Physics of fusion power

Lecture 8 : The tokamak continued

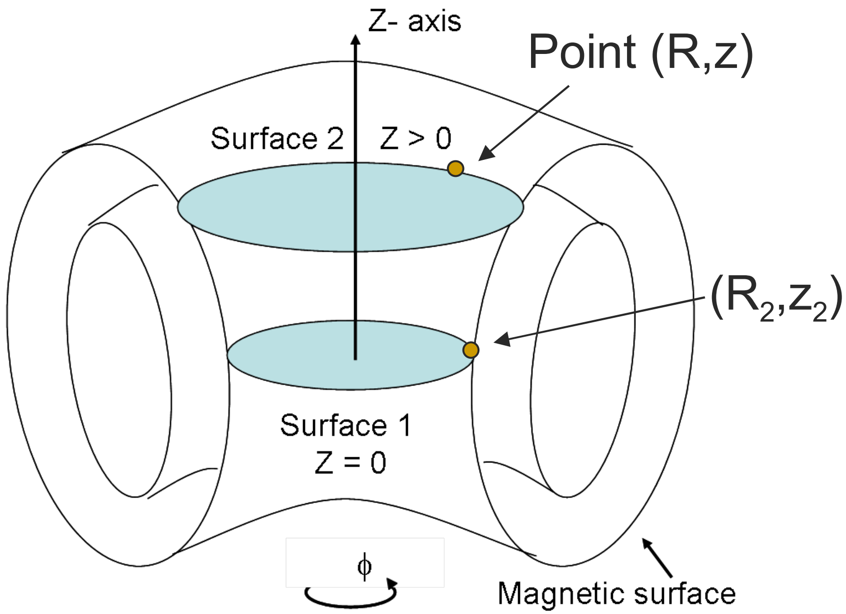
Poloidal flux

- The poloidal flux $\psi(R,z)$ is the flux through the circle with its centre at r = 0 lying in the z-plane and having (R,z) lying on its boundary

$$\nabla \cdot \mathbf{B} = 0$$

- Integrated over a volume enclosed by two of these circles and the magnetic surface yields

$$\psi(R, z) = \psi(R_2, z_2)$$



The poloidal flux is the flux through the blue areas. It is constant on a magnetic surface

Pitch of the field

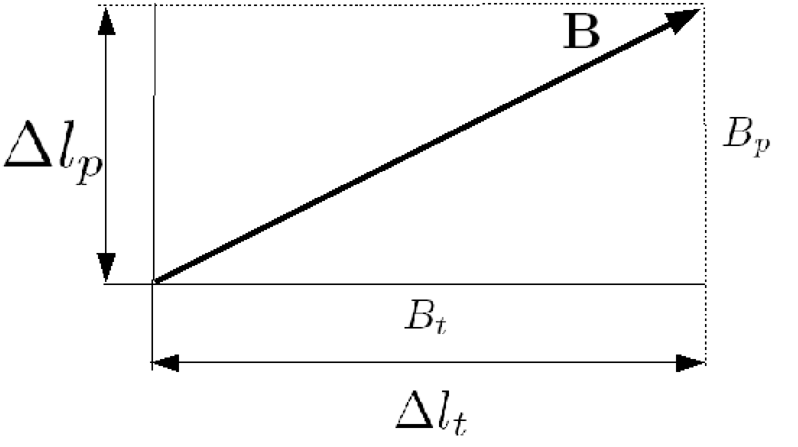
- Along the magnetic field

$$\frac{\Delta l_t}{\Delta l_p} = \frac{B_t}{B_p}$$

$$\mathrm{d}l_t = \frac{B_t}{B_p}\mathrm{d}l_p$$

- Consequently the length of the field line in toroidal direction is

$$l_t = \int \mathrm{d}l_t = \int \frac{B_t}{B_p}\mathrm{d}l_p$$



Pitch of the field line

Pitch of the magnetic field

- Length of the field

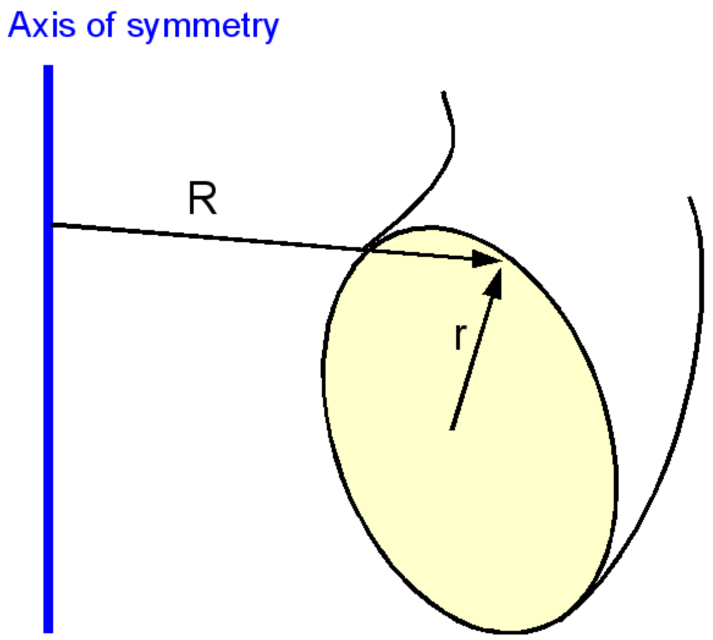
$$l_t = \int \mathrm{d}l_t = \int \frac{B_t}{B_p} \mathrm{d}l_p$$

