Physics of Fusion power

Lecture 7: Stellarator / Tokamak

Toroidal curvature has its price

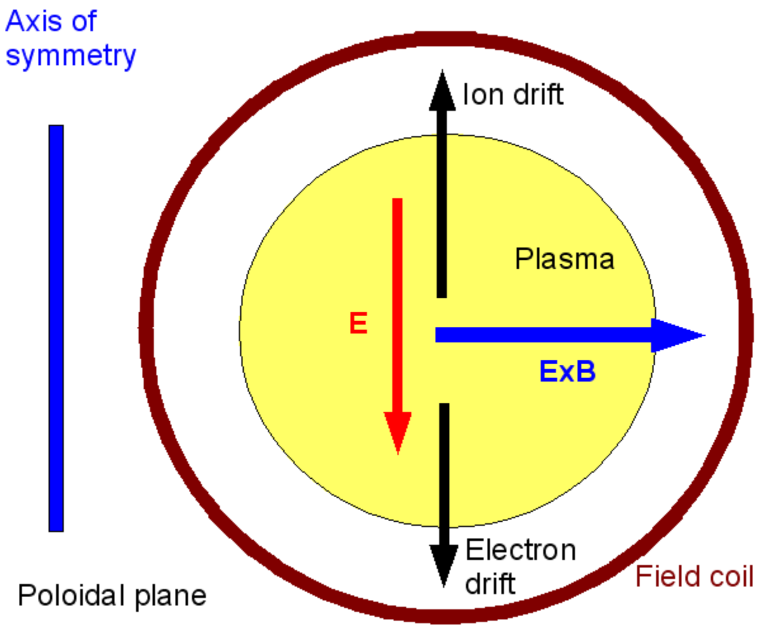
- Plasma wants to escape the magnetic field

$$F = -\mu \nabla B$$

- The ExB velocity

$$\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = -\frac{E_z}{B}\mathbf{e}_R$$

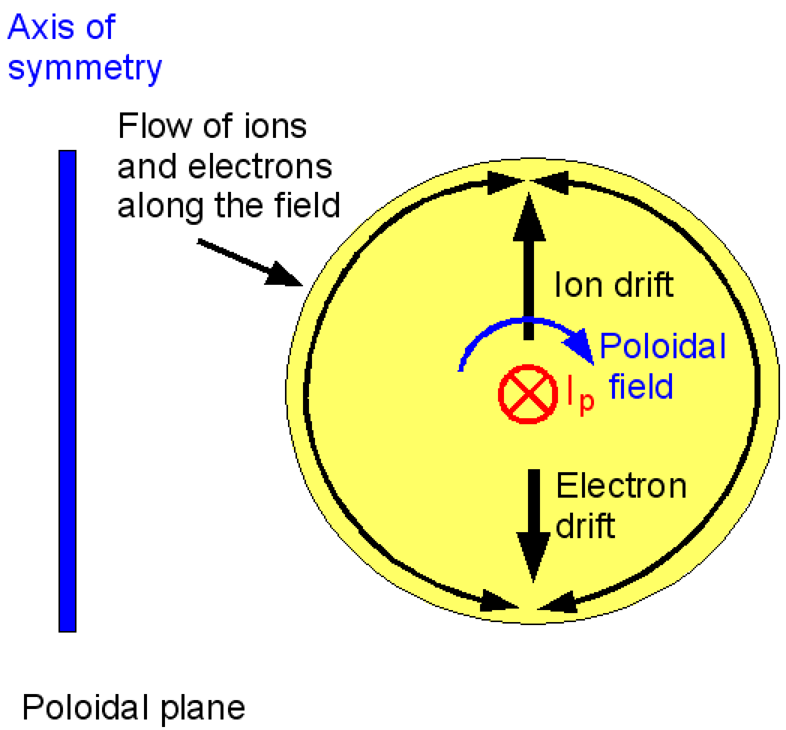
- Is directed outward and will move the plasma on the wall in a short timescale
- Remedy is the toroidal plasma current



Poloidal cut of the tokamak.

Remedy : a plasma current

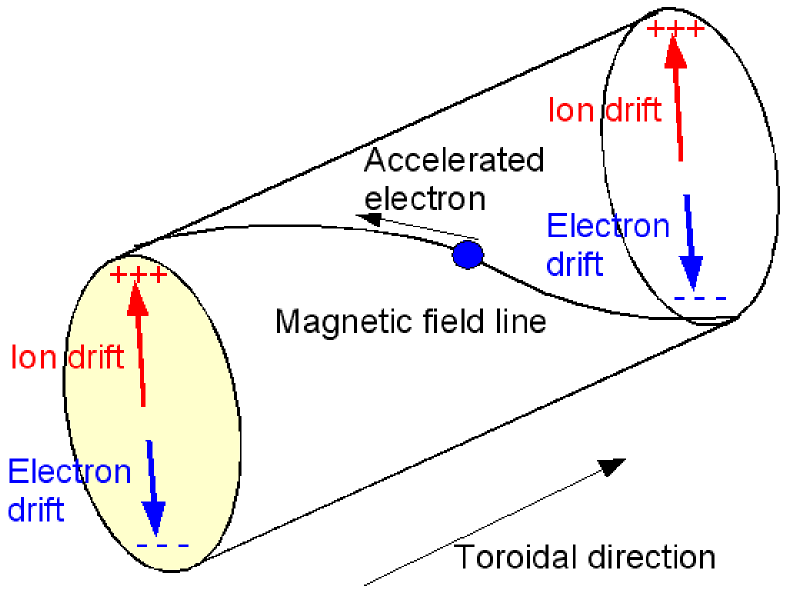
- A toroidal current in the plasma will generate a poloidal field
- Top and bottom are connected by the magnetic field line
- A vertical electric field would have a component along the field and leads to acceleration of the ions / electrons
- Drift will be balanced by a return flow along the field



Poloidal cut of the tokamak.

