$$

Stored energy



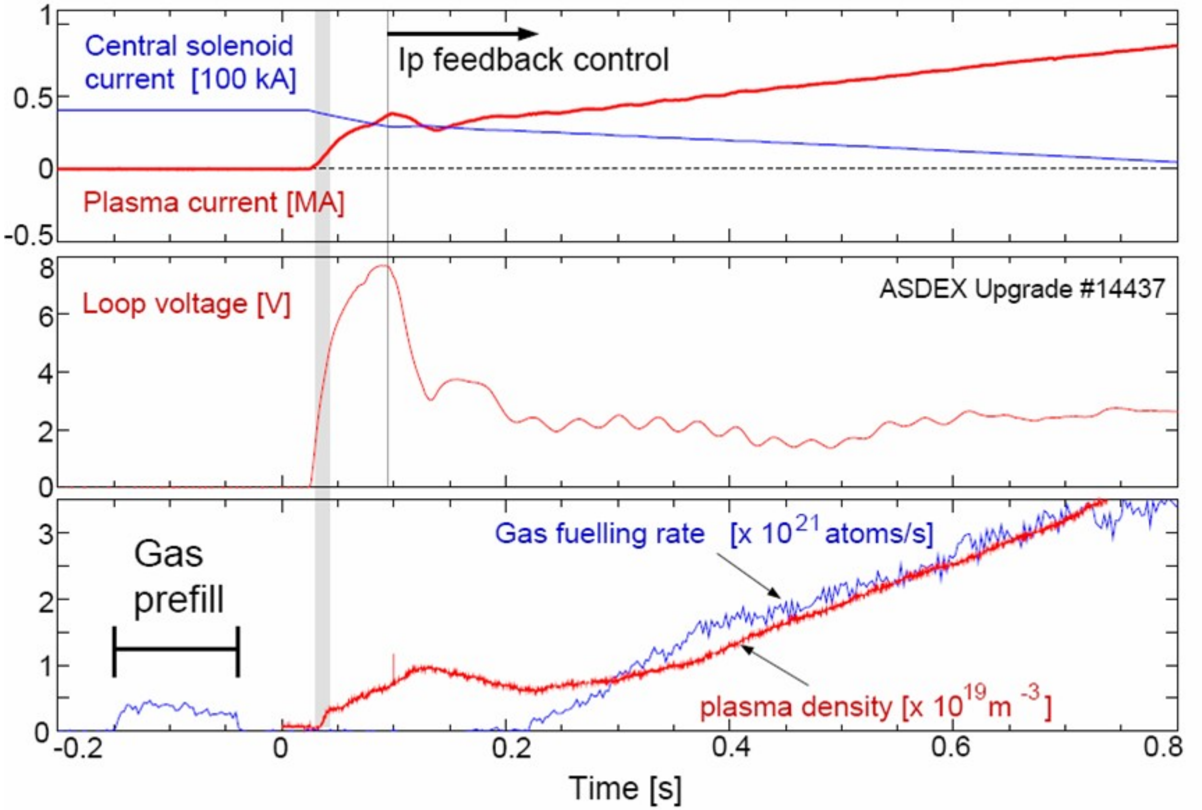
Magnetic coils

Easy and cheap

Allow for the determination of many key quantities

- Plasma current
- Loop voltage
- Stored energy
- Plasma shape

Density control



- Density is controlled in the same way
- Density is measured and controlled by a simple gas puff into the main chamber.

