Physics of fusion power

Lecture 11: Diagnostics / heating

Contents

Density measurement

- Interferometer
- Reflectometry
- Thomson scattering

Temperature measurements

- Thomson scattering
- Electron cyclotron emission
- Line radiation

Plasma heating

- Ohmic heating

Plasma frequency

- Dispersion relation of a wave propagating in a plasma

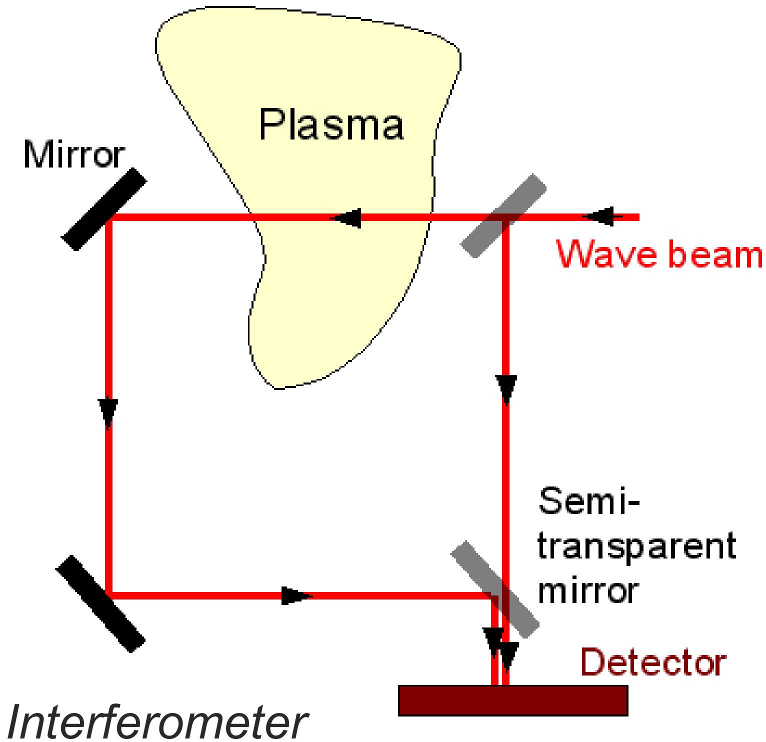
$$k^2 = \frac{1}{c^2}[\omega^2 - \omega_p^2] = \left(\frac{2\pi}{\lambda}\right)^2 \qquad \omega_p^2 = \frac{e^2}{\epsilon_0 m_e} n$$

- Phase difference over a path (number of wavelengths in the length of the path times 2 pi)

$$\Delta\phi = \frac{2\pi L}{\lambda} = \int_A^B k\mathrm{d}x = C - D\int_A^B n\mathrm{d}x$$

Measurement of the density

- Wave beam is split
- One leg goes through the plasma
- The other leg is used for reference
- Measuring the phase difference of the two beams gives the information on the line integral of the density



Meaning of the plasma frequency

- Relation for the wave vector

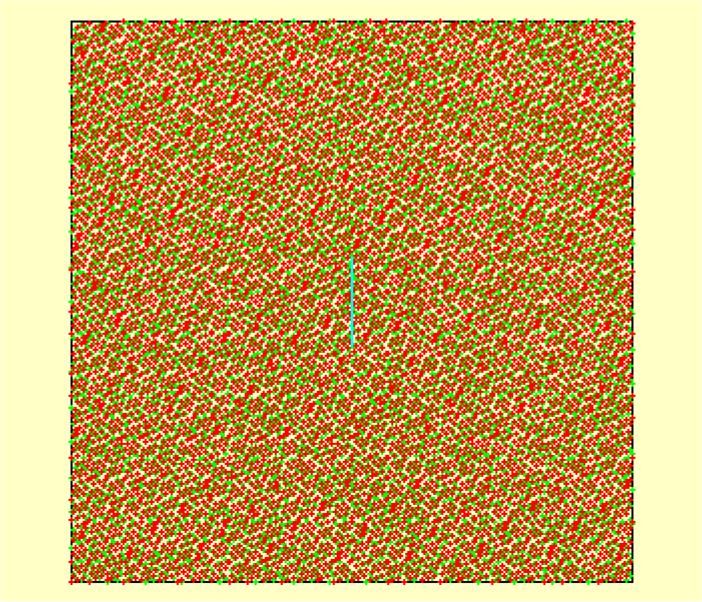
$$k^2 = \frac{1}{c^2}[\omega^2 - \omega_p^2] = \left(\frac{2\pi}{\lambda}\right)^2$$