Maybe easier to understand

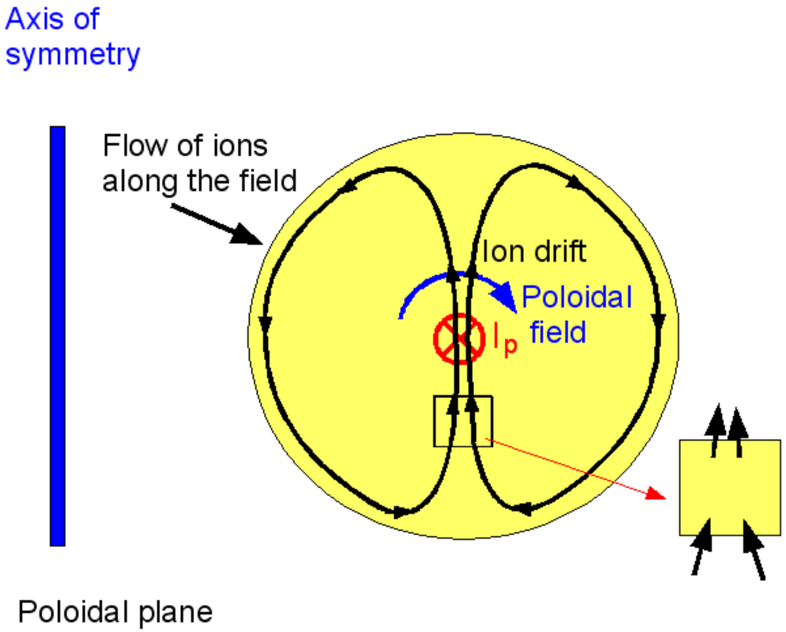
- For every toroidal angle the ions drift up and the electrons drift down
- A helical field line will therefore connected the regions of ‘positive and negative’ charge
- Electrons are accelerated along the field line, and neutrality can be maintained
- Note it does lead to parallel flows (with a toroidal component)



Attempt at a 3D view. The toroidal plasma is drawn as a cylinder

Same thing again

- Poloidal projection of the flow pattern (ions as well as electrons) is closed in the poloidal plane
- In every small volume the ions leaving the volume are replaced by the ions entering
- No charge separation
- Note, this is of course a cartoon picture.

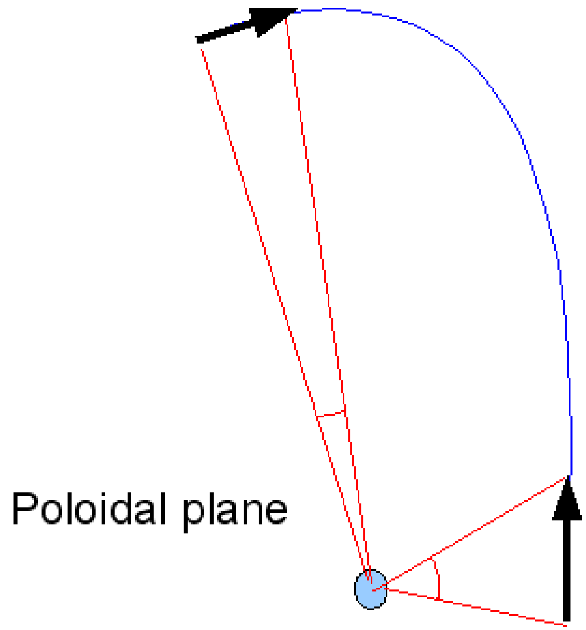


Do we really need the plasma current?

- It might at first appear obvious that the answer is yes since without current inside the plasma

$$\oint \mathrm{ds} \cdot \mathbf{B} = 0$$

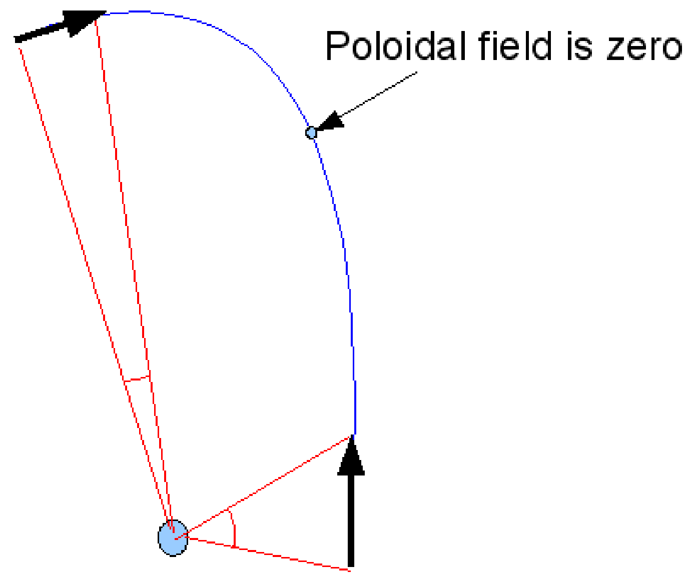
- But a positive as well as negative poloidal field does not necessarily mean that the field line on average does not go around poloidally



On average the field line can go around even if the enclosed current is zero.

Toroidal symmetry

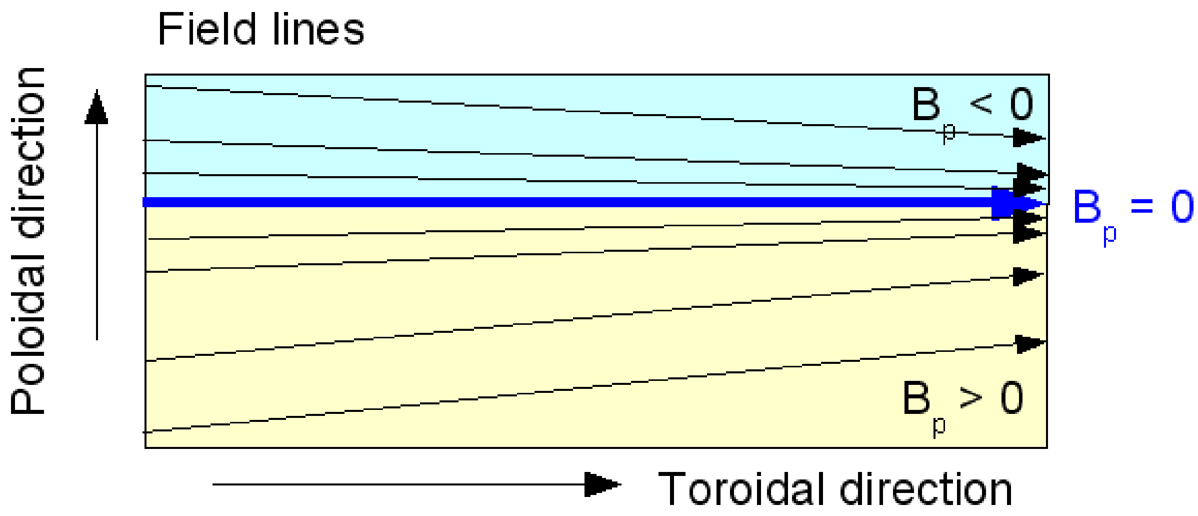
- With zero current at some point the poloidal field must be zero
- In the case of toroidal symmetry this field line closes upon itself
- Regions of positive and negative field are not connected
- A field line can not wind around poloidally
- Then top and bottom can not be connected



