Physics of fusion power

Lecture 12: Diagnostics / heating

Contents

Heating

- Ohmic
- Neutral beam

Transport

- Classical transport

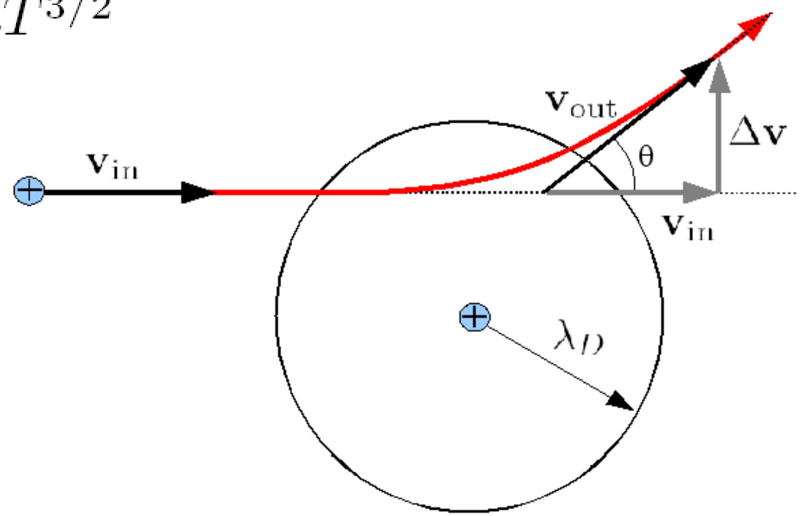
Collisions

- Previously derived

$$D = \frac{(\Delta\theta)^2}{2\tau} \propto \frac{1}{m^2 v^3} \propto \frac{1}{\sqrt{m}T^{3/2}}$$

- Velocity scaling surprising related to

$$\Delta\theta = \frac{\Delta v}{v} \qquad \Delta t = \frac{\lambda_D}{v}$$



- For larger velocity, also a larger change in velocity is necessary to deflect the particle over a given angle
- The contact time scales as 1/v

Mean velocity

- The mean electron velocity can be modelled by the equation

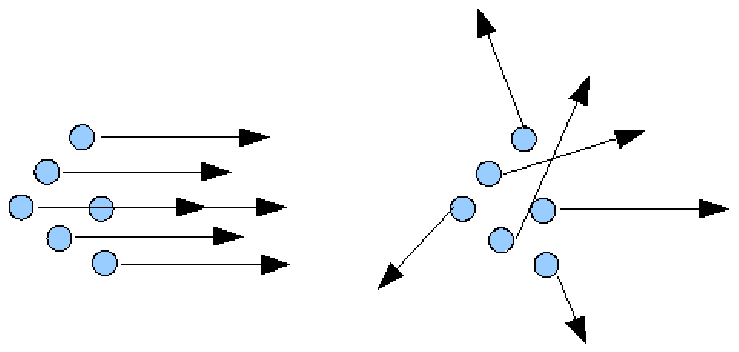
$$m_e \frac{\mathrm{d}u_e}{\mathrm{d}t} = -m_e \nu_e u_e$$

Collision frequency

- This has the solution

$$u_e = C \exp[-\nu_e t]$$

- Such that the velocity decays on a typical time given by 1 / collision frequency



Initially particles move in one direction

After some time the collisions lead to a random direction

Ohm's law

- Adding the electric field acceleration

$$m_e \frac{\mathrm{d}u_e}{\mathrm{d}t} = -eE - m_e \nu_e u_e \xrightarrow[\text{Solution}]{\text{Stationary}} u_e = -\frac{e}{m_e \nu_e} E$$

- The current follows Ohm's law

$$J = en[u_i - u_e] \approx -enu_e \longrightarrow \boxed{J = \frac{ne^2}{m_e \nu_e} E = \sigma E}$$

Ohm's law

- The conductivity scales as

$$\sigma \propto \frac{1}{\nu_e} \propto T^{3/2}$$

At the same electric field, a higher temperature will lead to a higher current

Heating power

- The change in energy (of one particle) follows from

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}mu_e^2\right] = m_e u_e \frac{\mathrm{d}u_e}{\mathrm{d}t} = -enu_e E - m_e \nu_e u_e^2$$

- The total energy per unit of volume is

$$W = \frac{1}{2}nmu_e^2$$

- Therefore the energy put in the plasma due to the electric field is

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -enu_e E = JE$$

Ohmic heating

- This energy change

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -enu_e E = JE$$

- Is the Ohmic heating power

$$P_{\mathrm{Ohmic}} = JE = \sigma E^2 = \frac{1}{\sigma} E^2 \qquad \sigma \propto \frac{1}{\nu_e} \propto T^{3/2}$$