- Yields: The natural plasma oscillation

$$\omega = \omega_p \rightarrow k = 0$$

- Wave cut-off

$$\omega < \omega_p \rightarrow k = \sqrt{- \ldots}$$



Wave reflection

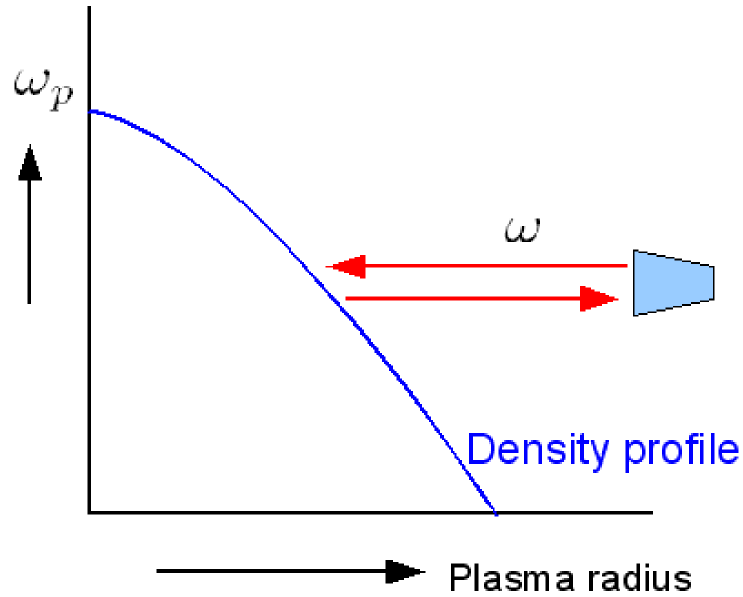
- At the cut-off the wave is reflected

$$\omega < \omega_p \rightarrow k = \sqrt{- \ldots}$$

- Only waves with a frequency larger than the plasma frequency can propagate

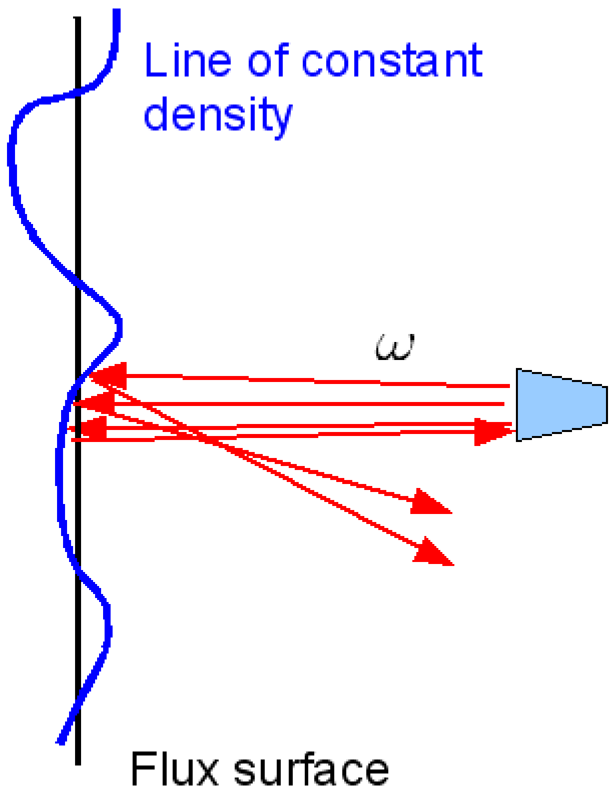
Second possibility

- A wave with a fixed frequency will be reflected somewhere in the plasma
- The phase difference between the ingoing wave and the reflected wave is determined by the length of the path and the wave vector in the plasma
- By sweeping the frequency (starting from a low value) one can determine the density profile
- Works well if the profile is sufficiently steep