With toroidal symmetry one field line can not wind around poloidally

Same thing again

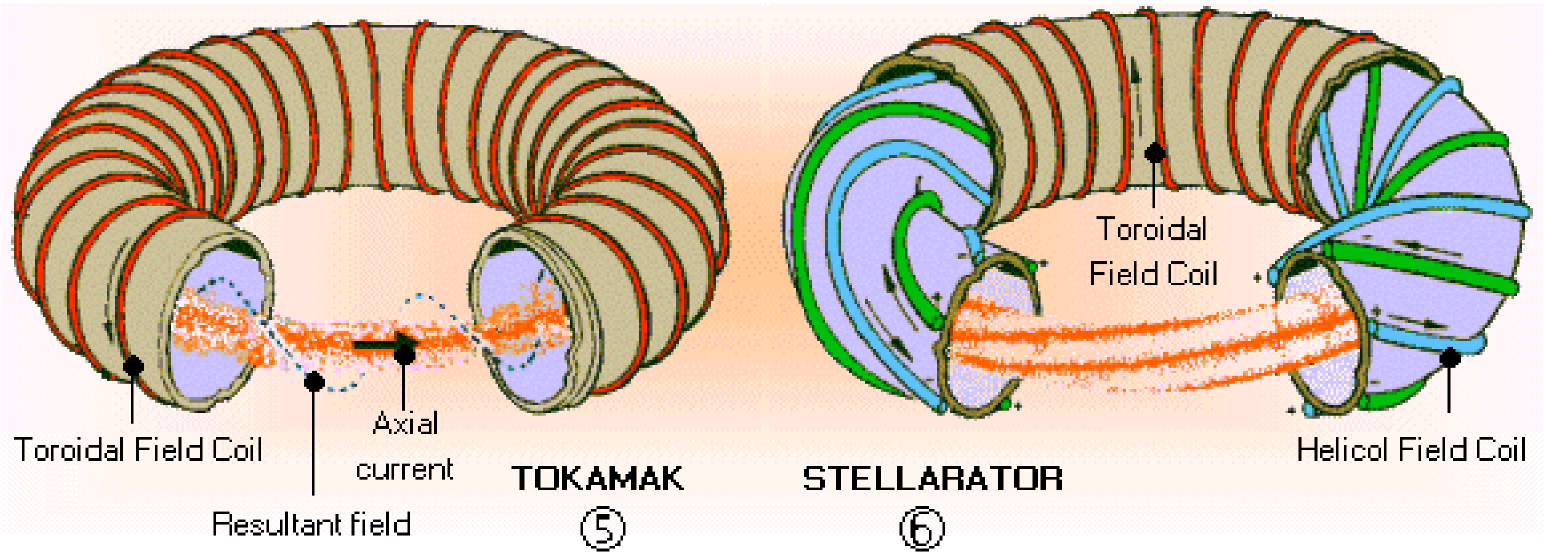
Poloidal winding of a zero current device with toroidal symmetry



- Field lines will move towards the field line with zero poloidal field
- For zero field the field line closes upon it self
- No magnetic field line can cross this line
- The field line can not wind around poloidally
- No flow from top to bottom is possible -> No equilibrium

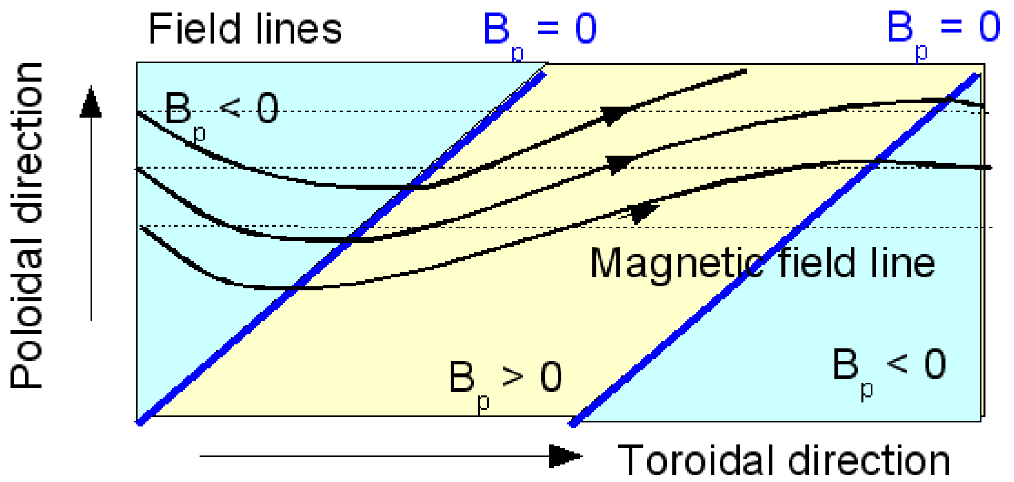
The stellarator

- If the field is not toroidally symmetric the motion in the toroidal direction will move the field line from regions of positive poloidal field into regions of negative field
- Then a net poloidal turn of the field line can be achieved
- Steady state operation is possible at the cost of greater complexity