At constant electric field the heating power goes up with temperature, but also the current is increased. It is the current that is limited by the kink stability limit, and at constant current the heating power decreases with temperature

$$P_{\mathrm{Ohmic}} = T^{-3/2}$$

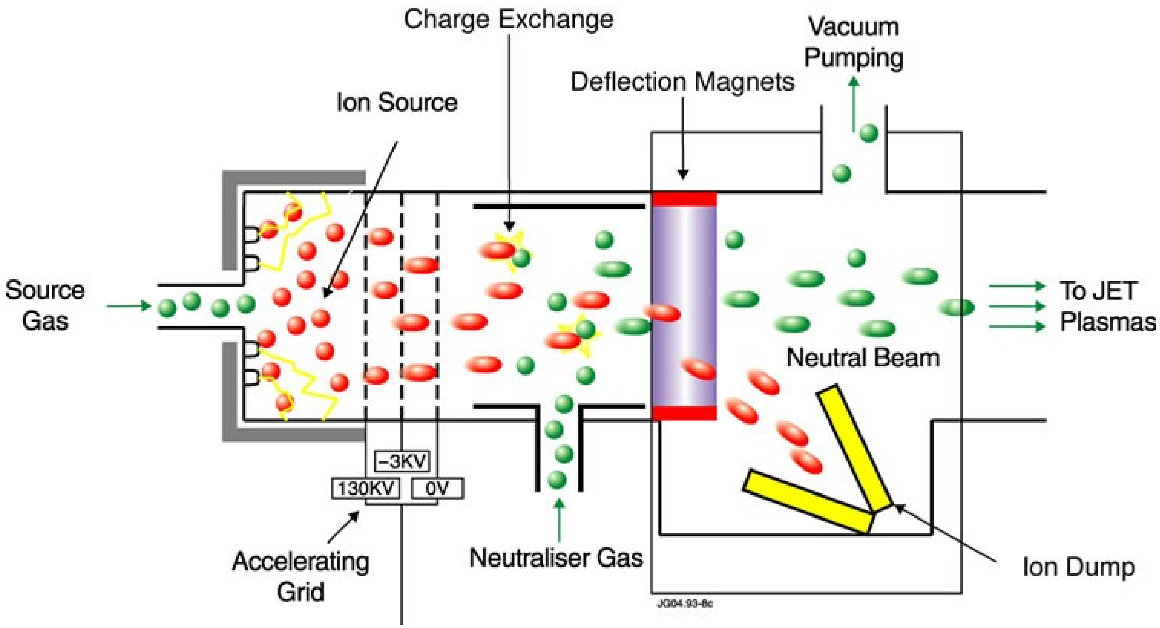
Key things to remember

- Current density follows Ohm's law with the conductivity increasing with temperature
- Ohmic heating power is the product of current and electric field
- Since it is the current that is limited in a tokamak reactor (kink) at higher temperatures one must use a smaller electric field.
- The Ohmic heating power then scales as

$$P_{\text{Ohmic}} = T^{-3/2}$$

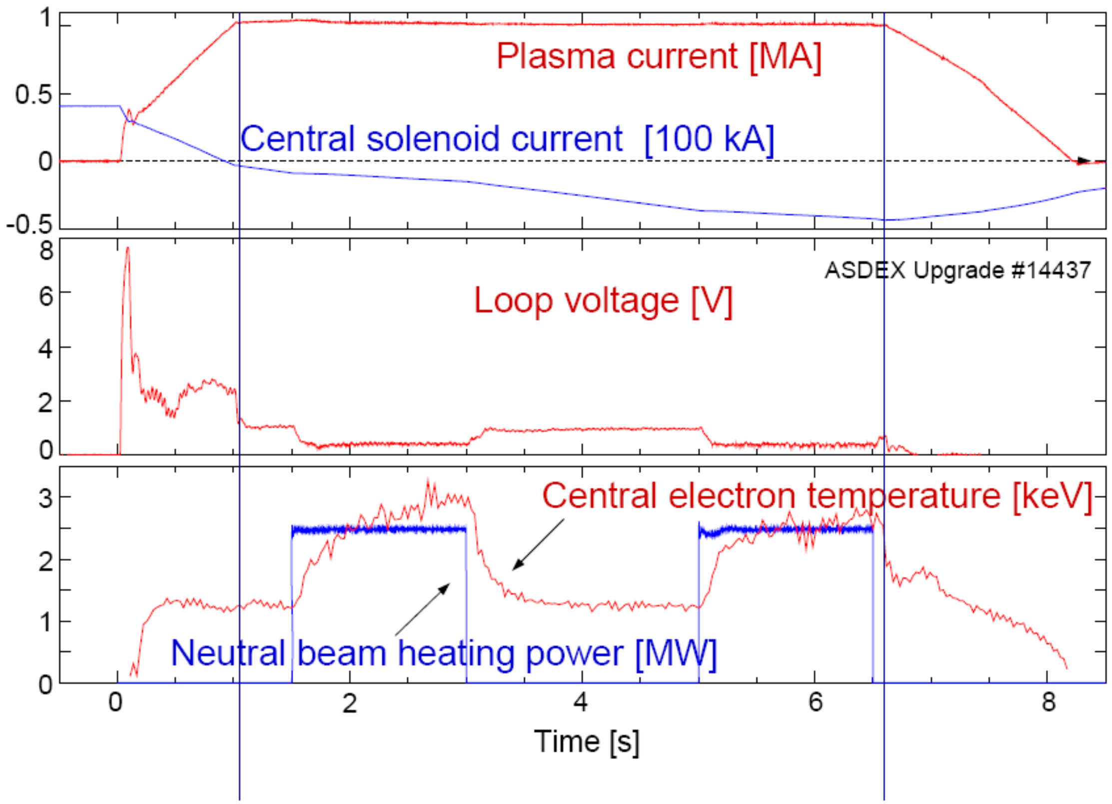
- A reactor can not be heated to the temperatures needed for fusion reactions to occur using only the Ohmic heating since it's efficiency decreases with increasing temperature

