

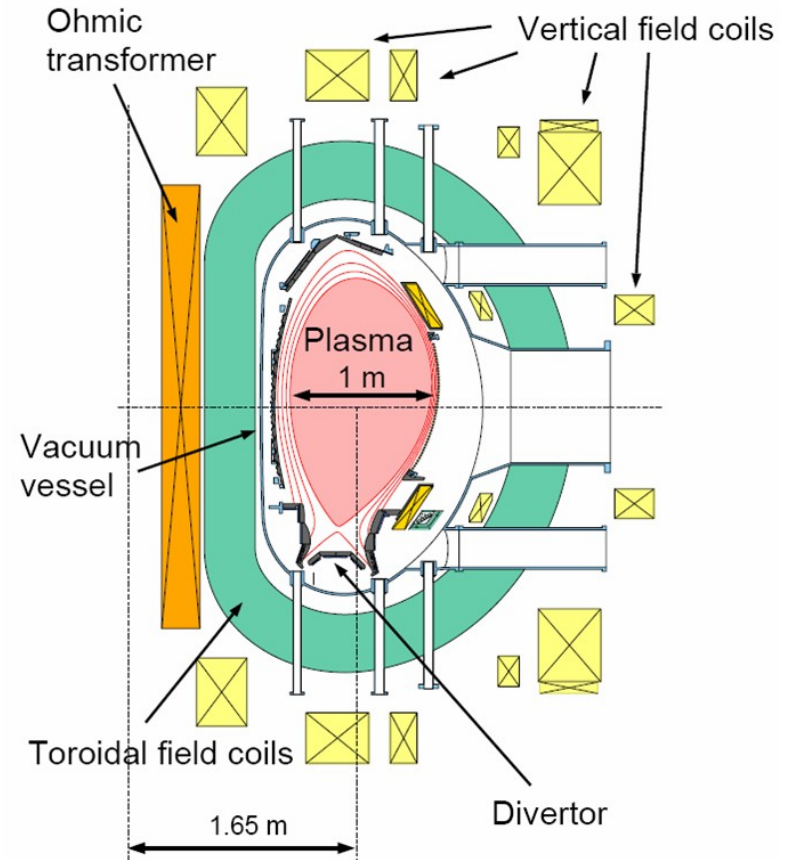


# Physics of fusion power

Lecture 9 : The tokamak continued

# Back to the picture

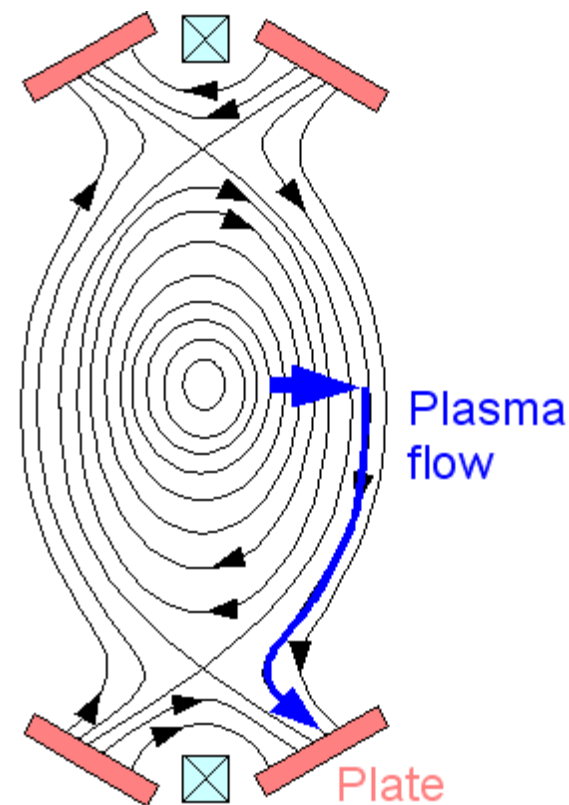
- This makes clear the amount of coils around the plasma
- The vertical coils can shape the plasma and control its position
- Note dominant shaping is the vertical elongation of the plasma