- In one poloidal turn

$$l_t = 2\pi r \frac{B_t}{B_p}$$

- Number of toroidal turns in one poloidal turn (safety factor q)

$$q \equiv \frac{l_t}{2\pi R} = \frac{r B_t}{R B_p}$$



Definition of the minor r and major R radius

Kink stability

$$q \equiv \frac{l_t}{2\pi R} = \frac{rB_t}{RB_p}$$

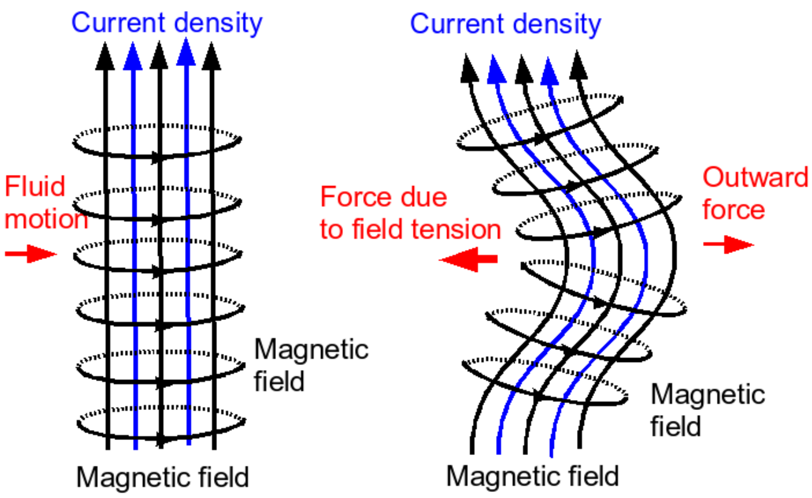
- Relation with the current

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$2\pi r B_p = \mu_0 I$$

$$q = \frac{2\pi r^2 B_t}{\mu_0 R I} = \frac{2AB_t}{\mu_0 R I}$$

- For stable operation the safety factor at the edge is chosen q > 3. The means a maximum current



Stability considerations of the screw-pinch also apply to the tokamak

Ratio of poloidal and poloidal field

- From the safety factor it follows

$$q = \frac{rB_t}{RB_p} = 3$$

- Therefore the ratio between the poloidal and toroidal field is

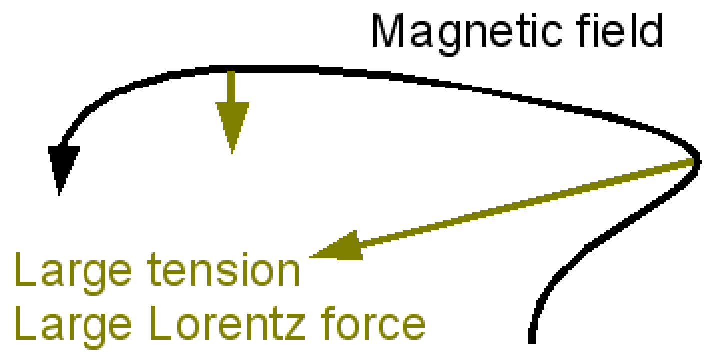
$$\frac{B_p}{B_t} = \frac{r}{3R} \approx 0.1$$

Magnetic surfaces

- Traced out by the magnetic field
- The pressure is constant on the surface
- The current lies inside the surface
- The poloidal flux is constant on a surface. The surfaces are therefore also called flux-surfaces

Plasma shape isn't obvious

- Bending of the magnetic field leads to a tension
- The magnetic field 'tries to avoid' sharper edges
- Naturally the plasma would remain circular
- The elongated shape must be imposed upon the plasma



Schematic Drawing magnetic field and tension force. The magnetic field does not appreciate being bend

Distance between the surfaces

- Magnetic field is divergence free

$$\nabla \cdot \mathbf{B} = 0$$

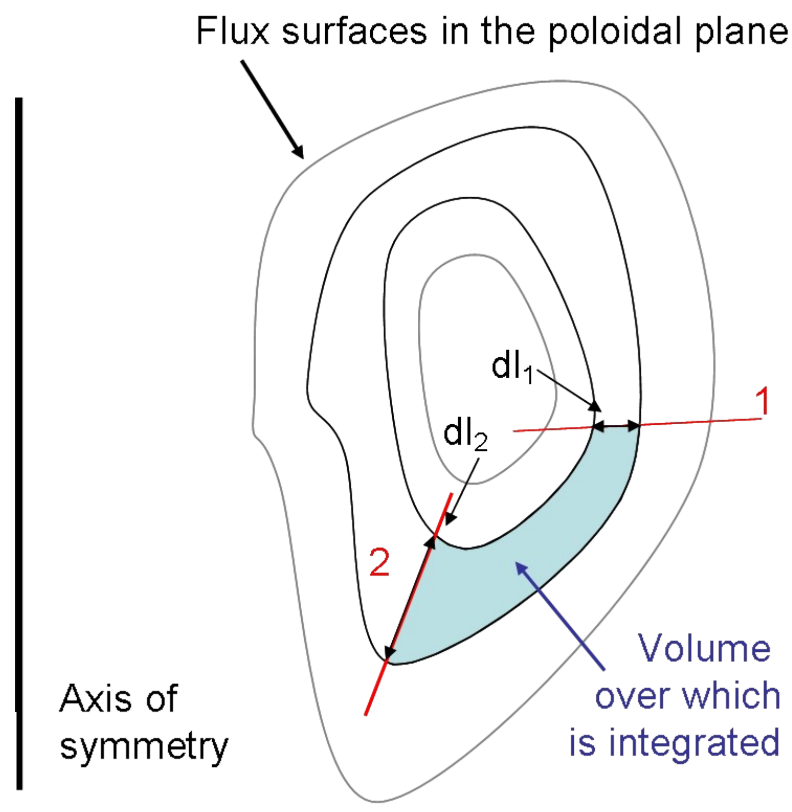
- Integrating over the indicated volume gives

$$-B_{p1} 2\pi R_1 \mathrm{d}l_1$$

$$+B_{p2} 2\pi R_2 \mathrm{d}l_2 = 0$$

- Inside the surface

$$B_{p2} = B_{p1} \frac{R_1}{R_2} \frac{\mathrm{d}l_1}{\mathrm{d}l_2} \rightarrow B_p \propto \frac{1}{R\mathrm{d}l}$$