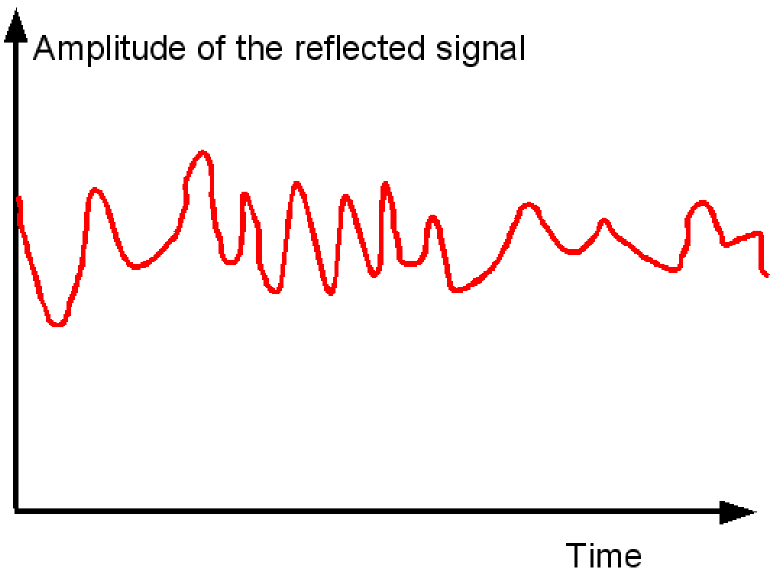
Small density perturbations

- The density is supposed to be constant on a magnetic surface
- If it is not part of the wave is scattered away from the antenna
- The amplitude of the reflected signal is then not constant in time (even for fixed frequency)



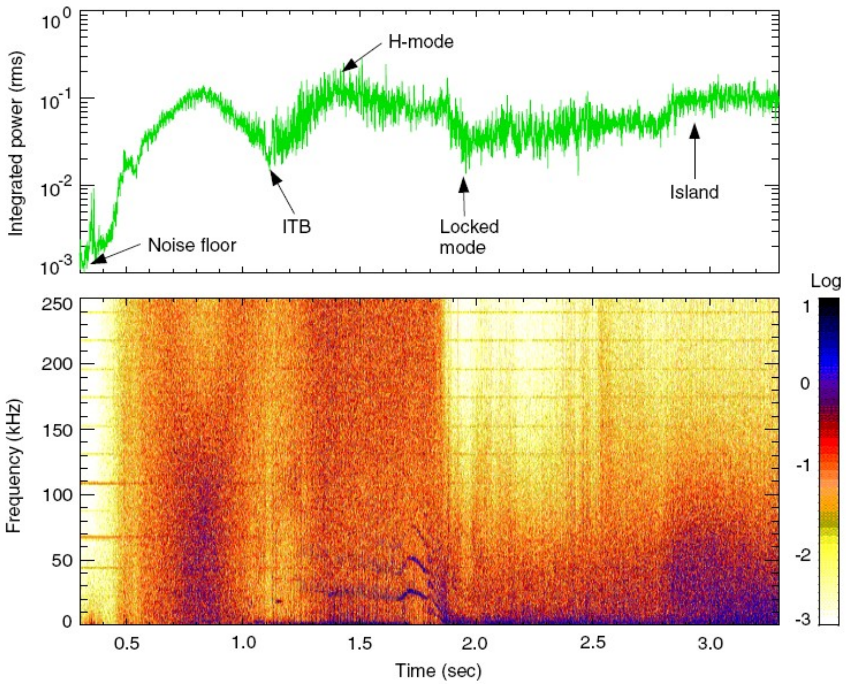
Amplitude of the reflected signal in time

- If there are small density fluctuations then the amplitude of the reflected signal varies in time
- Here drawn by hand
- One can do a Fourier analysis to find the typical frequencies
- This is often done for many small time windows such that one gets the frequency of the signal as a function of time