*Schematic Drawing of the poloidal cross section of the ASDEX Upgrade tokamak*

# Reason 1 For elongation: Divertor

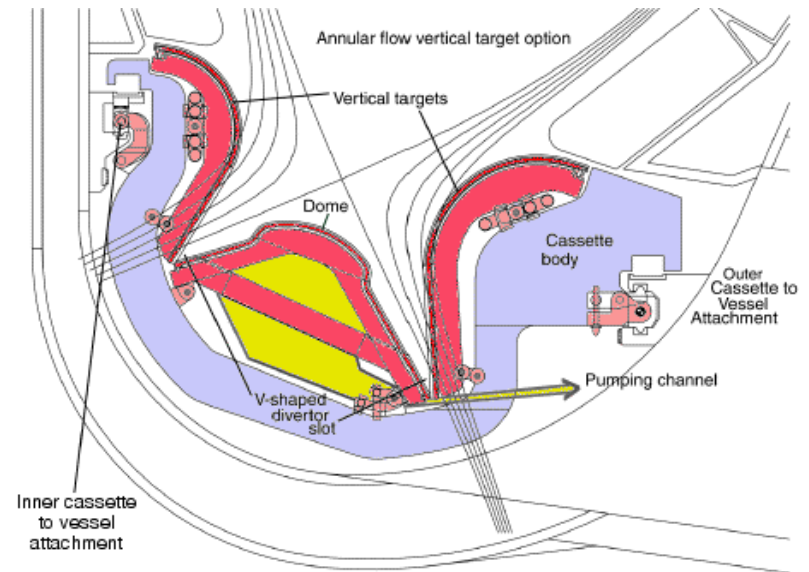
- Using a divertor the particles that leave the plasma flow along the magnetic field and hit the target plates
- These plates are far away from the plasma such that any impurity released at the plate has a smaller chance ending up in the plasma
- Furthermore, one can try to cool the plasma further through special arrangements in front of the plates



*Plasma flow in divertor configuration*

# The divertor

- A modern divertor design looks something like this
- Note that it has, as far as possible a closed structure. This to allow the efficient pumping of the neutral particles
- Note also that the angle between the magnetic field and the plate is as small as possible. This makes that the energy carried by the particles to the plate is distributed over the largest possible area



*Modern divertor design (ITER)*

# Reason II : Plasma elongation

- Distance to go around poloidally is larger

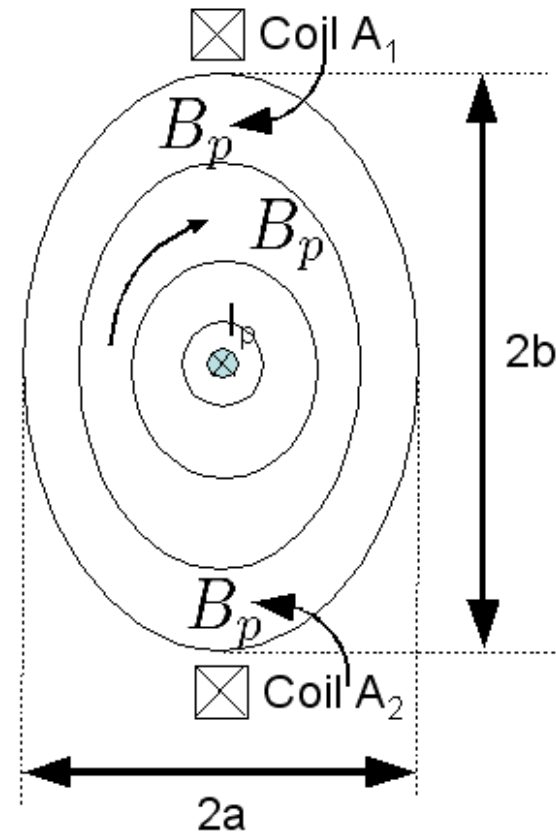
$$q = \frac{2\pi r^2 B_t}{\mu_0 R I} = \frac{2AB_t}{\mu_0 R I}$$