Wave in a plasma

- Using the Maxwell equations

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

- One can derive the wave equation in its standard form

$$\nabla \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \frac{\partial \mathbf{J}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$-\nabla \times (\nabla \times \mathbf{E}) = \mu_0 \frac{\partial \mathbf{J}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Waves in plasma

- Then substitute a plane wave

$$\mathbf{E} = \mathbf{E}_0 \exp[\mathrm{i}\mathbf{k}\cdot\mathbf{x} - \mathrm{i}\omega t]$$

- Suppose the equation is

$$\frac{\partial \mathbf{E}}{\partial t} + v\frac{\partial \mathbf{E}}{\partial x} = 0$$

$$-\mathrm{i}\omega\mathbf{E}_0 \exp[\mathrm{i}\mathbf{k}\cdot\mathbf{x} - \mathrm{i}\omega t] + \mathrm{i}k_x v\mathbf{E}_0 \exp[\mathrm{i}\mathbf{k}\cdot\mathbf{x} - \mathrm{i}\omega t] = 0$$

$$-\mathrm{i}\omega\mathbf{E}_0 + \mathrm{i}k_x v\mathbf{E}_0 = 0$$

- One can therefore obtain algebraic equations

$$\frac{\partial}{\partial t} \rightarrow -\mathrm{i}\omega \qquad \nabla = \frac{\partial}{\partial \mathbf{x}} \rightarrow \mathrm{i}\mathbf{k}$$

Waves in plasmas

- The wave equation simplifies

$$-\nabla \times (\nabla \times \mathbf{E}) = \mu_0 \frac{\partial \mathbf{J}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$-\mathrm{i}\mathbf{k} \times (\mathrm{i}\mathbf{k} \times \mathbf{E}_0) = -\mathrm{i}\omega\mu_0 \mathbf{J}_0 + (-\mathrm{i}\omega)^2 \frac{1}{c^2} \mathbf{E}_0$$

- With the choice $\mathbf{E}_0 \perp \mathbf{k}$

$$-k^2 \mathbf{E}_0 = -\mathrm{i}\omega\mu_0 \mathbf{J}_0 - \frac{\omega^2}{c^2} \mathbf{E}_0$$

Response of the plasma

- Current is calculated from the electron response

$$\mathbf{J} = en[\mathbf{u}_i - \mathbf{u}_e] \approx -en\mathbf{u}_e$$

- Using the equation of motion

$$m_e \frac{\mathrm{d}\mathbf{u}_e}{\mathrm{d}t} = -e\mathbf{E}$$

$$-\mathrm{i}\omega m_e \mathbf{u}_{e0} = -e\mathbf{E}_0$$

$$\mathbf{J}_0 = -en\frac{e\mathbf{E}_0}{\mathrm{i}m_e\omega} = \mathrm{i}\frac{ne^2}{m_e\omega}\mathbf{E}_0$$

- Relation between current and electric field

Wave equation

- Substituting the expression for the current

$$-k^2\mathbf{E}_0 = \frac{ne^2\mu_0}{m_e}\mathbf{E}_0 - \frac{\omega^2}{c^2}\mathbf{E}_0$$

$$\frac{\omega_p^2}{c^2} \longrightarrow \omega_p^2 = \frac{ne^2\mu_0 c^2}{m_e} = \frac{ne^2}{\epsilon_0 m_e} \longleftarrow c^2 = \frac{1}{\epsilon_0\mu_0}$$

- The equation can then be written in the form

$$\left[\frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2} - k^2\right]\mathbf{E}_0 = 0$$

$$k^2 = \frac{1}{c^2}[\omega^2 - \omega_p^2] = \left(\frac{2\pi}{\lambda}\right)^2$$

Phase difference

- The wave vector determines the phase difference

$$k^2 = \frac{1}{c^2}[\omega^2 - \omega_p^2] = \left(\frac{2\pi}{\lambda}\right)^2$$

$$\exp[\mathrm{i}\mathbf{k}\cdot\mathbf{x}] \longrightarrow \Delta\phi = \mathbf{k}\cdot(\mathbf{x}_B - \mathbf{x}_A) \to \int_A^B \mathbf{k}\cdot\mathrm{d}\mathbf{x}$$