Relation with the poloidal flux

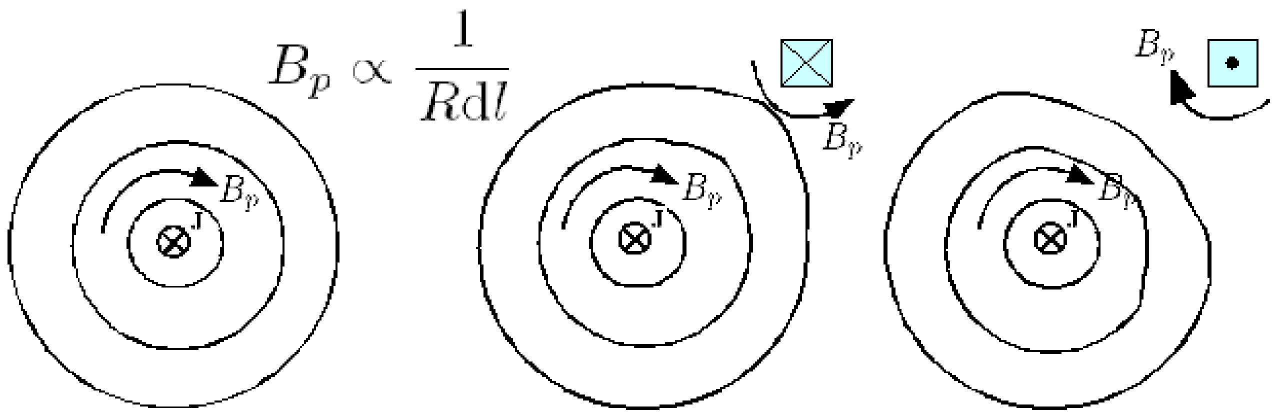
- The poloidal flux is constant on each of the surfaces

$$\delta\psi = |\nabla\psi(1)|\mathrm{d}l_1 = |\nabla\psi(2)|\mathrm{d}l_2 \longrightarrow \frac{\mathrm{d}l_1}{\mathrm{d}l_2} = \frac{|\nabla\psi(2)|}{|\nabla\psi(1)|}$$

- This yields for the poloidal field

$$B_{p2} = B_{p1}\frac{R_1}{R_2}\frac{\mathrm{d}l_1}{\mathrm{d}l_2} \longrightarrow B_{p2} = B_{p1}\frac{R_1|\nabla\psi(2)|}{R_2|\nabla\psi(1)|} \longrightarrow B_p \propto \frac{1}{R}|\nabla\psi|$$

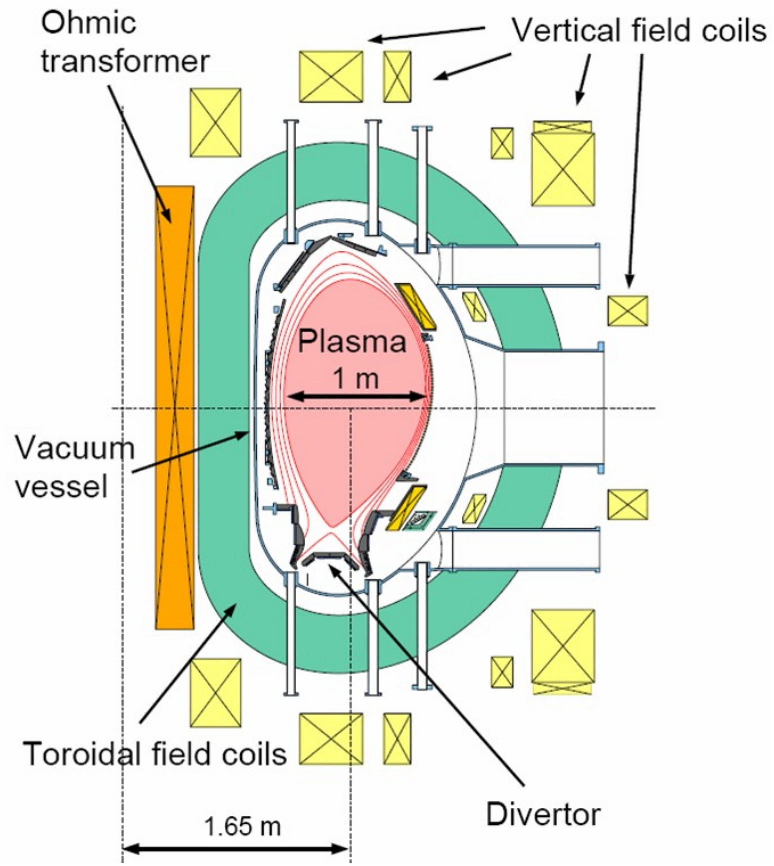
Plasma shaping



- Can be understood from the relation between poloidal field and distance between the surfaces
- A current in a coil outside the plasma will change the poloidal field
- If it weakens the poloidal field of the current the distance between the surfaces increases
- If it enhances the field the distance decreases