Additional forms of heating



- Several types of waves can be absorbed by the plasma (will not be discussed further)
- One can inject neutral particles with high energies into the plasma
- This is known as 'Neutral beam heating'
- This is the most efficient form of heating applied today

A real discharge again



start-up current flat-top phase ramp-down

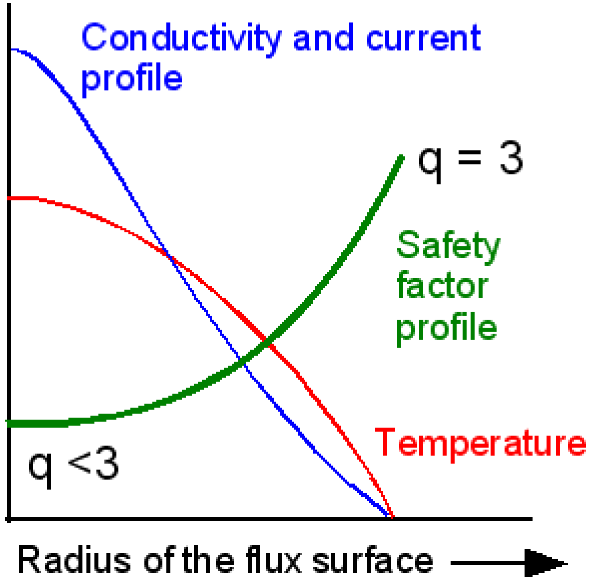
Current profile

- Temperature / conductivity / current profile is peaked

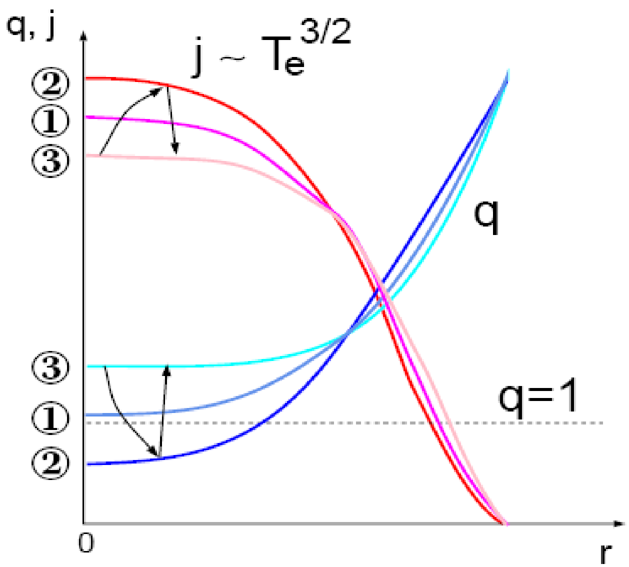
$$q = \frac{2\pi r^2 B_t}{\mu_0 R I} = \frac{2 A B_t}{\mu_0 R I}$$

- Safety factor is smaller at smaller radius
- A critical value of 3 at the edge does lead to the stability of the plasma as a whole but not locally

$$\sigma \propto \frac{1}{\nu_e} \propto T^{3/2}$$



Saw-tooth instability / Internal kink



Temperature and current increase in the centre -> q drops below 1

An internal kink becomes unstable and throws out particles density and current -> q again above 1