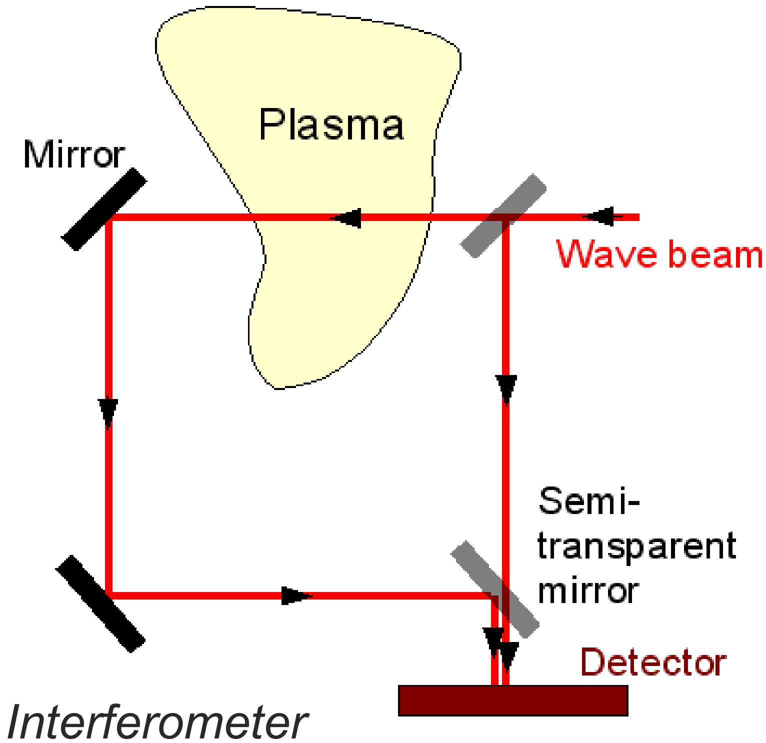
- For high wave frequencies

$$k = \sqrt{\frac{1}{c^2}[\omega^2 - \omega_p^2]} = \frac{\omega}{c}\sqrt{1 - \frac{\omega_p^2}{\omega^2}} \approx \frac{\omega}{c}\left[1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2}\right]$$

$$\Delta\phi = \frac{\omega}{c}\int_A^B \left(1 - \frac{e^2}{2\epsilon_0 m_e \omega^2} n\right)\mathrm{d}x = C + D\int_A^B n\,\mathrm{d}x$$

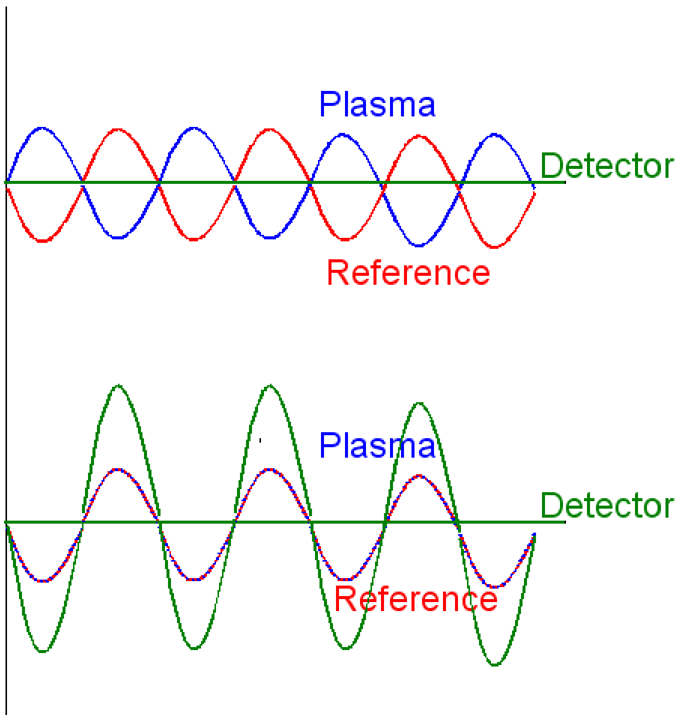
Measurement of the density

- Wave beam is split
- One leg goes through the plasma
- The other leg is used for reference
- Measuring the phase difference of the two beams gives the information on the line integral of the density



