Physics of fusion power

Lecture 4: Cylindrical concepts

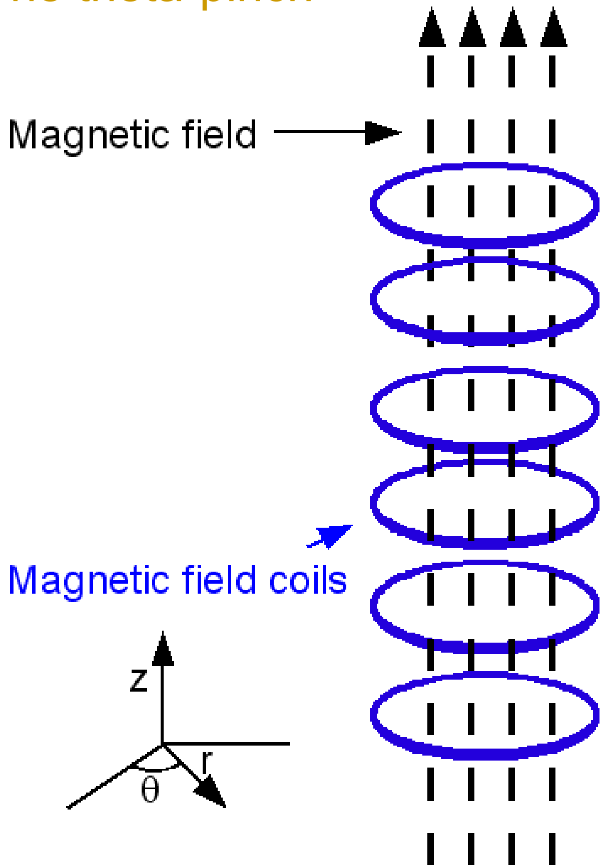
Magnetic field

If no magnetic field is used for confinement

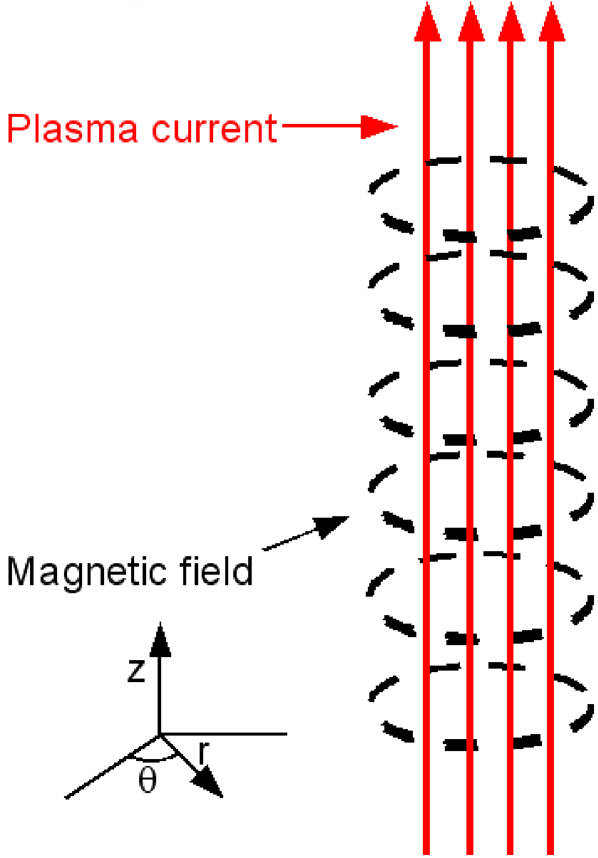
- The energy confinement time is unrealistically small
- The material walls will have to withstand the plasma pressure
- The wall would have to stand an enormous heat flux due to the large flux of energetic particles to the wall.

Cylindrical concepts

The theta-pinch



The z-pinch



Why discuss cylindrical them?

- Historically correct
- Allows to introduce several concepts in an simple geometry

Theta pinch

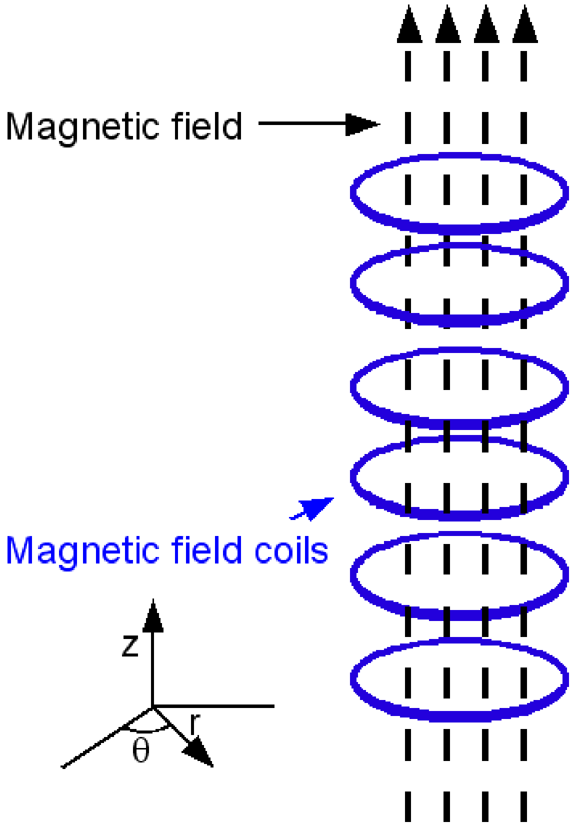
- Straight magnetic field no tension.