Back to the picture

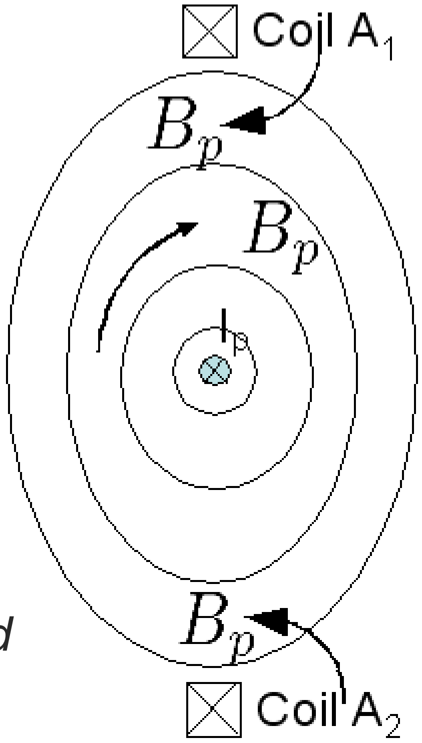
- This makes clear the amount of coils around the plasma
- The vertical coils can shape the plasma and control its position
- Note dominant shaping is the vertical elongation of the plasma



Schematic Drawing of the poloidal cross section of the ASDEX Upgrade tokamak

Dominant shaping : elongation

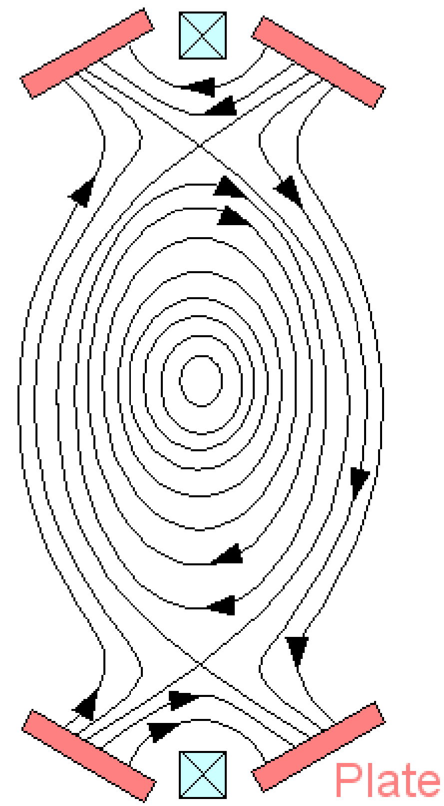
- Dominant shaping is the elongation of the plasma
- This is achieved by two coils on the top and bottom of the plasma with a current in the direction of the plasma current



Elongation is generated by two field coils at the top and bottom of the plasma

Reason 1 for plasma elongation

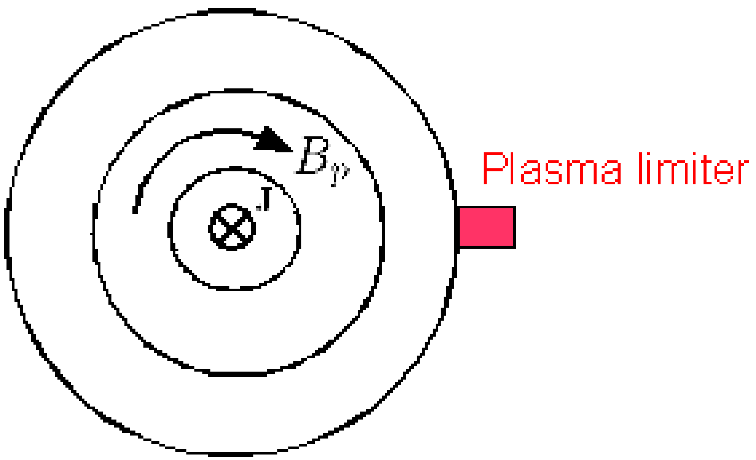
- Plasma can be diverted onto a set of plates
- Close to the coils the field of the coils dominates
- In between the field is zero resulting in a purely toroidal field line
- This shows up as an X-point in the figure of the magnetic surfaces
- Surfaces outside the one with the X-point are not close with the field ending on the plates



Shaping coils allow for plasma to be diverted onto the divertor-plates

Plasma limiter

- Without divertor the plasma needs to be limited by a material (referred to as limiter)
- The plasma touching the limiter is still several 1000 of Kelvin
- Sputtering or melting leads to the release of material into the plasma
- These unwanted components are referred to as impurities



Schematic picture of a plasma limiter

Impurities are no good news

- Given a fixed electron density, impurities dilute the fuel

$$n_e = n_D + n_T + Zn_I$$

Density of the impurity with charge Z

- Acceleration of electrons by the ions in the plasma lead to radiation losses known as 'Bremstrahlung'