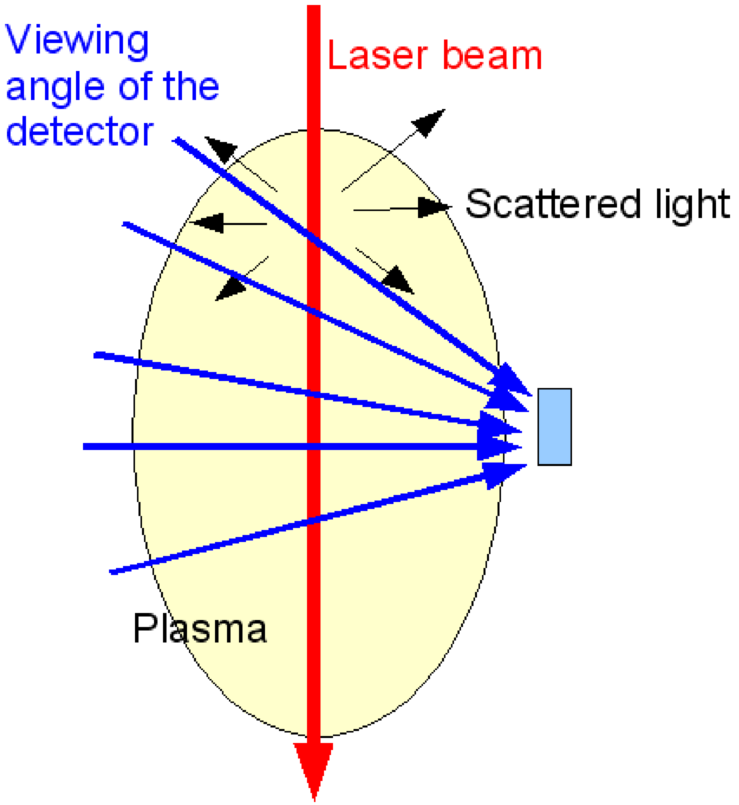
Rapid density fluctuations

- Rapid oscillations of the density layer are observed (measured with constant wave frequency)
- This means that the plasma is not quiet
- The MHD solution is not complete
- Fluctuations due to small scale instabilities do exist



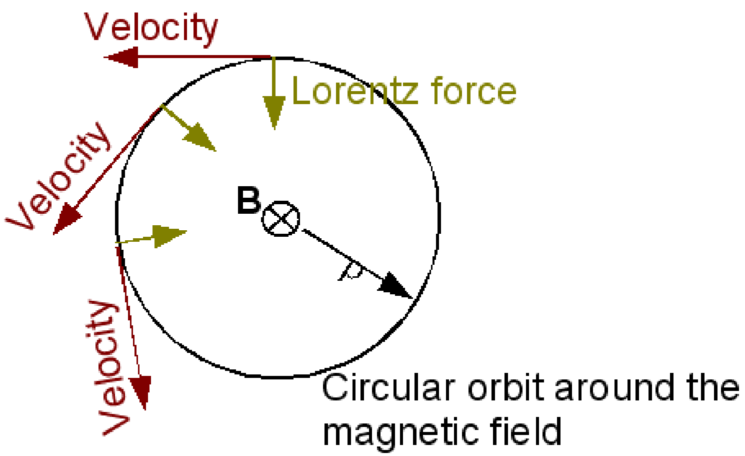
Thomson scattering

- Light is also scattered of particles.
- The amount is simply proportional to the number of particles
- Viewing the scattered light of an injected laser beam under allows one to determine how many particles there are in the scatter volume
- This allows for a measurement of the density



Electron cyclotron emission

- The Lorentz force accelerates the electrons
- Acceleration of a charge leads to the emission of radiation
- The radiation emitted will have a frequency that matches the cyclotron frequency of the electrons
- The emitted radiation is almost that of a radiating black body and therefore carries the information of the Temperature