$$A = \pi ab = \pi a^2 \kappa \quad \kappa = \frac{b}{a}$$

For the same plasma current

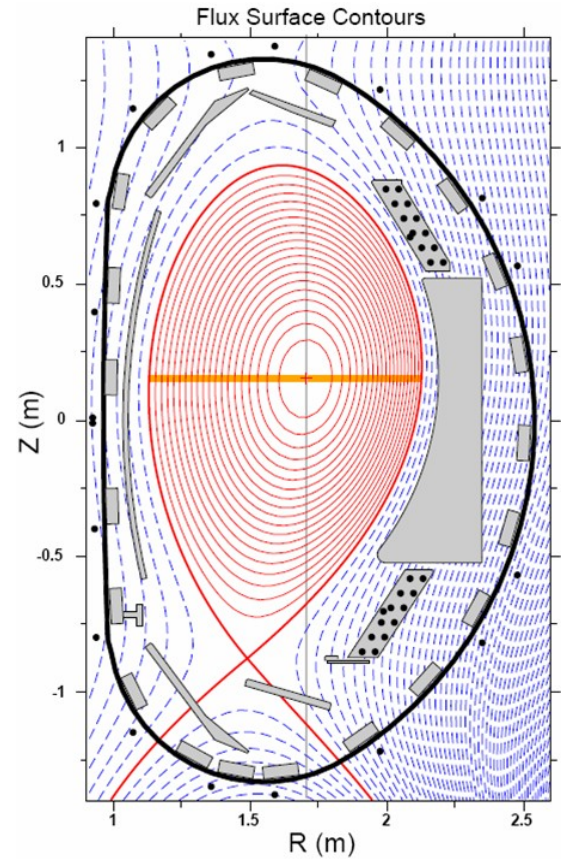
$$q_{\text{elip}} = q_{\text{circ}} \kappa$$

- If  $q = 3$  is the limit of operation one can run a larger current in an elliptically shaped plasma



# Why more current?

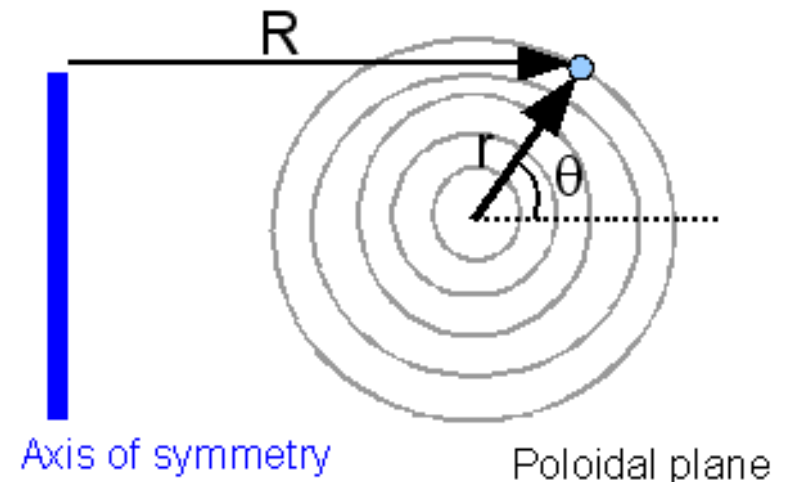
- One of the reasons is related to the equilibrium
- The surfaces of the tokamak are shifted outward



# Outward shift ???

- The surfaces of the tokamak are shifted outward
- This effect will be investigated starting from circular concentric surfaces, i.e. no outward shift
- The pressure is constant on a magnetic surface

$$p = p(r)$$



*Circular concentric magnetic surfaces*

# Outward force

- Force = pressure x area
- Considering the inboard and outboard side the areas are not the same