Same thing again

Poloidal winding of a zero current device *without* *toroidal symmetry*

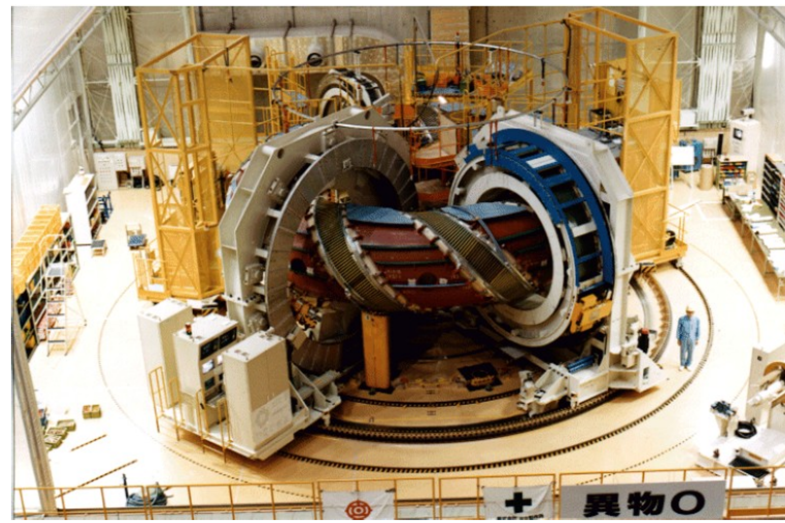


- Without toroidal symmetry to toroidal field can move the field line from the region of positive poloidal to negative poloidal field
- With the correct shaping of the surfaces one can impose a net transform of the field line
- Top and bottom can be connected
- An equilibrium exists

Large Helical Device (LHD,Japan)

Specifications of LHD devices

	Phase I (II)
Major radius	3.9 m
Coil minor radius	0.975 m
Plasma radius	0.5–0.65
Plasma volume	20–30 m^3
I/m	2/10
i (0)/i (a)	$<$ 0.5/1
Helical ripple	0.2
Bo/Bmax	3/6.6 T (4/9.2 T)
Helical coil current	5.85 MAT (7.8 MAT)
LHe temp.	4.4 K (1.8 K)
Poloidal coil (IV/IS/OV)	5.0/−4.5/ − 4.5 MAT
Magnetic energy	0.9 GJ (1.6 GJ)
Plasma duration	10 s
Repetition time	5 min
Heating power	
ECRH	10 MW
NBI	15 MW (20 MW)
ICRF	3 MW (12 MW)

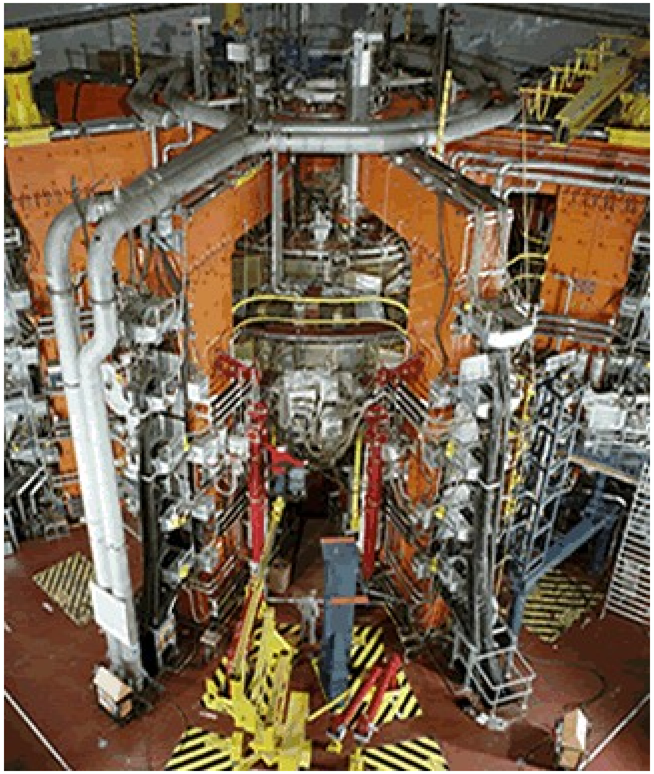


Stellarator

- Inside the device it looks something like this
- Picture from LHD in JAPAN

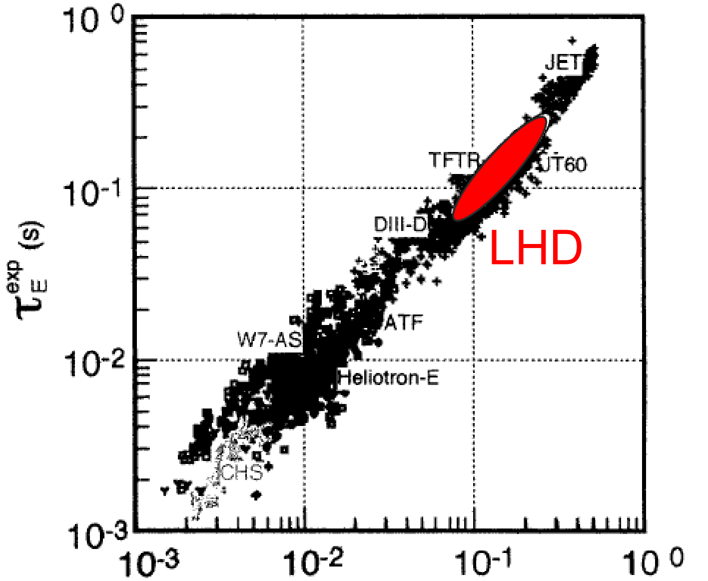
Larges tokamak: JET (EU,UK)

- Major radius 3 m
- Minor radius 1. m
- Magnetic field < 4 T
- Plasma volume 100 m^3
- Plasma current < 7 MA
- Plasma duration 10 s



Comparison of confinement time

- Confinement times of LHD are below those of the large tokamaks
- This is mostly due to the smaller plasma volume