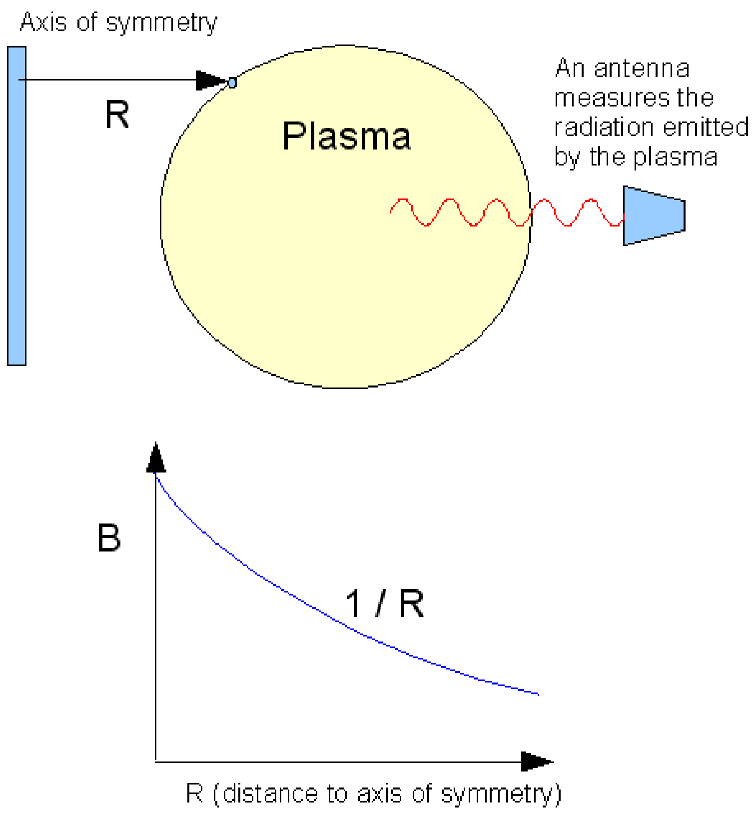


Electron cyclotron emission

- The radiation has a frequency

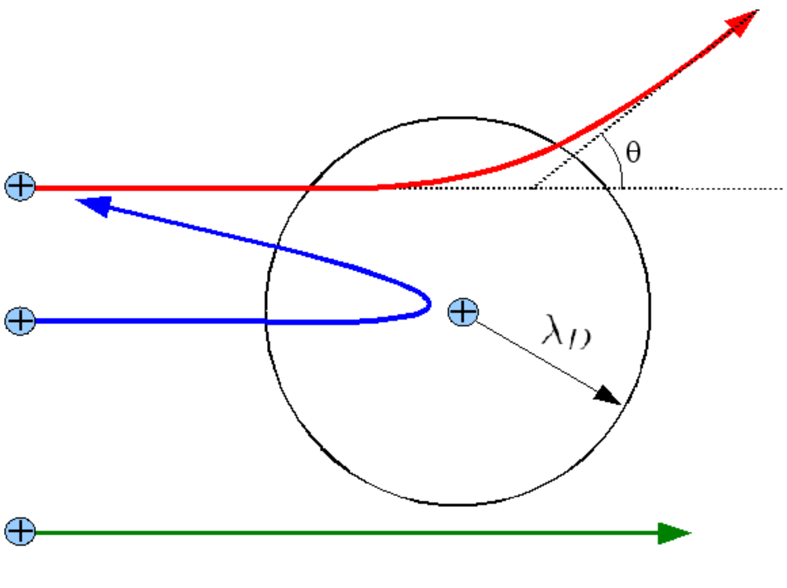
$$\omega_c = \frac{eB}{m}$$

- The magnetic field strength is dominantly determined by the toroidal field
- This field varies as 1 / R where R is the distance to the axis of symmetry
- Consequently the frequency of the emitted radiation is linked with a well defined position in the plasma



Collisions

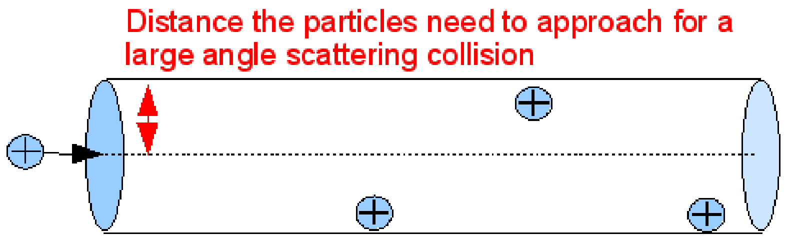
- Coulomb interaction between electrons and ions leads to the scattering of particles -> Collisions
- Interaction occurs only within the Debye sphere
- The angle of deflection depends on how close the particles are approaching each other.