Measurement of the density

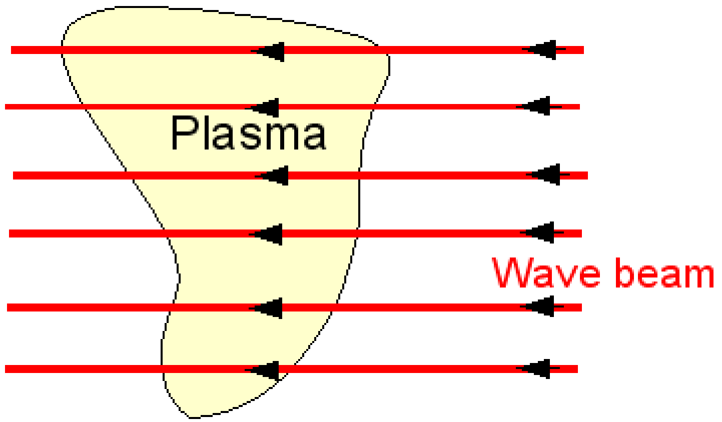
- At the detector the phase difference between the reference and plasma beam determines the signal
- Every time one ‘removes’ a wavelength from the plasma the signal goes through a maximum
- Note : again one integrates the change in time



Signal at the detector

Density profile

- The interferometer measures the line integrated density
- To obtain the profile one can use more than one beam and reconstruct the profile
- The reconstruction in general is somewhat inaccurate
- Profile can not be very accurately determined



Many chords through the plasma allow for the construction of the density profile

Meaning of the plasma frequency

- Relation for the wave vector

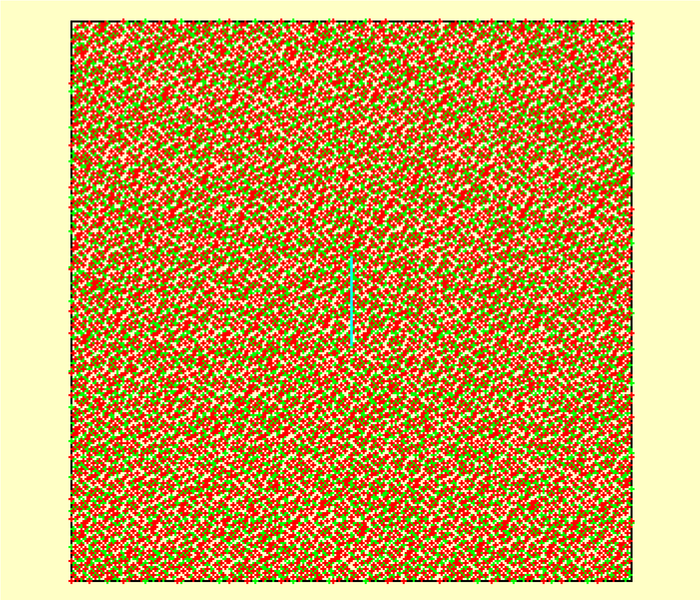
$$k^2 = \frac{1}{c^2}[\omega^2 - \omega_p^2] = \left(\frac{2\pi}{\lambda}\right)^2$$