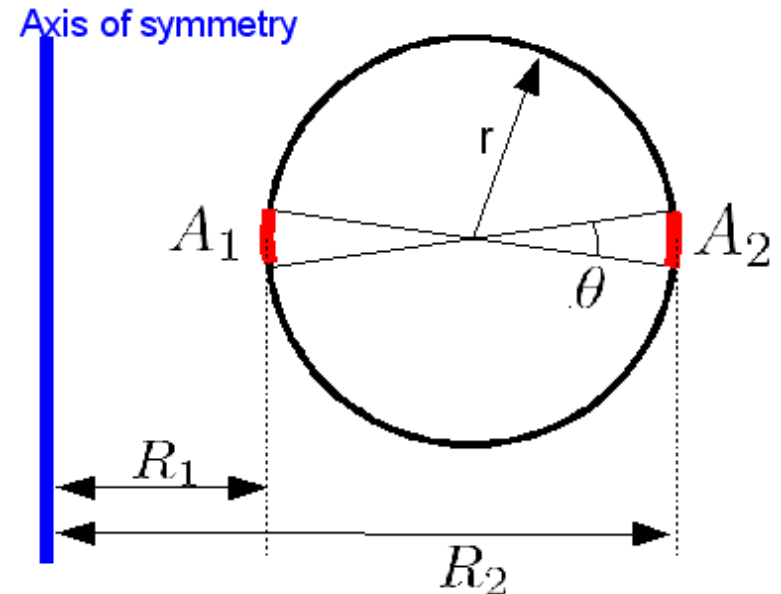
$$A_1 = 2\pi R_1 r d\theta$$

$$A_2 = 2\pi R_2 r d\theta$$

- This lead to an outward force

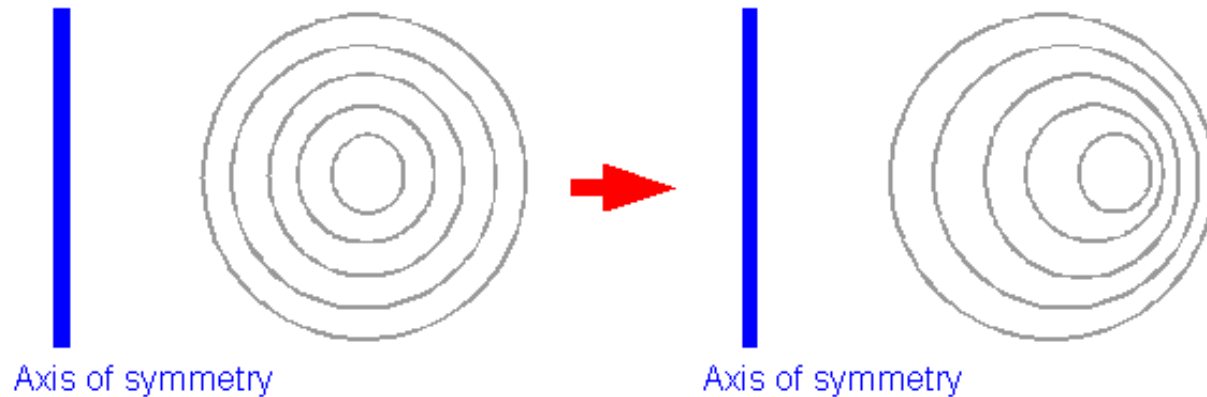
$$F_1 + F_2 = 2\pi(R_2 - R_1)p r d\theta$$

- Equivalent to inflating a tire





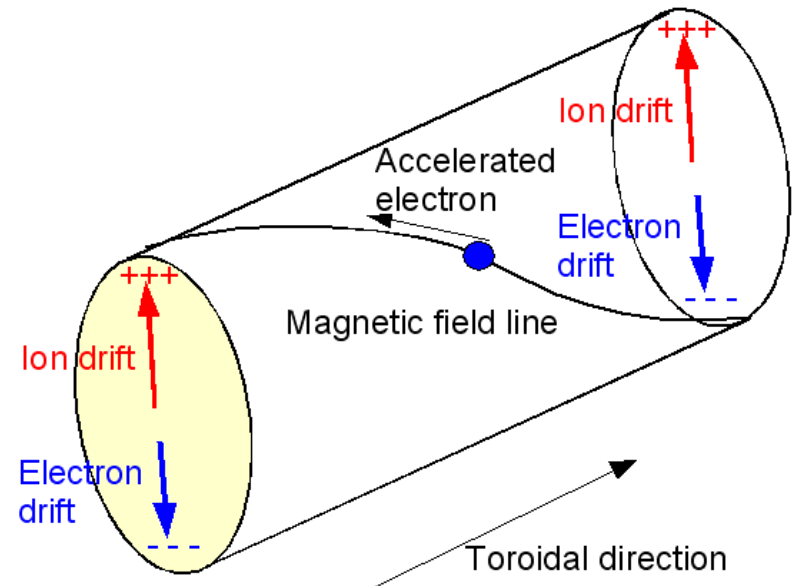
# The force leads to an outward shift



- The pressure force leads to an outward shift
- This shift compresses the surfaces on the outboard side
- The poloidal magnetic field increases (distance between surfaces is smaller)
- The increased tension and magnetic pressure can then balance the outward force