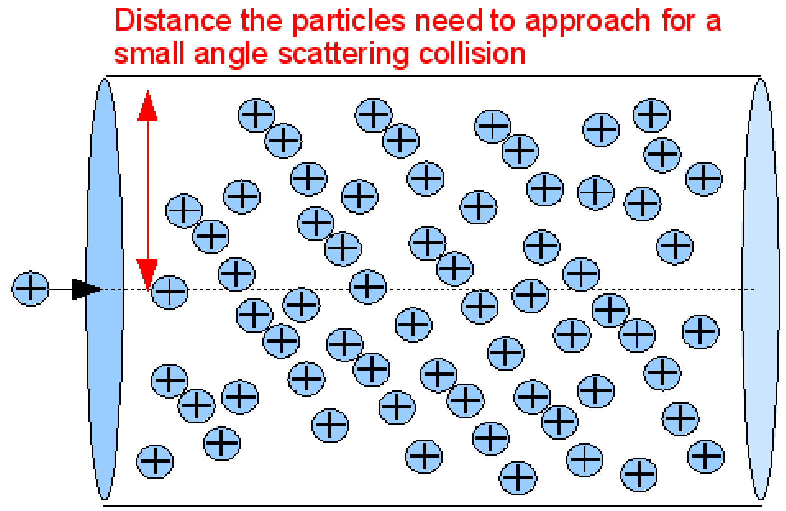


Collisions between the ions

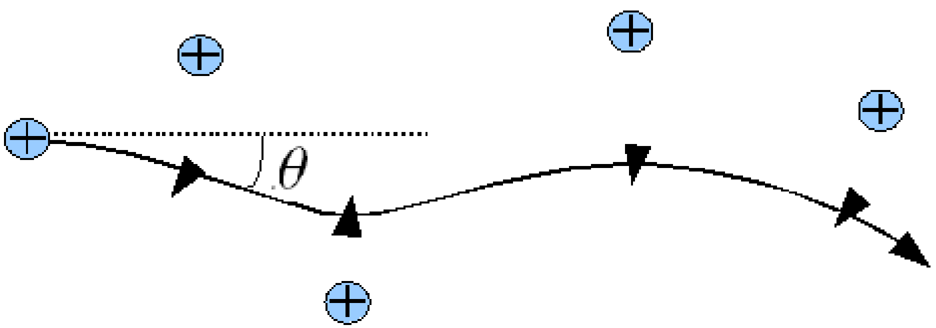
Small / Large angle scattering

- Deflection depends on how close the particles approach each other
- For a given time a particle moving through the plasma will collide with all particles in a cylinder
- For large angle scattering collisions the distance must be small and so is the radius of the cylinder
- For small angle scattering collisions the radius is much larger
- It turns out that small angle scattering collisions dominate





Random walk of the velocity



- The small angle scatterings lead to a random walk.
- This can be described by a diffusion coefficient of the angle the velocity makes with a chosen reference direction

$$\langle \Delta\theta \rangle = 0$$

Change due to one collision of the angle that gives the direction of the velocity

$$D = \frac{\langle \Delta\theta \Delta\theta \rangle}{2\tau}$$

Diffusion coefficient

Typical time between to collisions

Diffusion of the velocity

- Diffusion coefficient

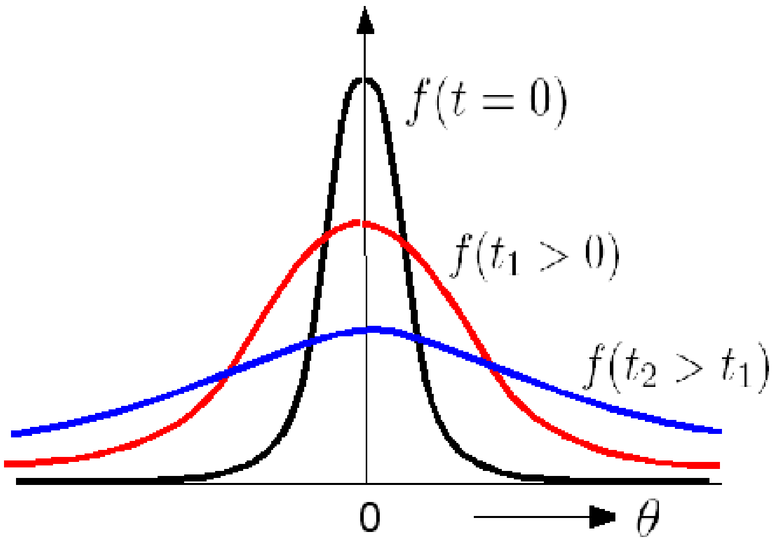
$$D = \frac{\langle \Delta\theta \Delta\theta \rangle}{2\tau}$$