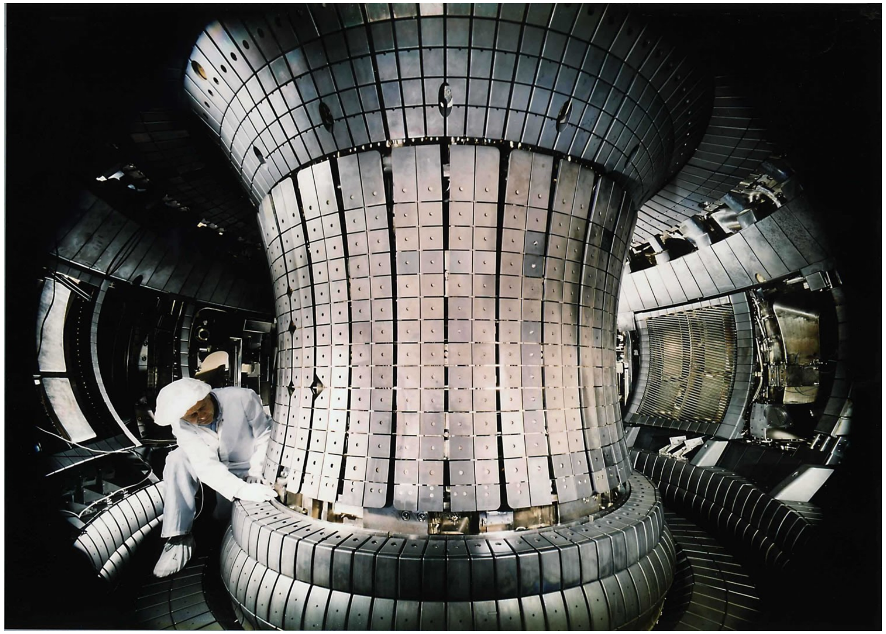
$$P_{\mathrm{rad}} \propto n_e^2 Z_{\mathrm{eff}} \sqrt{T}$$

Effective charge

- The radiation scales with the average charge. High Z impurities enhance the radiation
- High Z-impurities also lead to energy loss through line radiation

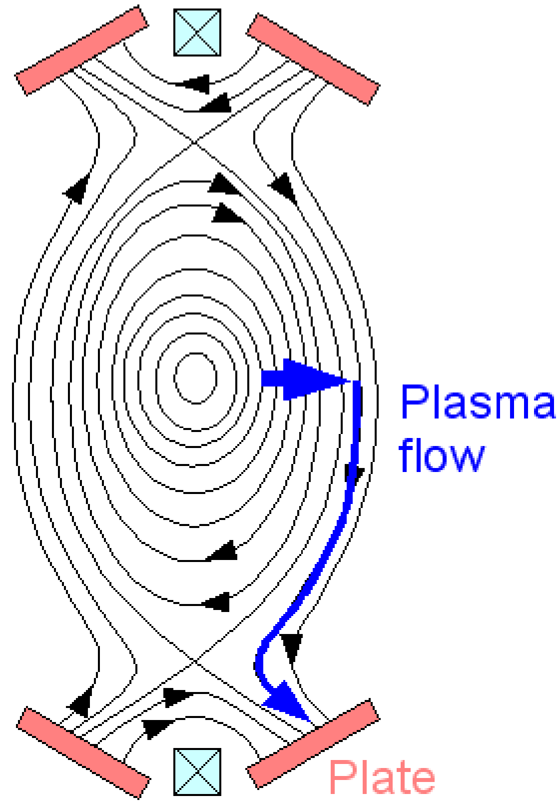
Preventing impurities

- Plasma facing components have to be chosen carefully
- Carbon / Beryllium have a low Z
- Carbon does not melt but has the problem that it binds well with Tritium (contamination of the machine)
- Tungsten has very high Z, but takes the heat loads very well



Divertor

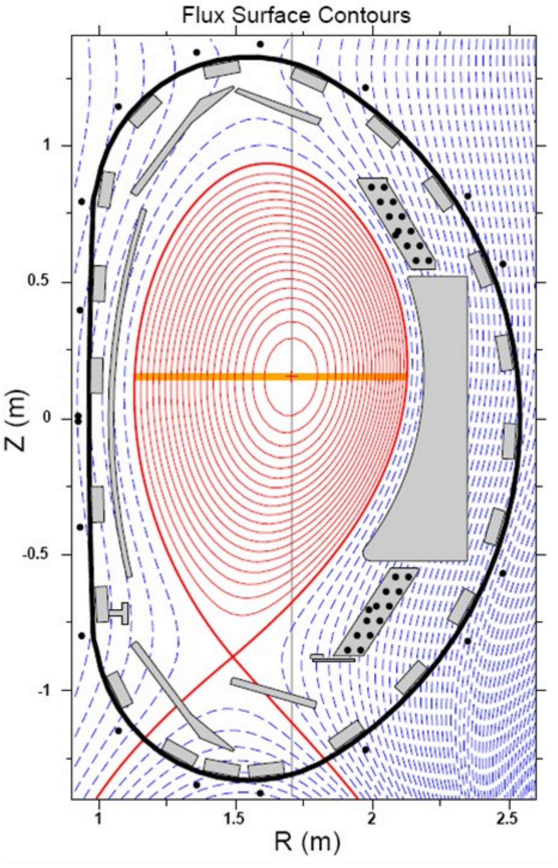
- Using a divertor the particles that leave the plasma flow along the magnetic field and hit the target plates
- These plates are far away from the plasma such that any impurity released at the plate has a smaller chance ending up in the plasma
- Furthermore, one can try to cool the plasma further through special arangements in front of the plates