# Pfirsch Schluter current

- Of course a current is necessary to change the magnetic field
- This current is in fact the toroidal component of the parallel plasma flow that compensates for the drift



*Attempt at a 3D view. The toroidal plasma is drawn as a cylinder*

# After some mathematics

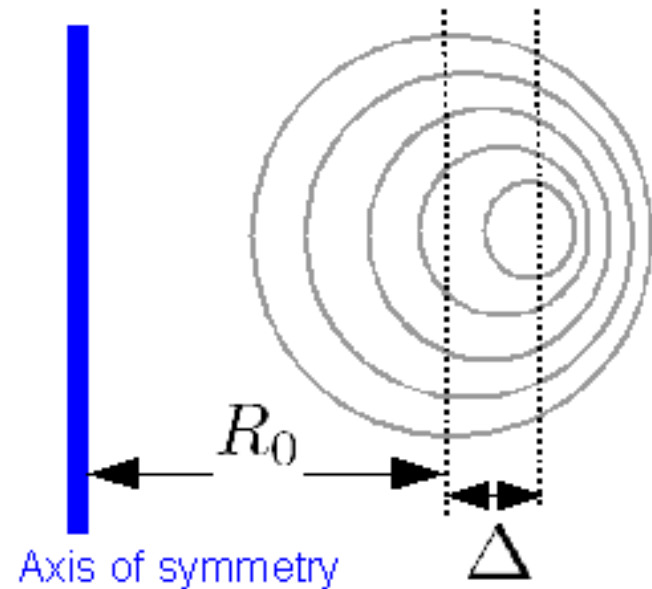
- The shift can be estimated to be roughly equal to

$$\begin{aligned}\frac{\Delta}{r} &= \frac{\langle p \rangle}{B_p^2 / 2\mu_0} \frac{r}{R_0} \\ &= \beta_p \frac{r}{R_0}\end{aligned}$$

- Where

$$\beta_p = \frac{\langle p \rangle}{B_p^2 / 2\mu_0}$$

- Is the plasma beta connected with the poloidal field



*Picture of the Shafranov shift*

# But the shift can be too large

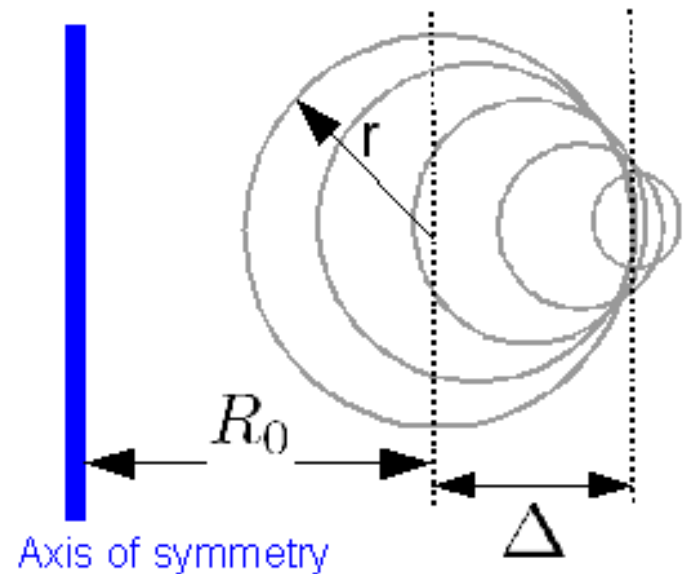
- The solution stops to give a physically reasonable equilibrium when the shift is equal to the plasma radius

$$\frac{\Delta}{r} = \beta_p \frac{r}{R_0} = 1$$

- From which it follows

$$\beta_p < \frac{R_0}{r}$$

- Loosely speaking it is only the poloidal field that confines the plasma



# Relation between the beta values

- The relation

$$\beta_p = \frac{\langle p \rangle}{B^2 / 2\mu_0} \frac{B^2}{B_p^2} \approx \frac{B_t^2}{B_p^2} \beta \longrightarrow \frac{B_t^2}{B_p^2} \beta = \beta_p < \frac{R_0}{r}$$