- Define a function that determines the number of particles moving in a direction given by the angle θ

$$\mathrm{d}N = f(\theta)\mathrm{d}\theta$$

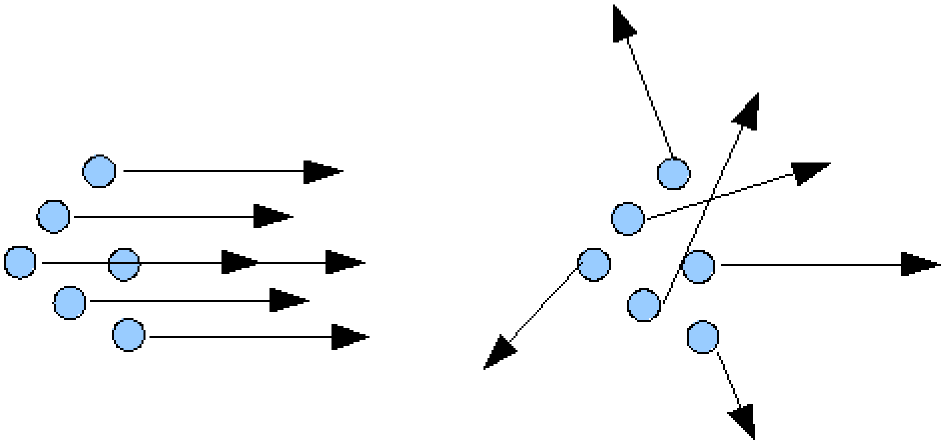
- Collisions lead to a diffusion in the angle described by

$$\frac{\partial f}{\partial t} = D \frac{\partial^2 f}{\partial \theta^2}$$



Initial distribution of particles moving mostly in the same direction (same angle) are scattered by collisions which randomize the angle of propagation

Diffusion of the velocity direction



Initially particles move in one direction

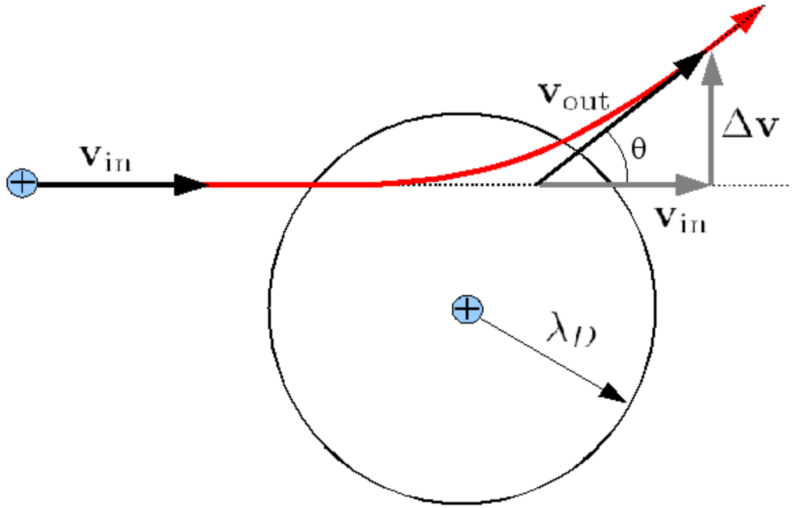
After some time the collisions lead to a random direction

We will now calculate the scaling of the diffusion coefficient with the velocity of the particles as well as the mass.

$$D = \frac{\langle \Delta\theta \Delta\theta \rangle}{2\tau}$$

We will then look at what this means for the plasma resistivity and the plasma heating due to the current

Collisions



$\mathbf{v}_{in}$
$\mathbf{v}_{out}$
θ
$\Delta\mathbf{v}$
$\mathbf{v}_{in}$
λ_D

Resistivity

Temperature dependence