$$\nabla\left(p+\frac{B^2}{2\mu_0}\right)-\frac{\mathbf{B}\cdot\nabla\mathbf{B}}{\mu_0}=0$$

$$B_z\frac{\partial B_z}{\partial z}=0$$

- Equation gives constant total pressure

$$\nabla\left(p+\frac{B^2}{2\mu_0}\right)=0$$



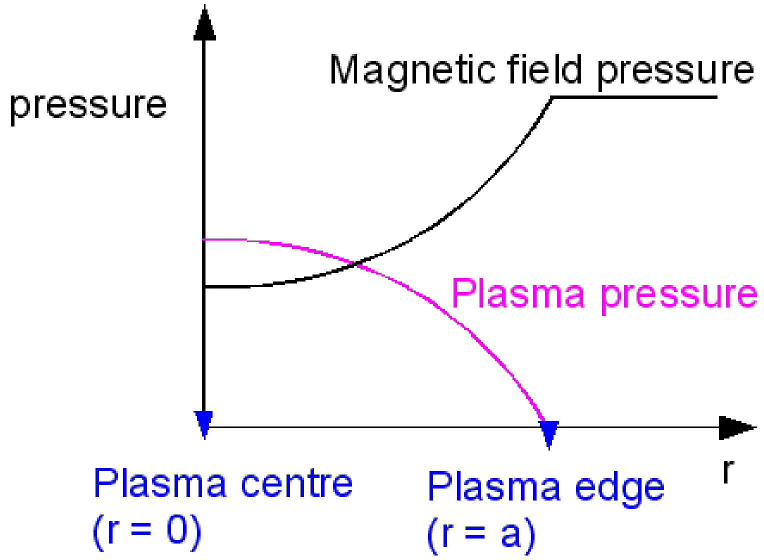
Theta pinch

- Total pressure is constant

$$\nabla\left(p+\frac{B^2}{2\mu_0}\right)=0$$

$$p+\frac{B^2}{2\mu_0}=constant$$

- Magnetic field is reduced inside the plasma, i.e. the plasma is diamagnetic
- Note that any pressure profile is possible



Drawing of the plasma pressure and the magnetic field pressure as a function of the radius (r = 0 Is the centre of the plasma)

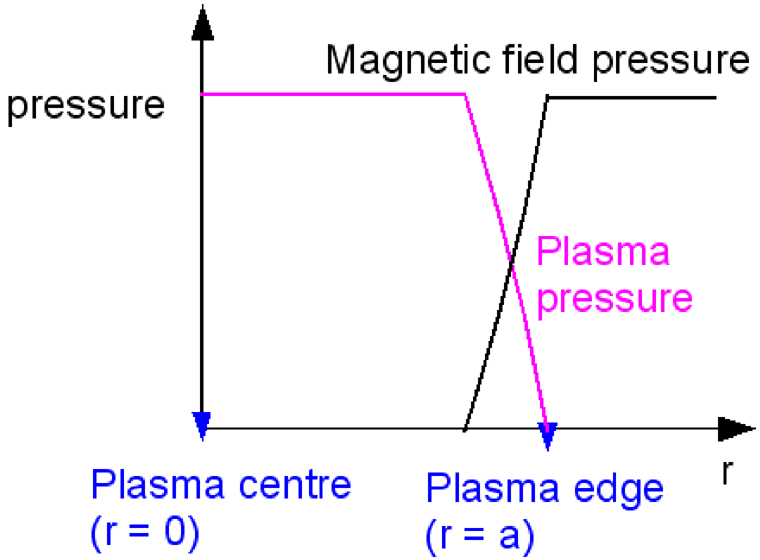
Theta pinch

- The maximum pressure is limited however

$$p + \frac{B^2}{2\mu_0} = constant$$

$$p_0 + \frac{B_0^2}{2\mu_0} = \frac{B^2(r=a)}{2\mu_0}$$

$$\left.\frac{p_0}{B_a^2/2\mu_0}\right|_{\max} = 1$$



Drawing of the plasma pressure and the magnetic field pressure as a function of the radius for the case in which the maximum pressure is reached