- Directly yields a maximum beta well below 1

$$\beta_{\max} = \frac{B_p^2}{B_t^2} \frac{R_0}{r} \approx 3\%$$

Ratio of the field strengths is roughly 0.1

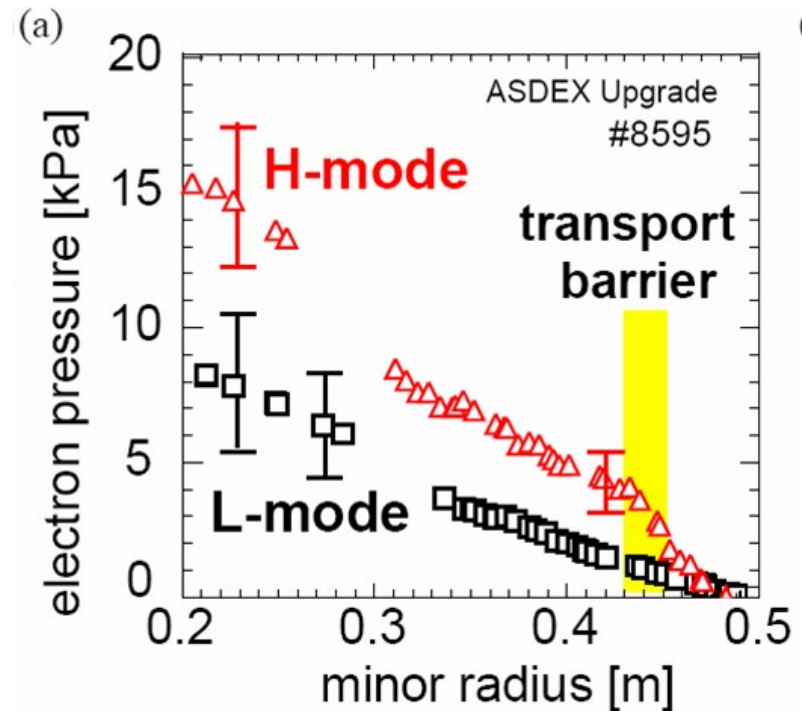
Typical value 3

# [ Reason 2 for elongation ]

- Elongation allows for a larger plasma current (kink stability limit)
- The larger current leads to a strong magnetic field on the outboard side of the plasma
- The larger poloidal field allows for a higher kinetic pressure

# Reason III : Plasma elongation

- A transition phenomenon is observed in Divertor plasmas known as the L (low) to H (high confinement) transition
- In this transition a steep pressure profile is generated at the plasma edge
- Not very well understood
- **Confinement improvement is roughly a factor 2 !!!!**