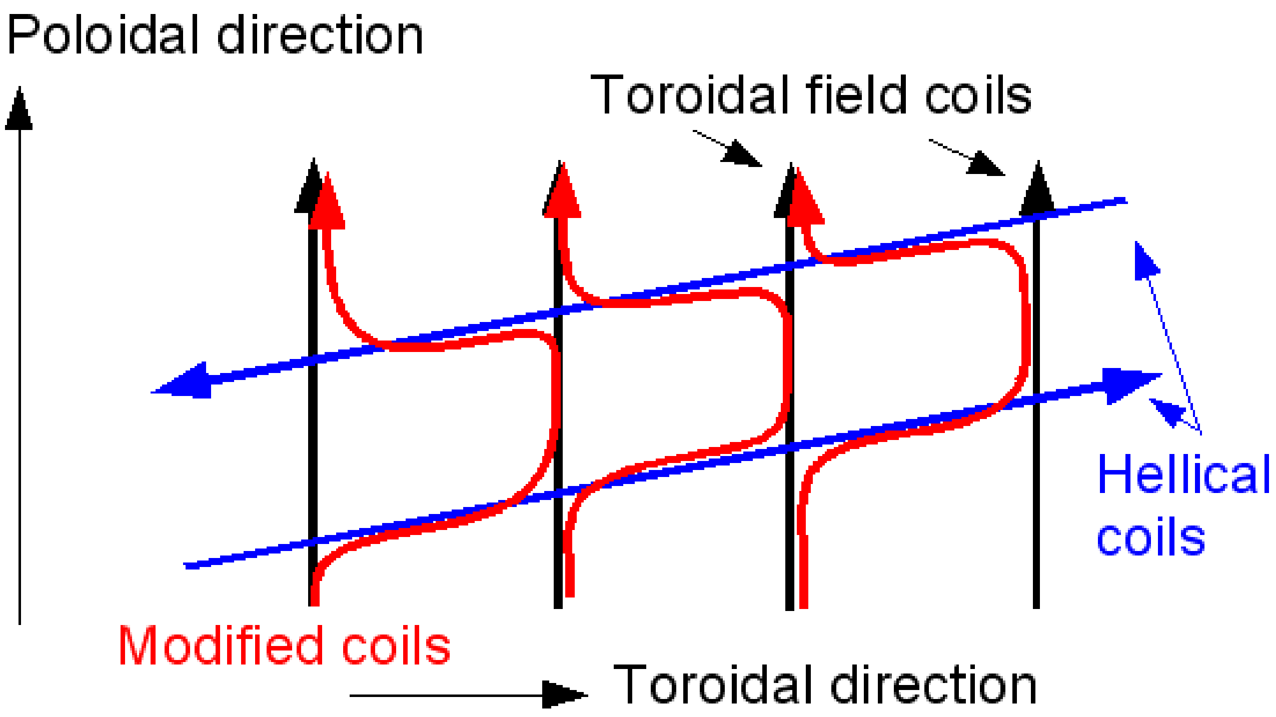
$$\tau_E^{ISS95} = 0.08\, a_p^{2.21}\, R_0^{0.65}\, P_{abs}^{-0.59}\, \bar{n}_e^{0.51}\, B_0^{0.83}\, ι_{2/3}^{0.40}$$

τ_E^{ISS95} (s), a_p (m), R_0 (m), P_{abs}(MW), $\bar{n}_e$(10^{19}m^{-3}), B_0 (T)

Confinement time of tokamaks and stellarators compared

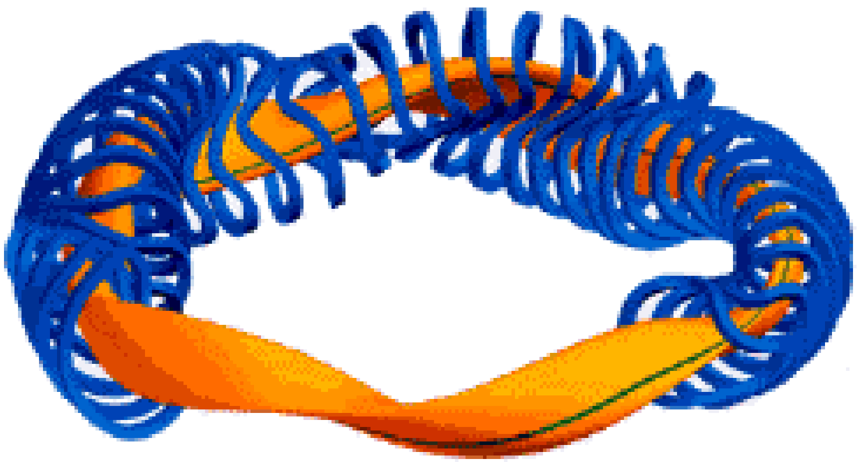
Hellical coils can be simplified

- The picture shows how the combination of helical coils and toroidal field coils can be changed to use modular coils