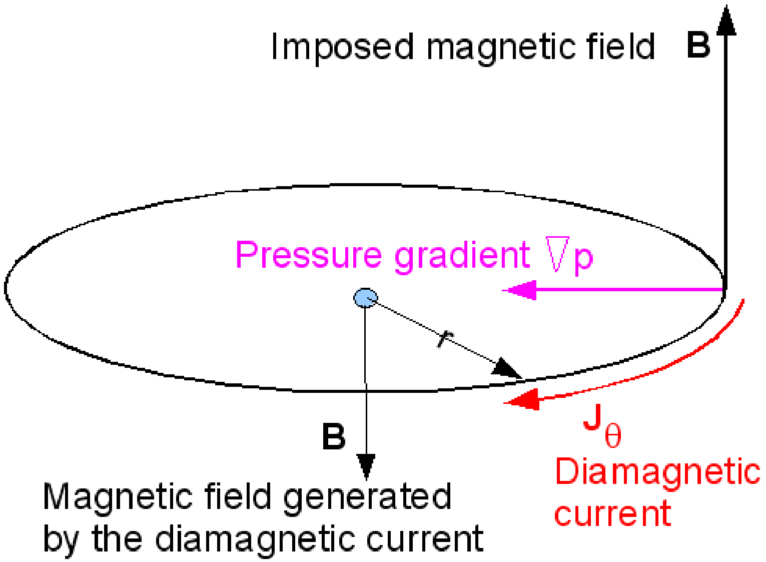
Theta pinch

- A current is needed to change the magnetic field

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

$$\mathbf{J}_\perp = \frac{\mathbf{B} \times \nabla p}{B^2}$$

- Current is in the θ direction.
- Generates a magnetic field in the opposite direction
- Hence its name: diamagnetic current



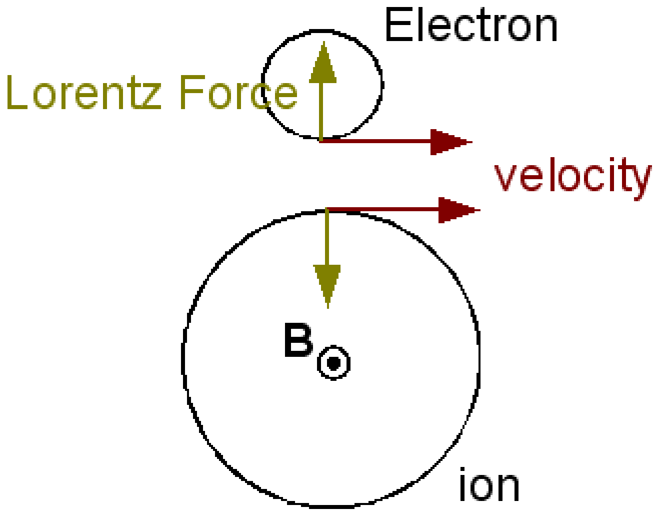
Diamagnetic current J generated by the pressure gradient

Aren’t the particles automatically confined ?

- Charged particles gyrate around the magnetic field lines
- Note since the Lorentz force depends on the charge

$$\mathbf{F} = Z_s e \mathbf{v} \times \mathbf{B}$$

- Electrons and ions gyrate in the opposite direction



Ions and electrons gyrate around the magnetic field line in the opposite direction.

Theta pinch

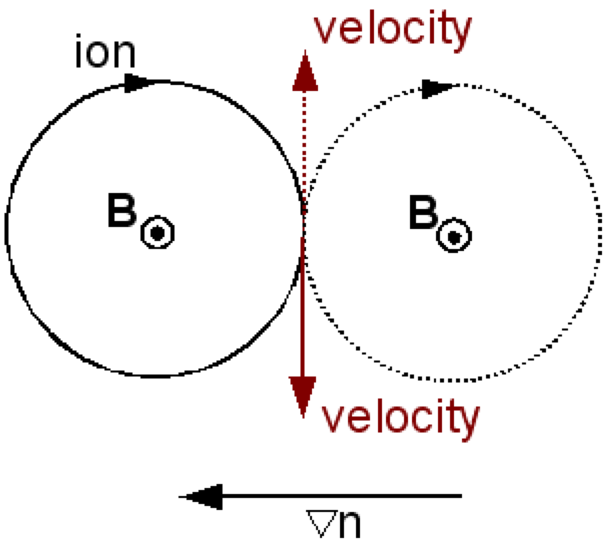
- Diamagnetic current is automatically generated