# Equilibrium / Vertical instability

- Magnetic field due to the coil follows form

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

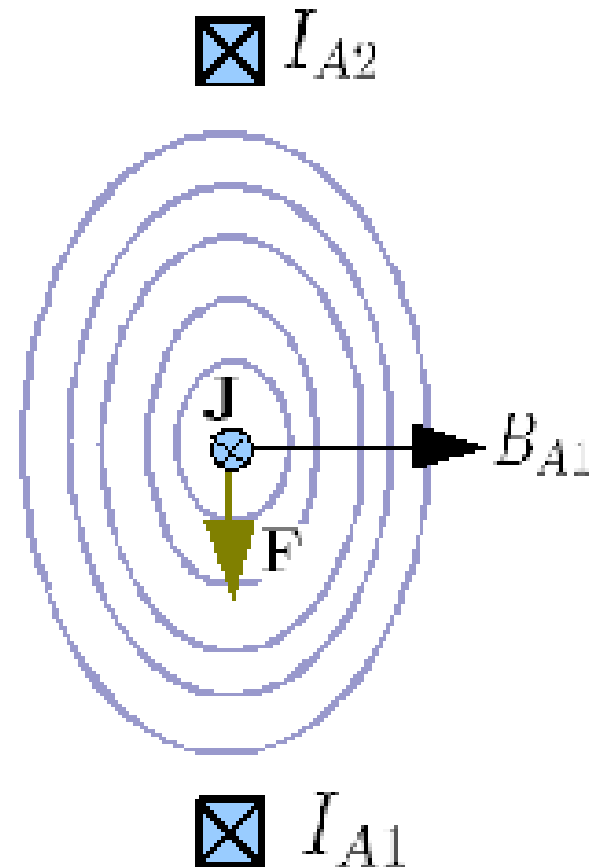
- Assume  $d \ll R$  one finds

$$2\pi d B_{A1} = \mu_0 I_{A1}$$

$$B_{A1} = \frac{\mu_0 I_{A1}}{2\pi d}$$

- This leads to a force on the plasma

$$\mathbf{F} = \mathbf{J} \times \mathbf{B}$$



# Vertical stability

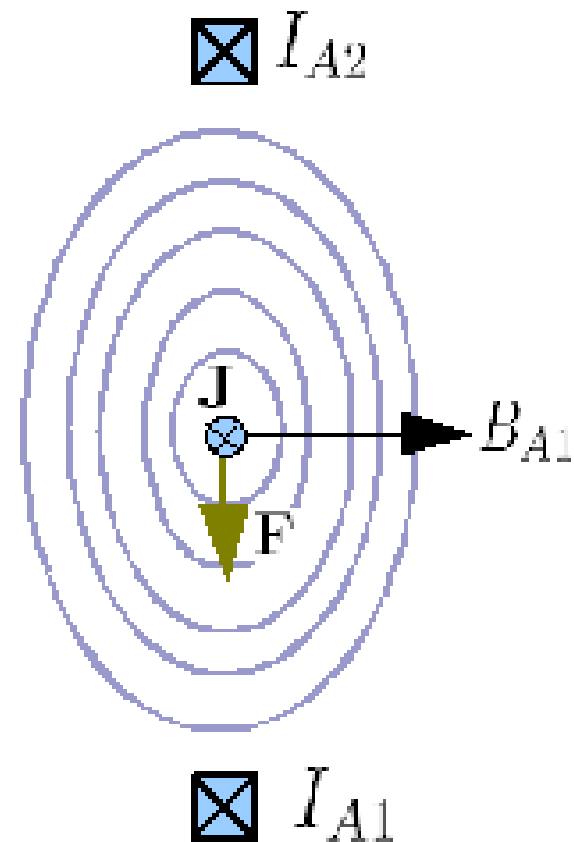
- Integrating the force

$$\begin{aligned}\mathbf{F}_T &= \int d^3V \mathbf{J} \times \mathbf{B} \\ &= -2\pi R \int d^2A J B_{A1} \mathbf{e}_z \\ &= -2\pi R \frac{\mu_0 I_{A1}}{2\pi d} I_p \mathbf{e}_z\end{aligned}$$

- Thus

$$\mathbf{F}_{T1} = -\frac{\mu_0 R I_{A1} I_p}{d} \mathbf{e}_z$$

$$\mathbf{F}_{T2} = \frac{\mu_0 R I_{A2} I_p}{d} \mathbf{e}_z$$



# Vertical stability

- Forces

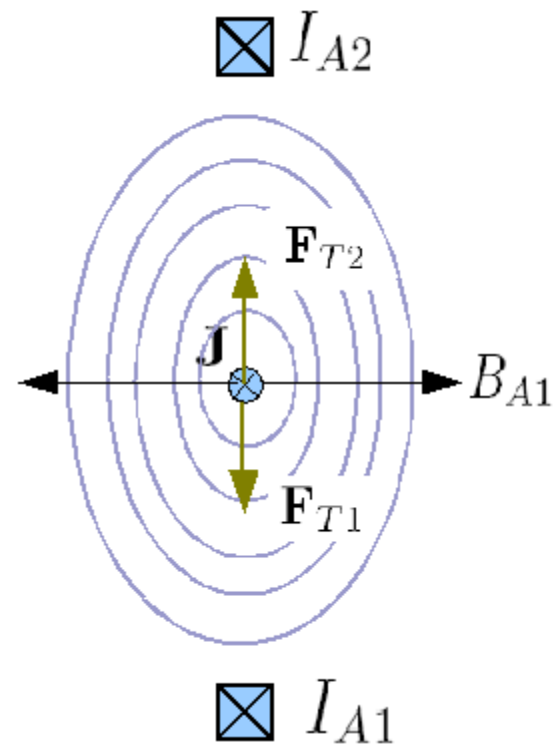
$$\mathbf{F}_{T1} = -\frac{\mu_0 R I_{A1} I_p}{d} \mathbf{e}_z$$