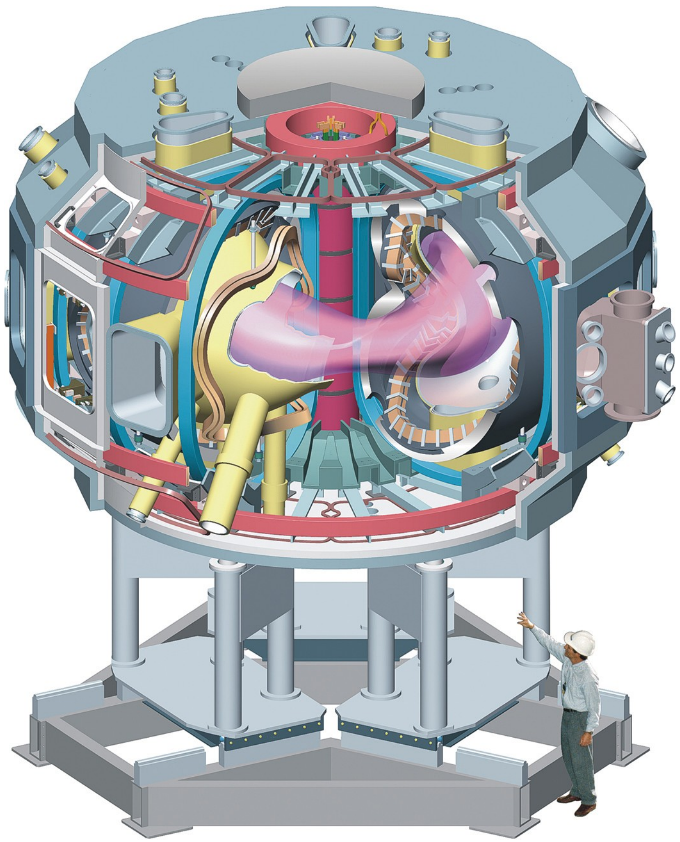


Applied in W7X

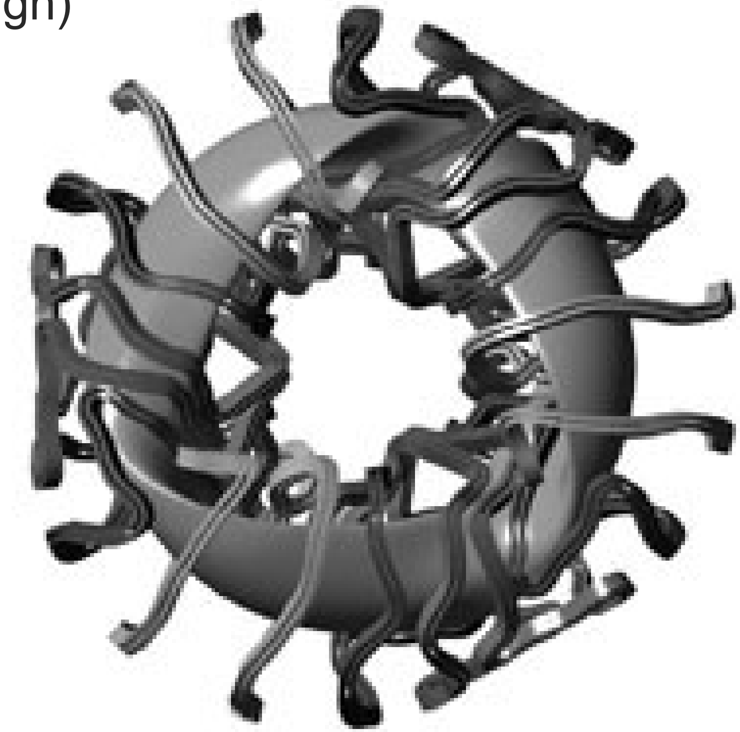
- Modular coils of W7x
- There is a large disadvantage in the use of the modular coils. They are highly bend and therefore there are large force on them
- In general it is difficult to build a compact device with a big plasma. The poloidal field one imposes from the outside decays rapidly with distance from the coils



Compact stellarator NCSX princeton



Compact stellarotors are a challenge. Note there is a plasma current in this device (not driven by a transformed though)



Tokamak versus stellarator

Advantage of the stellarator

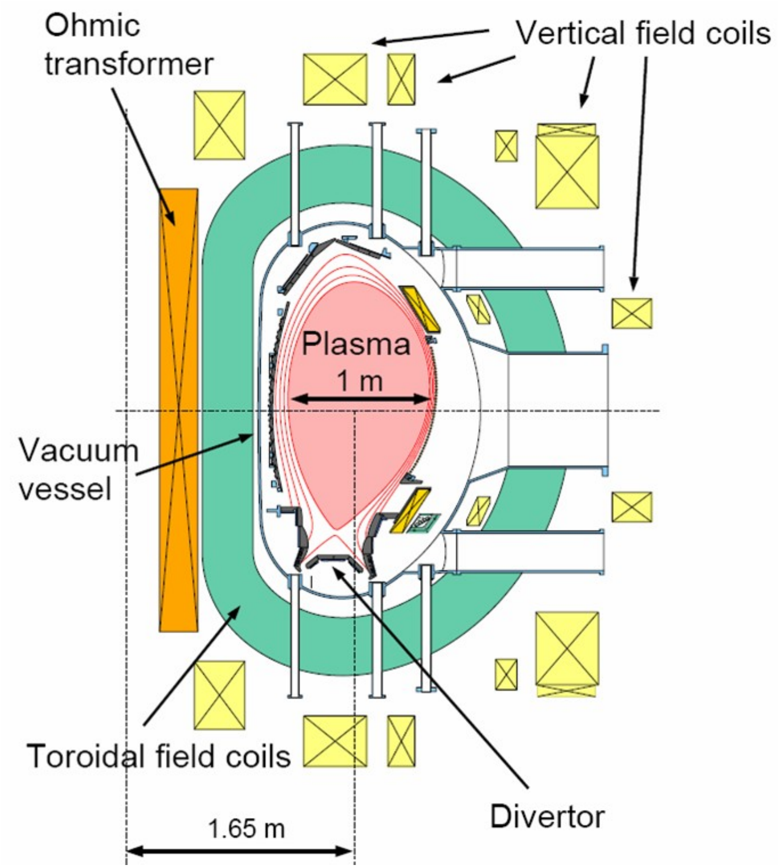
- Stationary plasma operation
- No current in the plasma, and therefore no current driven instabilities

Disadvantage

- Complex magnetic field coils
- Curved coils lead to large forces (strong supporting structures)
- Difficult to make compact devices

A tokamak

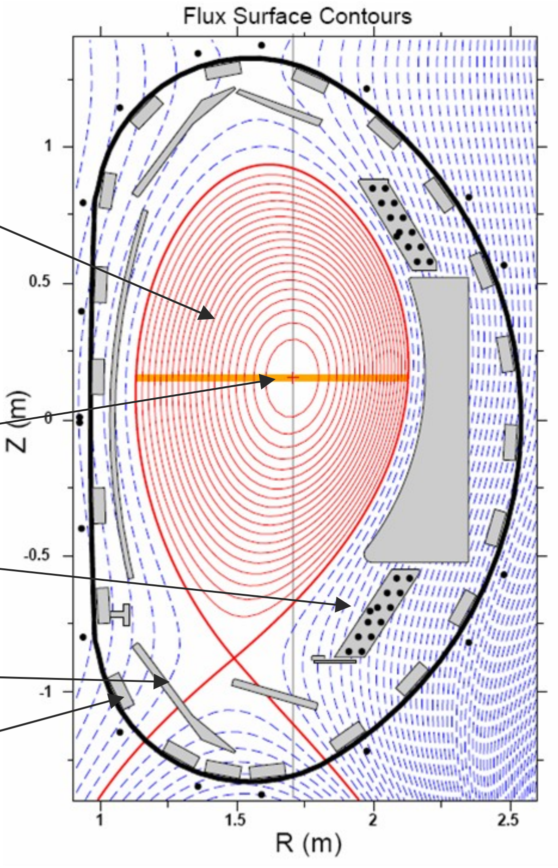
- Plasma (purple) Notice the shape
- Surrounded by plates
- Vessel (pumps)
- Coils mostly outside vessel (finite reaction time)
- Ohmic transformer / toroidal field coils (green)