$$J = ev_\perp n(x+\rho) - ev_\perp n(x-\rho)$$

$$\approx 2ev\rho\frac{\partial n}{\partial x}$$

$$\rho = \frac{mv_\perp}{eB} \qquad \langle 2mv_\perp^2\rangle = 2T$$

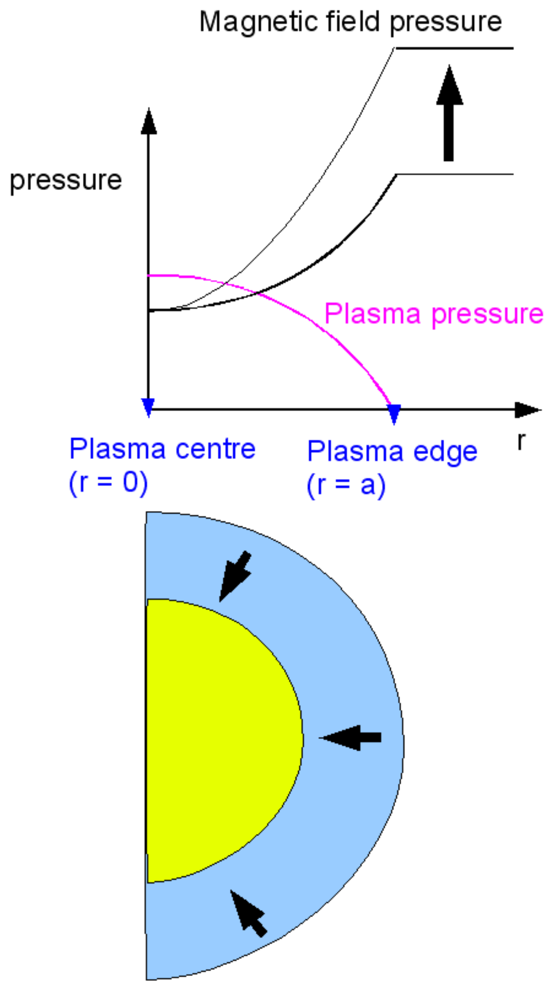
$$\boxed{J \approx 2\frac{T}{B}\frac{\partial n}{\partial x} \rightarrow \mathbf{J}_\perp = \frac{\mathbf{B}\times\nabla p}{B^2}}$$



Two gyrating ions. Due to the density gradient there are more particles moving down compared with up

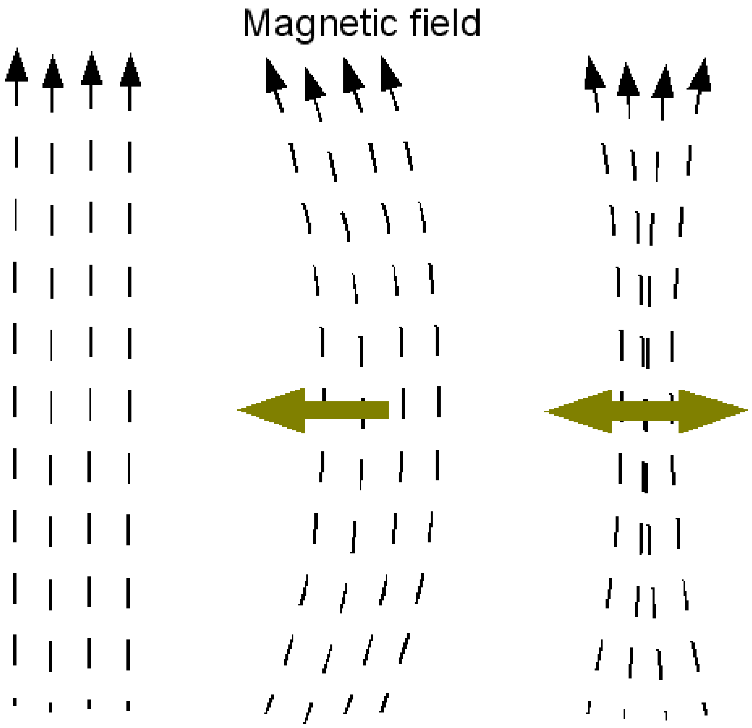
Heating of the theta pinch

- Ramp up the magnetic field by ramping the current in the coils
- The magnetic field pressure will increase and is no longer balanced by the plasma pressure
- The plasma is compressed
- Compression leads to work against the pressure gradient force which will heat the plasma



The theta pinch is stable

- The magnetic field acts like a rubber tube
- Bending it will lead to a magnetic field tension, and consequently to a force that wants to make the field straight again
- Squeezing it will lead to an increase in the magnetic field pressure and consequently to a force that wants to prevent the squeezing