Plasma flow in divertor configuration

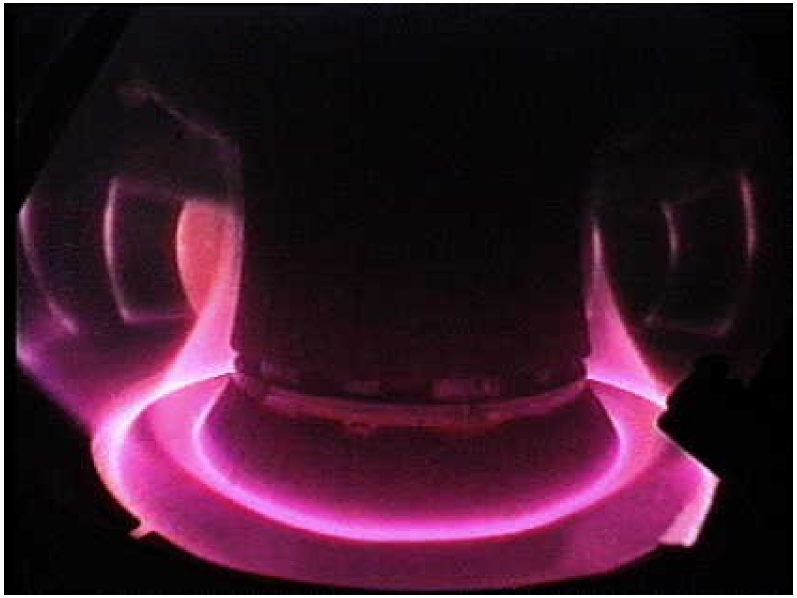
Divertor

- The divertor has a disadvantage : it takes space
- In general only one divertor is used, usually at the bottom (easier to construct)



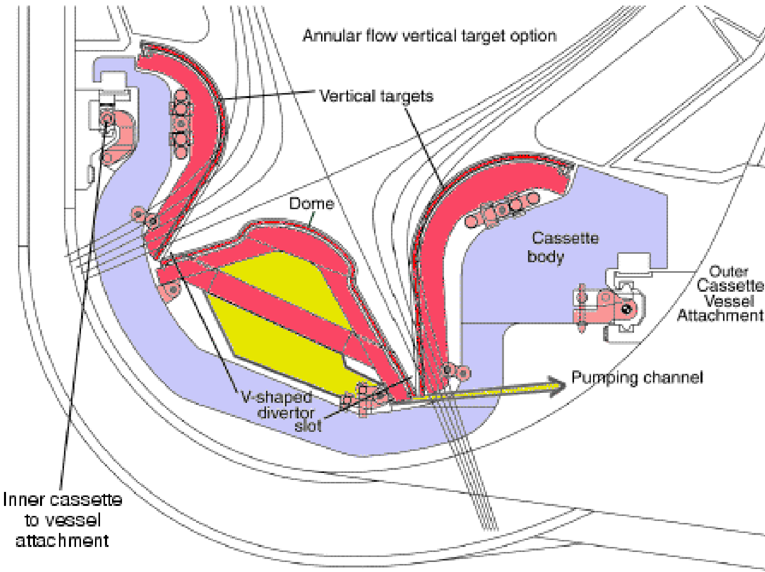
Picture of the plasma

- Shows that most of the line radiation (one of the lines of Hydrogen) comes from the divertor structure
- Real plasma so hot that it does not have Hydrogen line radiation
- So thin that you look right through it



The divertor

- A modern divertor design looks something like this
- Note that it has, as far as possible a closed structure. This to allow the efficient pumping of the neutral particles
- Note also that the angle between the magnetic field and the plate is as small as possible. This makes that the energy carried by the particles to the plate is distributed over the largest possible area



Modern divertor design (ITER)

Reason II : Plasma elongation

- Distance to go around poloidally is larger

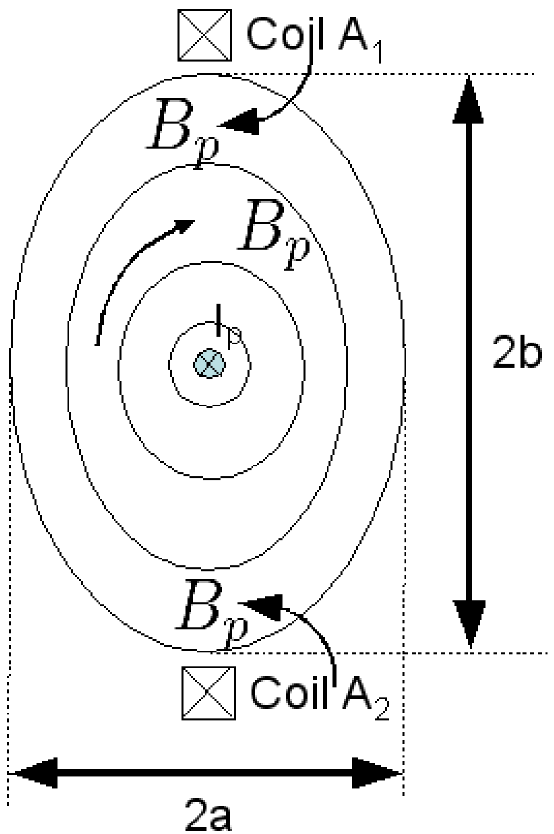
$$q = \frac{2\pi r^2 B_t}{\mu_0 R I} = \frac{2 A B_t}{\mu_0 R I}$$