Schematic Drawing of the poloidal cross section of the ASDEX Upgrade tokamak

The tokamak

- Magnetic surfaces are the surfaces traced out by the magnetic field
- They are nested (best confinement)
- Centre is shifted outward
- Large passive coils
- Magnetic field ends on a set of plates
- Large set of small coils for diagnostic purposes



Schematic Drawing of the poloidal cross section of the ASDEX Upgrade tokamak

Pitch of the field

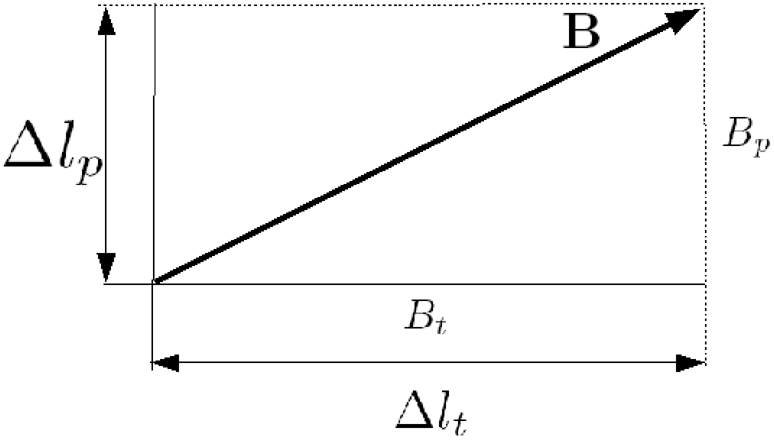
- Along the magnetic field

$$\frac{\Delta l_t}{\Delta l_p} = \frac{B_t}{B_p}$$

$$\mathrm{d}l_t = \frac{B_t}{B_p}\mathrm{d}l_p$$

- Consequently the length of the field line in toroidal direction is

$$l_t = \int \mathrm{d}l_t = \int \frac{B_t}{B_p}\mathrm{d}l_p$$



Pitch of the field line

Pitch of the magnetic field

- Length of the field

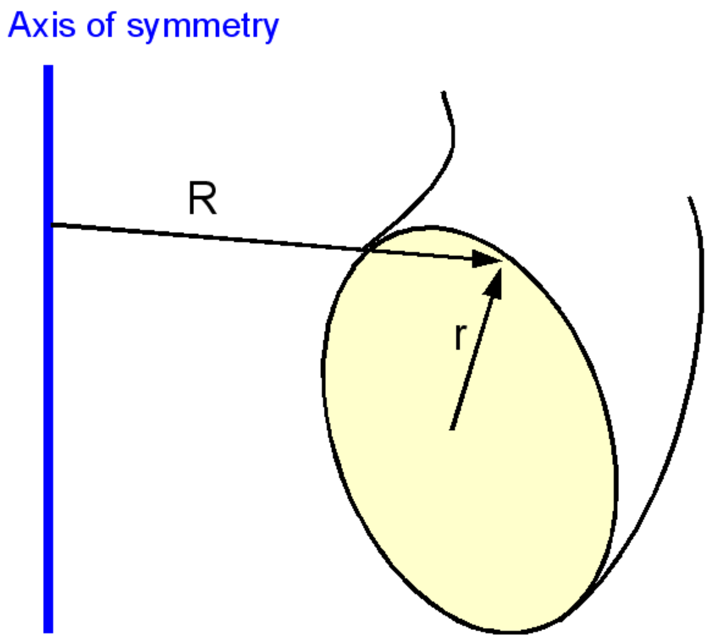
$$l_t = \int \mathrm{d}l_t = \int \frac{B_t}{B_p} \mathrm{d}l_p$$

- In one poloidal turn

$$l_t = 2\pi r \frac{B_t}{B_p}$$

- Number of toroidal turns in one poloidal turn (safety factor q)

$$q \equiv \frac{l_t}{2\pi R} = \frac{r B_t}{R B_p}$$



Definition of the minor r and major R radius

Kink stability

$$q \equiv \frac{l_t}{2\pi R} = \frac{rB_t}{RB_p}$$

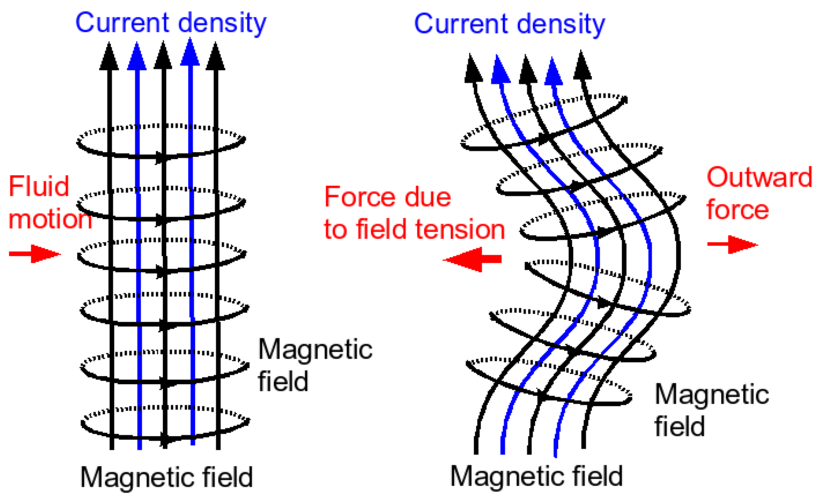
- Relation with the current

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$2\pi r B_p = \mu_0 I$$

$$q = \frac{2\pi r^2 B_t}{\mu_0 R I} = \frac{2AB_t}{\mu_0 RI}$$

- For stable operation the safety factor at the edge is chosen q > 3. The means a maximum current