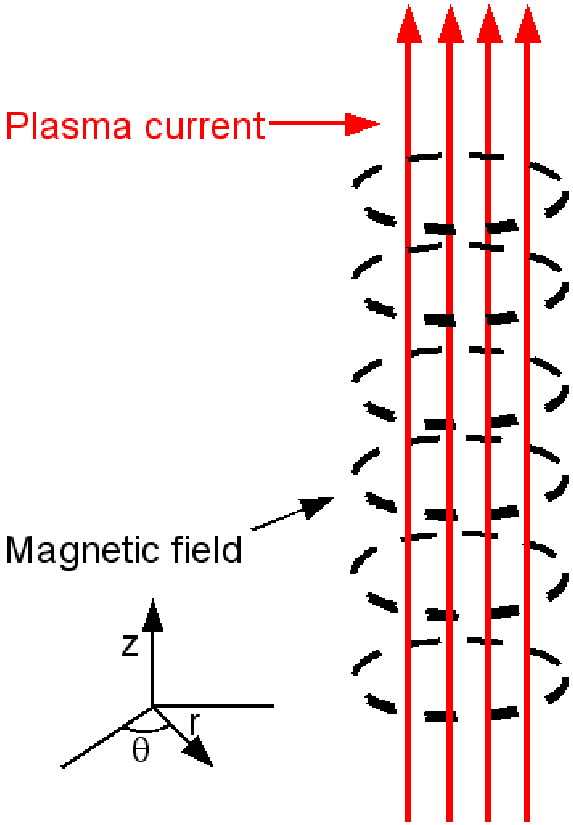
The arrow is the restoring force

Theta pinch

- Plasma with finite pressure in a magnetic field leads to diamagnetic current
- This current is ‘automatically’ generated and simply reflects that the particles are confined
- Because of diamagnetism the maximum plasma beta is 1.
- The theta pinch is stable and can confine any pressure profile
- End losses prevent this concept to be useful for a reactor.

Z-pinch

- A strong current is generated in the z-direction
- This current generates a magnetic field in the θ direction
- JxB force is then fully determined
- Pressure gradient must balance the JxB force and is then also fully determined by the current



Z-pinch magnetic field

- Current is the source of the magnetic field

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

$$2\pi r B_\theta = \int \mathrm{d}\mathbf{S} \cdot \nabla \times \mathbf{B} = \mu_0 \int \mathrm{d}\mathbf{S} \cdot \mathbf{J} = \mu_0 \pi r^2 J \qquad \boxed{r < a}$$

$$2\pi r B_\theta = \mu_0 \pi a^2 J \qquad \boxed{r > a}$$

- Field in the θ-direction : $B_\theta = \dfrac{\mu_0 J \min(r^2, a^2)}{2\, r}$

Z-pinch -profiles

- Pressure profile follows form the force balance

$$\mathbf{J} \times \mathbf{B} = -J B_\theta \mathbf{e}_r$$

$$\frac{\mathrm{d}p}{\mathrm{d}r}\mathbf{e}_r = \mathbf{J} \times \mathbf{B} = -\frac{\mu_0 J^2 r}{2}\mathbf{e}_r$$

$$p(r) = -\frac{\mu_0 J^2 r^2}{4} + C$$

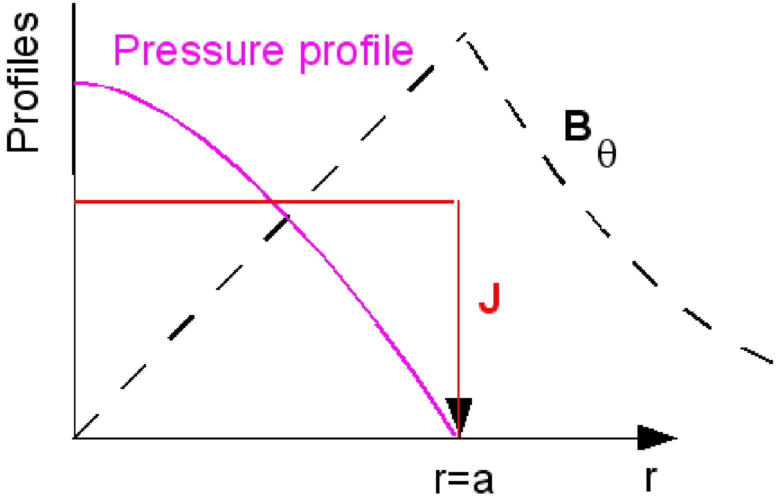
- The pressure profile is parabolic

$$p(r) = \frac{\mu_0 J^2}{4}(a^2 - r^2)$$

Z-pinch profiles

- For uniform current profile the pressure must be parabolic
- The maximum pressure scales with the current squared
- Magnetic field increases with radius -> magnetic pressure confines the plasma
- But field lines are also circular. Field line tension is as important

$$p(r) = \frac{\mu_0 J^2}{4}(a^2 - r^2)$$



Radial profiles of pressure current and magnetic field for a z-pinch

Z-pinch : plasma beta

- Taking the pressure in the centre and the magnetic field at the edge

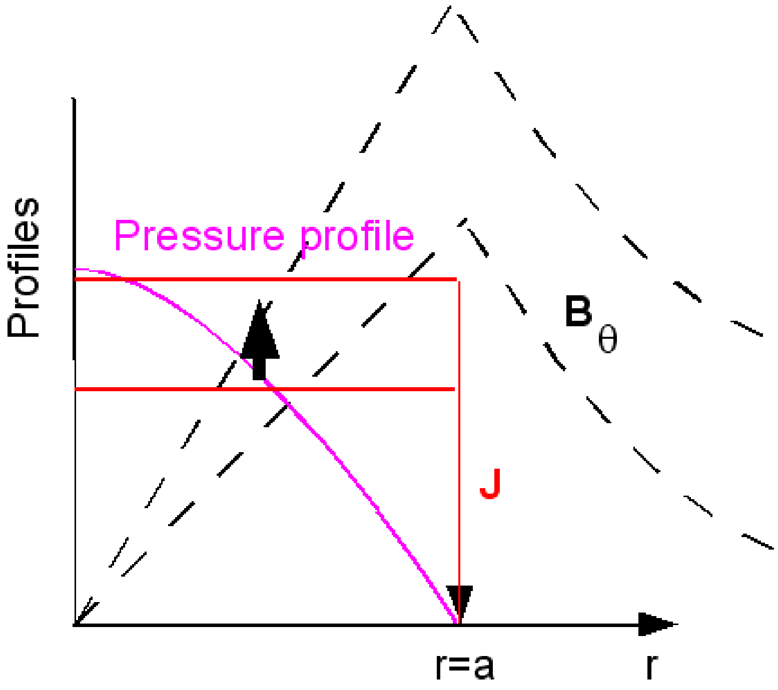
$$p(r) = \frac{\mu_0 J^2}{4}(a^2 - r^2) \qquad B_\theta = \frac{\mu_0 J \min(r^2, a^2)}{2\, r}$$