$$A = \pi a b = \pi a^2 \kappa \quad \kappa = \frac{b}{a}$$

For the same plasma current

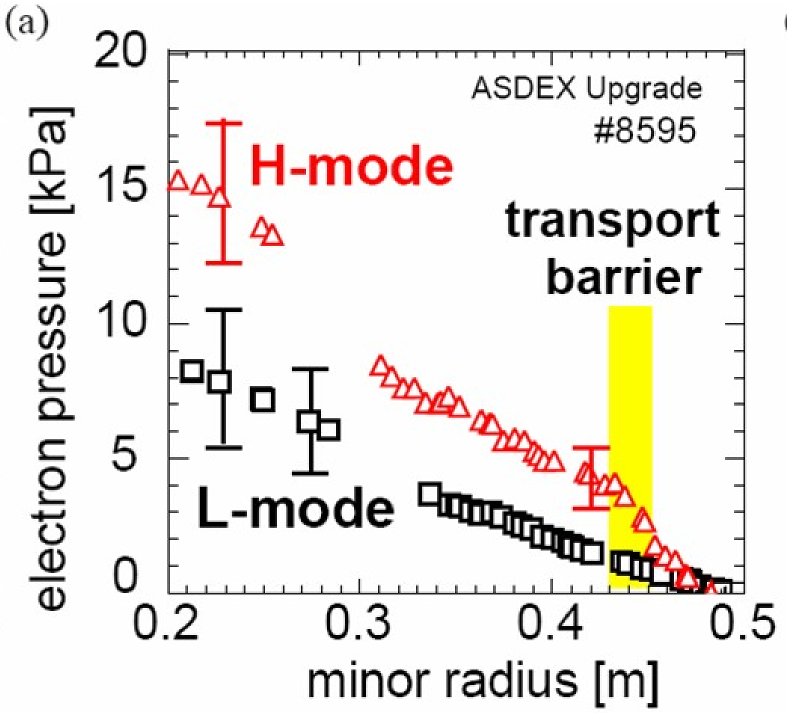
$$q_{\text{elip}} = q_{\text{circ}} \kappa$$

- If q = 3 is the limit of operation one can run a larger current in an elliptically shaped plasma



Reason III : Plasma elongation

- A transition phenomenon is observed in Divertor plasmas known as the L (low) to H (high confinement) transition
- In this transition a steep pressure profile is generated at the plasma edge
- Not very well understood
- Confinement improvement is roughly a factor 2 !!!!



Equilibrium / Vertical instability

- Magnetic field due to the coil follows form

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

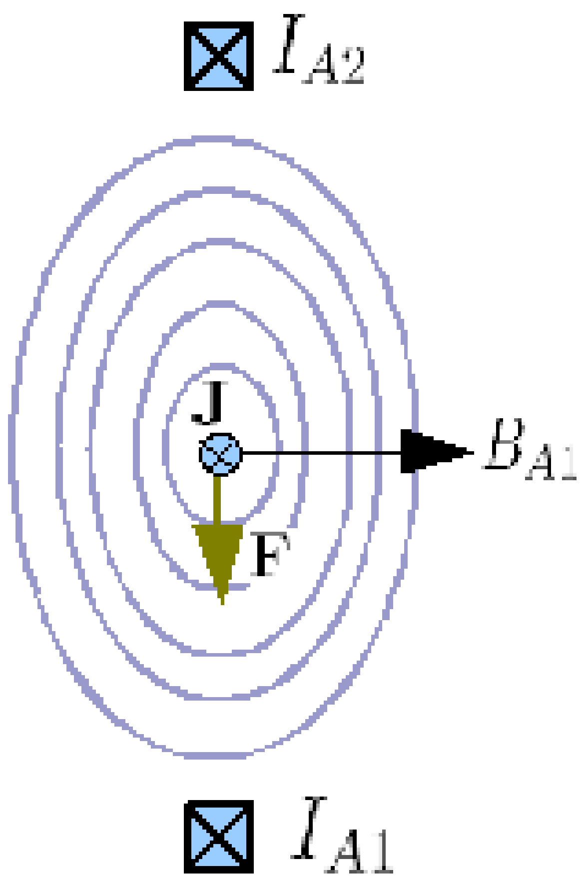
- Assume d<<R one finds

$$2\pi d B_{A1} = \mu_0 I_{A1}$$

$$B_{A1} = \frac{\mu_0 I_{A1}}{2\pi d}$$

- This leads to a force on the plasma

$$\mathbf{F} = \mathbf{J} \times \mathbf{B}$$



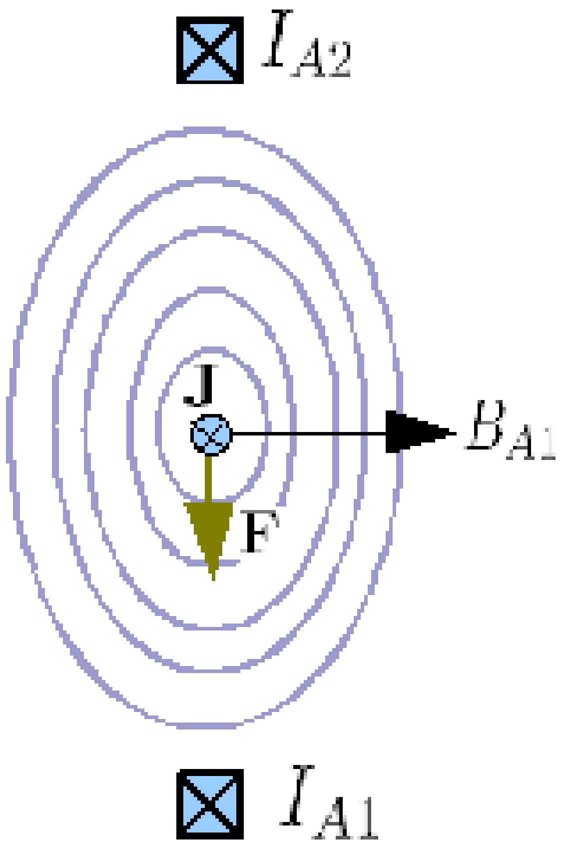
Vertical stability

- Integrating the force

$$\mathbf{F}_T = \int \mathrm{d}^3V\, \mathbf{J} \times \mathbf{B}$$
$$= -2\pi R \int \mathrm{d}^2A\, J B_{A1} \mathbf{e}_z$$
$$= -2\pi R \frac{\mu_0 I_{A1}}{2\pi d} I_p\, \mathbf{e}_z$$