Stability considerations of the screw-pinch also apply to the tokamak

Ratio of poloidal and poloidal field

- From the safety factor it follows

$$q = \frac{rB_t}{RB_p} = 3$$

- Therefore the ratio between the poloidal and toroidal field is

$$\frac{B_p}{B_t} = \frac{r}{3R} \approx 0.1$$

Pressure and current

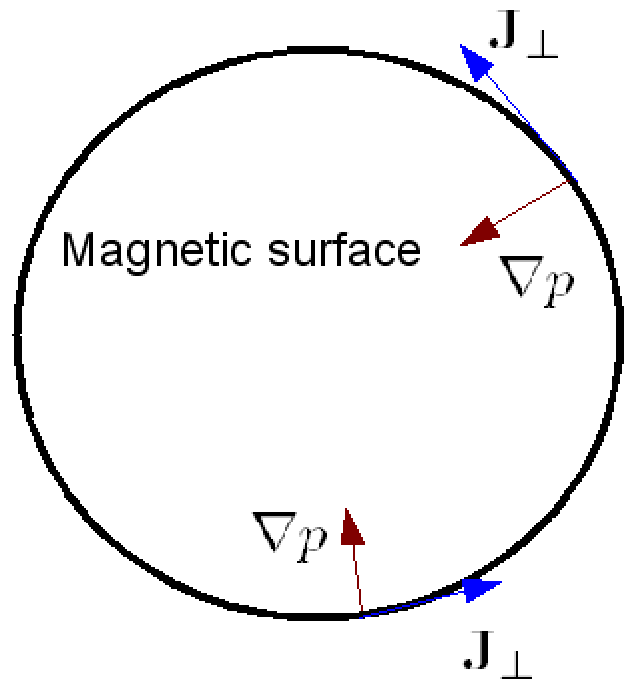
- From the force balance

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

- Taking the inner product with the magnetic field

$$\mathbf{B} \cdot \nabla p = 0$$

- The pressure gradient is perpendicular to the surface
- Pressure is constant on a surface



Pressure is constant on the magnetic surface, and the current lies inside the surface

Pressure and current

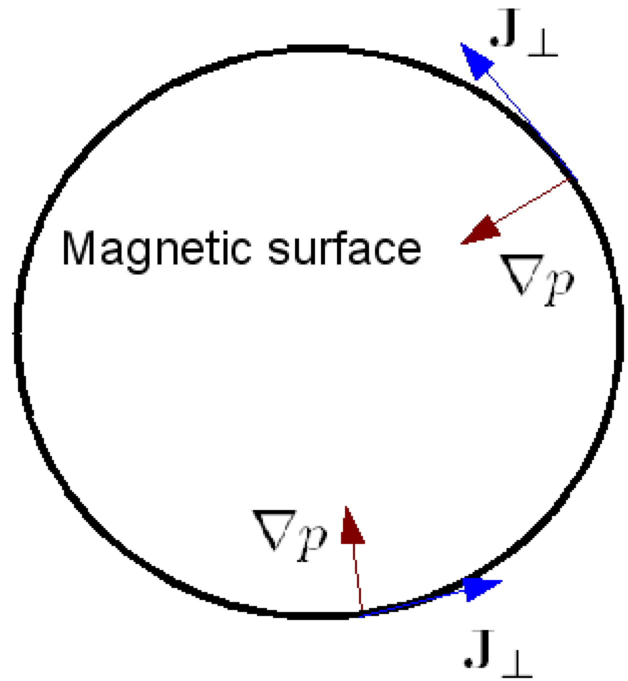
- Again using the force balance

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

- Taking the cross product with the magnetic field

$$\mathbf{J}_\perp = \frac{\mathbf{B} \times \nabla p}{B^2}$$

- Since the pressure gradient is perpendicular to the surface the current lies inside the surface



Pressure is constant on the magnetic surface, and the current lies inside the surface

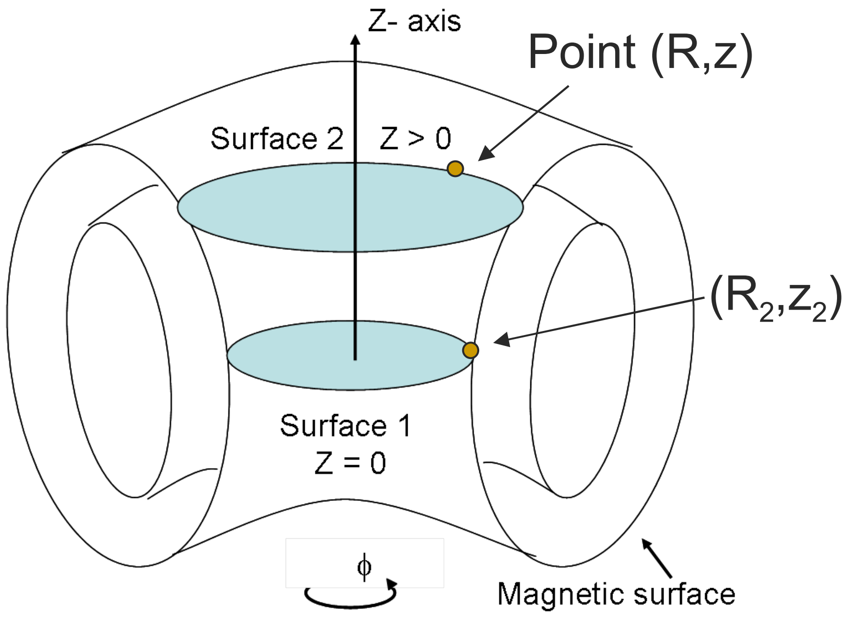
Poloidal flux

- The poloidal flux $\psi(R,z)$ is the flux through the circle with its centre at r = 0 lying in the z-plane and having (R,z) lying on its boundary

$$\nabla \cdot \mathbf{B} = 0$$

- Integrated over a volume enclosed by two of these circles and the magnetic surface yields

$$\psi(R,z) = \psi(R_2,z_2)$$



The poloidal flux is the flux through the blue areas. It is constant on a magnetic surface

Magnetic surfaces

- Traced out by the magnetic field
- The pressure is constant on the surface
- The current lies inside the surface
- The poloidal flux is constant on a surface. The surfaces are therefore also called flux-surfaces