- One finds

$$\beta = 2$$

Z-pinch heating

- Now on ramps the current, but the effect is again the compression of the magnetic field
- Besides the heating due to compression, the current will also dissipate heat when the plasma resistivity is finite

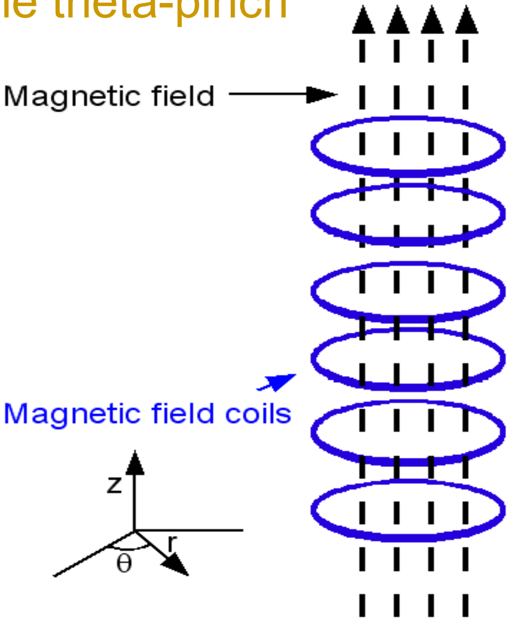


Ramping of the current will increase the magnetic field which will compress the plasma

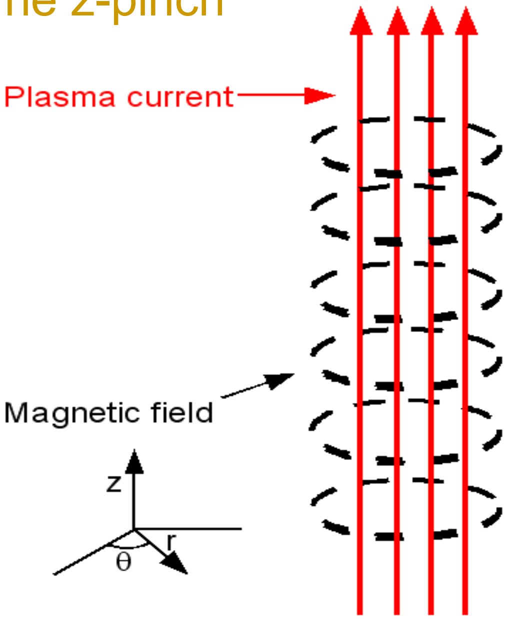
Confinement in the Z-pinch

- The z-pinch confines the particles. NO end losses

The theta-pinch

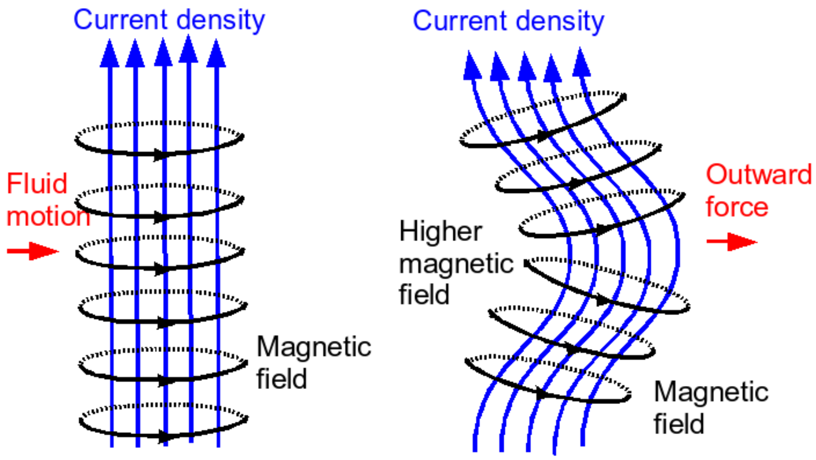


The z-pinch



Z-pinch stability

- Bend the plasma channel
- The field line density on the inside goes up (higher magnetic field strength) on the outside it decreases (lower magnetic field strength)
- The gradient in the magnetic field strength thus generated leads to a force that enhances the perturbation
- Equilibrium is not stable