$$\mathbf{F}_{T2} = \frac{\mu_0 R I_{A2} I_p}{d} \mathbf{e}_z$$

- Equilibrium requires

$$I_{A1} = I_{A2} = I_A$$

- Such that the forces balance



# Vertical stability

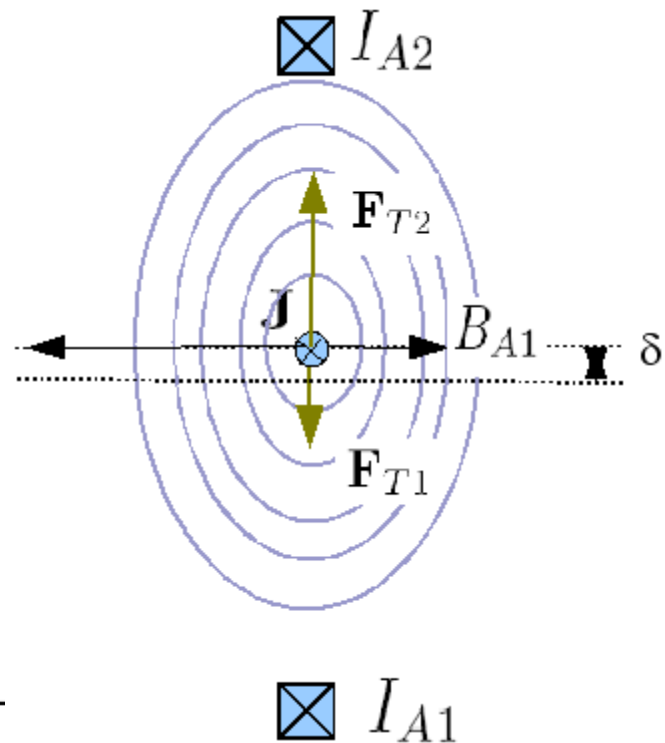
- The forces

$$\mathbf{F}_{T1} = -\frac{\mu_0 R I_{A1} I_p}{d} \mathbf{e}_z$$

$$\mathbf{F}_{T2} = \frac{\mu_0 R I_{A2} I_p}{d} \mathbf{e}_z$$

- Are in equilibrium when the coil currents are the same.
- But when the plasma is shifted upward by a small amount  $\delta$

$$\mathbf{F}_T = \mu_0 R I_A I_p \left[ \frac{1}{d - \delta} - \frac{1}{d + \delta} \right] \mathbf{e}_z$$



# Vertical instability

- Small shift  $\delta \ll d$

$$\mathbf{F}_T = \mu_0 R I_A I_p \left[ \frac{1}{d - \delta} - \frac{1}{d + \delta} \right] \mathbf{e}_z$$

$$F_T = \frac{2\mu_0 R I_A I_p}{d^2} \delta \left[ \frac{1}{d + x} = \frac{1}{d} \frac{1}{1 + x/d} \approx \frac{1}{d} \left[ 1 - \frac{x}{d} \right] \right]$$

- When total mass of the plasma is M

$$M \frac{d^2 \delta}{dt^2} = \frac{2\mu_0 R I_A I_p}{d^2} \delta \quad \delta = C \exp(\gamma t)$$

$$\gamma = \sqrt{\frac{2\mu_0 R I_A I_p}{M d^2}}$$

Growth rate of the vertical instability

# Back to the picture

- Plasma vertical instability with growth rates of the order  $10^6 \text{ s}^{-1}$
- For this reason the passive coils have been placed in the plasma
- When the plasma moves it changes the flux through the coils which generates a current that pushes the plasma back
- Growth rate is reduced to the decay time of the current in the coils (ms)