- Yields: The natural plasma oscillation

$$\omega = \omega_p \rightarrow k = 0$$

- Wave cut-off

$$\omega < \omega_p \rightarrow k = \sqrt{- \ldots}$$



Wave reflection

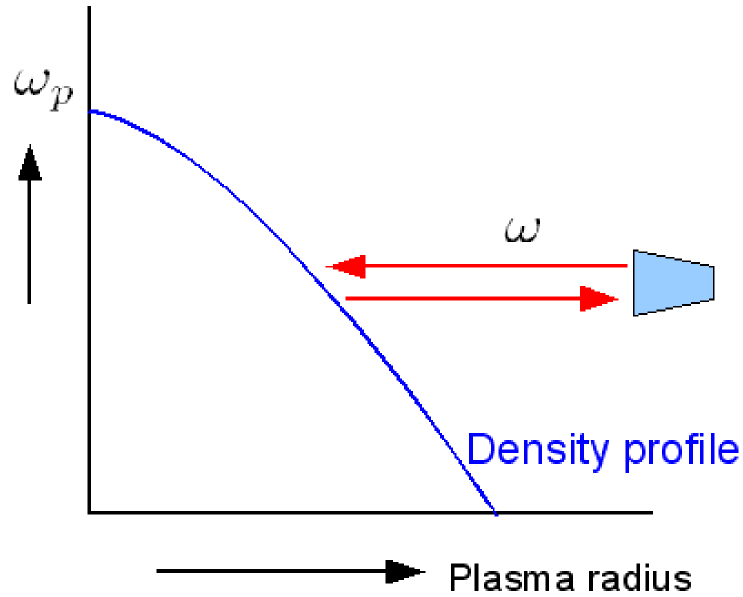
- At the cut-off the wave is reflected

$$\omega < \omega_p \rightarrow k = \sqrt{- \ldots}$$

- Only waves with a frequency larger than the plasma frequency can propagate

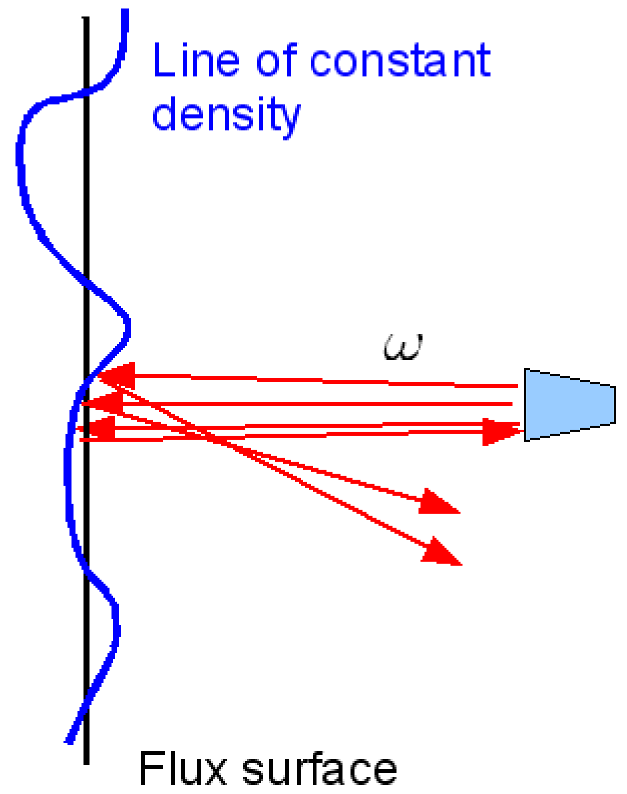
Second possibility

- A wave with a fixed frequency will be reflected somewhere in the plasma
- The phase difference between the ingoing wave and the reflected wave is determined by the length of the path and the wave vector in the plasma
- By sweeping the frequency (starting from a low value) one can determine the density profile
- Works well if the profile is sufficiently steep



Small density perturbations

- The density is supposed to be constant on a magnetic surface
- If it is not part of the wave is scattered away from the antenna
- The amplitude of the reflected signal is then not constant in time (even for fixed frequency)



Rapid density fluctuations

- Rapid oscillations of the density layer are observed (measured with constant wave frequency)
- This means that the plasma is not quiet
- The MHD solution is not complete
- Fluctuations due to small scale instabilities do exist