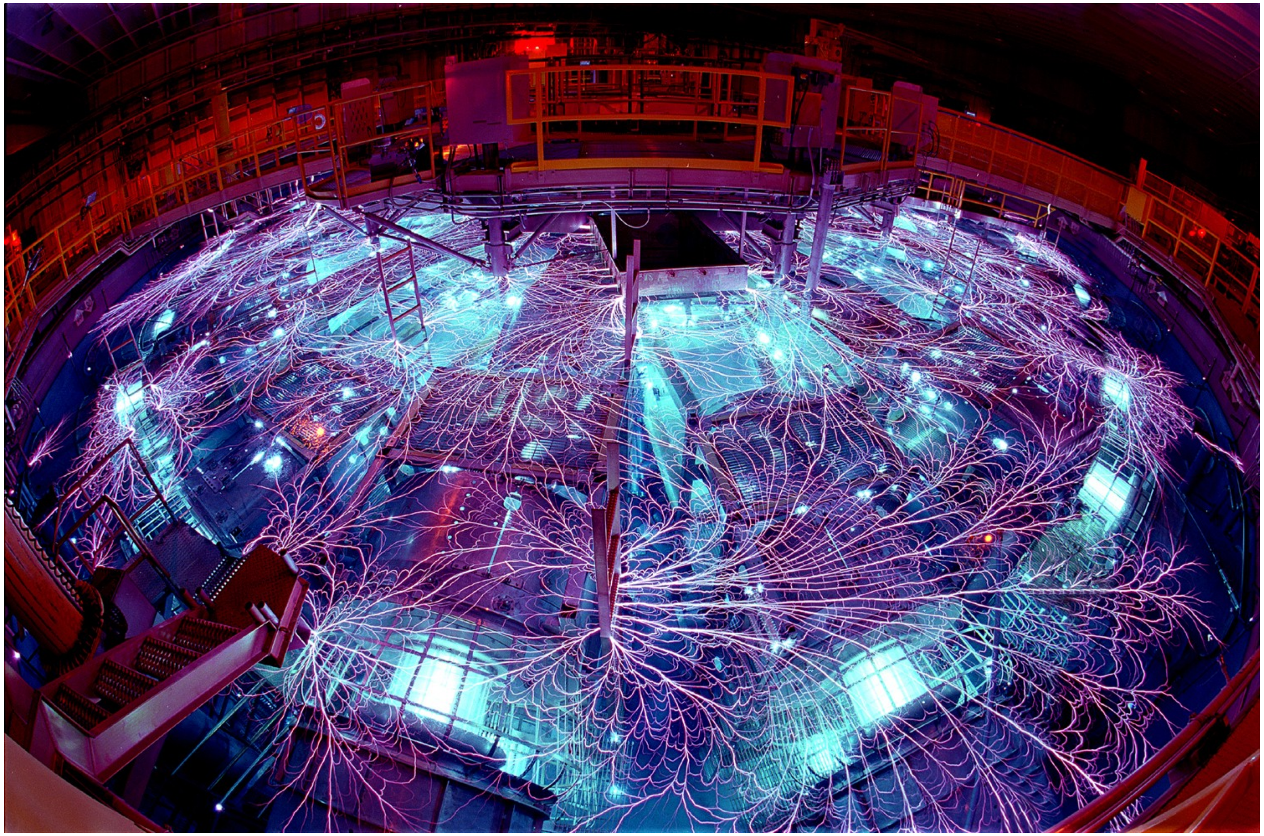


The Z-pinch is unstable. Most relevant instability is the kink

Z-pinch summary

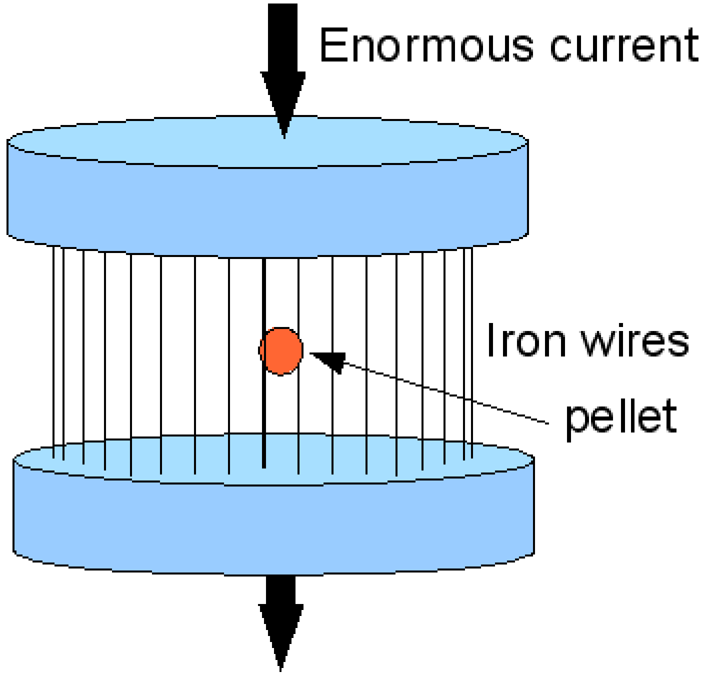
- Through the use of a current an equilibrium can be generated
- No end losses in this equilibrium
- It can be efficiently heated through compression as well as the dissipation due to a finite resistivity
- Plasma beta is 2
- BUT It isn’t stable
- Z-pinch is still used as X-ray source