- Thus

$$\mathbf{F}_{T1} = -\frac{\mu_0 R I_{A1} I_p}{d} \mathbf{e}_z$$
$$\mathbf{F}_{T2} = \frac{\mu_0 R I_{A2} I_p}{d} \mathbf{e}_z$$



Vertical stability

- Forces

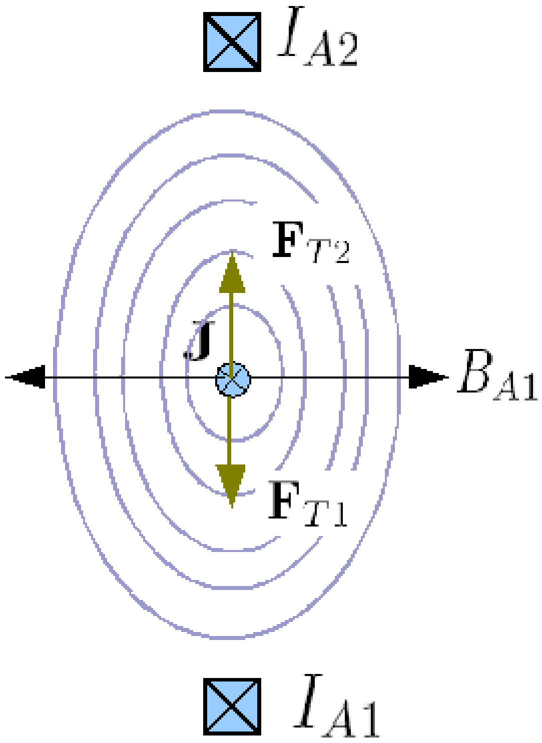
$$\mathbf{F}_{T1} = -\frac{\mu_0 R I_{A1} I_p}{d}\mathbf{e}_z$$

$$\mathbf{F}_{T2} = \frac{\mu_0 R I_{A2} I_p}{d}\mathbf{e}_z$$

- Equilibrium requires

$$I_{A1} = I_{A2} = I_A$$

- Such that the forces balance



Vertical stability

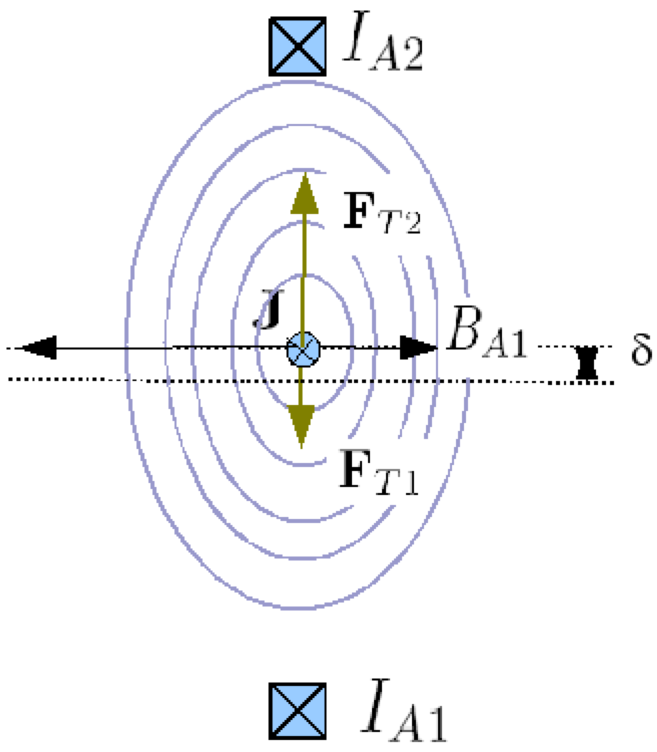
- The forces

$$\mathbf{F}_{T1} = -\frac{\mu_0 R I_{A1} I_p}{d}\mathbf{e}_z$$

$$\mathbf{F}_{T2} = \frac{\mu_0 R I_{A2} I_p}{d}\mathbf{e}_z$$

- Are in equilbrium when the coil currents are the same.
- But when the plasma is shifted upward by a small amount δ

$$\mathbf{F}_T = \mu_0 R I_A I_p \left[\frac{1}{d-\delta} - \frac{1}{d+\delta}\right]\mathbf{e}_z$$



Vertical instability

- Small shift δ << d

$$\mathbf{F}_T = \mu_0 R I_A I_p \left[\frac{1}{d-\delta} - \frac{1}{d+\delta} \right] \mathbf{e}_z$$

$$\frac{1}{d+x} = \frac{1}{d}\frac{1}{1+x/d} \approx \frac{1}{d}\left[1 - \frac{x}{d}\right]$$

$$F_T = \frac{2\mu_0 R I_A I_p}{d^2}\delta$$

- When total mass of the plasma is M

$$M\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = \frac{2\mu_0 R I_A I_p}{d^2}\delta \longleftarrow \delta = C\exp(\gamma t)$$

$$\gamma = \sqrt{\frac{2\mu_0 R I_A I_p}{M d^2}}$$

Growth rate of the vertical instability

Back to the picture

- Plasma vertical instability with growth rates of the order 10^6 s^{-1}
- For this reason the passive coils have been placed in the plasma
- When the plasma moves it changes the flux through the coils which generates a current that pushes the plasma back
- Growth rate is reduced to the decay time of the current in the coils (ms)