Z-machine (Sandia national lab)



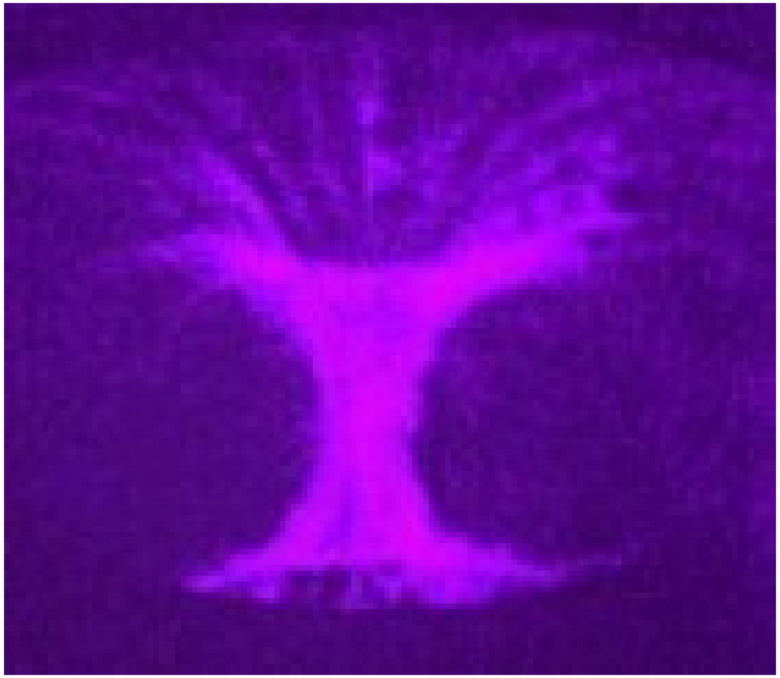
Use of the z-pinch

- An enormous current is sent through a set of iron wires
- The wires first melt and finally form a plasma
- This plasma produces X-rays which implode the pellet
- Also the different current channels contract each other
- Power 80 times the worlds energy consumption generated (for 1 billionth of a second



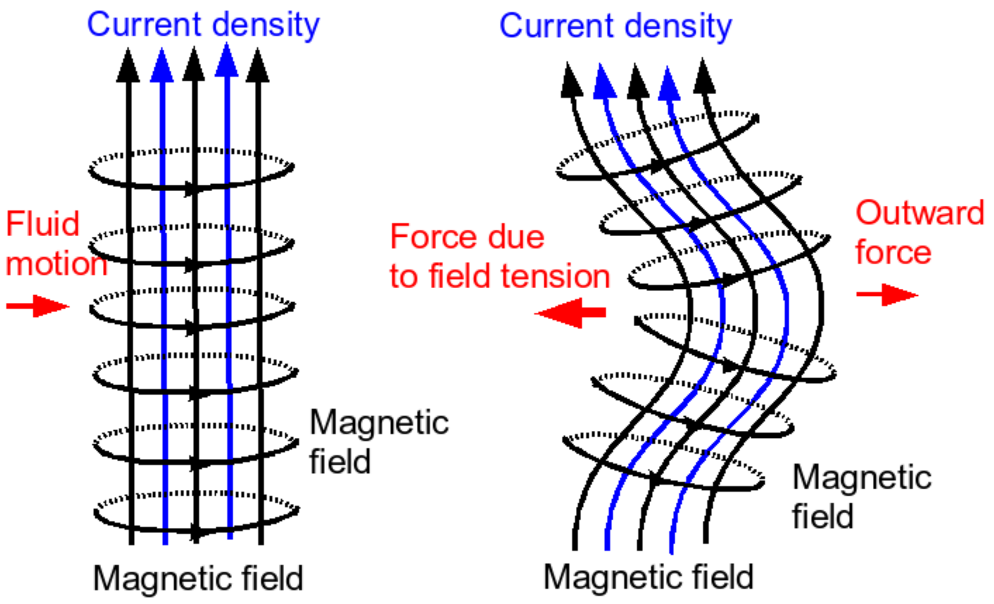
Z-machine

- The different current channels attract each other
- Leads to an inward motion
- Here photographed with a exposure time of 1 ns



Screw pinch : stabilize the Z-pinch

- Combination of the Z-pinch with the theta-pinch can lead to stabilization



- But the helical field lines will then again lead to end losses