Using mutual information to quantify spatial correlation between simultaneous spacecraft measurements of solar wind plasma turbulence

R.T.Wicks* **S.C.Chapman*** **R.O.Dendy*†**

* Centre for Fusion, Space and Astrophysics, Physics Department, University of Warwick, Coventry, CV4 7AL
† UKAEA Culham Division, Culham Science Centre, Abingdon, OX14 3DB

Abstract

Here we analyse data obtained during periods from 1998 onwards when the WIND and ACE spacecraft were simultaneously in the upstream solar wind, and explore a range of spatial scales sufficient to determine correlation properties (see, for example, [1]). Nonlinear correlation is quantified by calculating the mutual information between measurements of magnetic field $\boldsymbol{B}$, flow velocity $\boldsymbol{v}$, and density ρ from spatially separated spacecraft. Studies of spatiotemporal correlation between coupled nonlinear signals in the solar wind and magnetosphere-ionosphere [2],[3] have shown that the mutual information [4], and associated recurrence plot techniques, can be helpful in this context. This enables us to compare the relative degree of correlation between different solar wind bulk parameters, and also derived quantities such as Elsässer variables. The ordering of mutual information with respect to spacecraft separation and signal propagation relative to the background magnetic field direction is discussed in relation to current models and understanding of anisotropic solar wind plasma turbulence.

Introduction to Mutual Information

- Mutual Information (MI) is a quantitative measure of the amount of Shannon [4] entropy, $H(A)$, shared between two timeseries.

$$H(A) = -\sum_{a_i \in A} P(a_i)\log_2\left(P(a_i)\right)$$

- Normalised MI is defined as [5]:

$$NMI(A;B) = \frac{H(A)+H(B)}{H(A;B)} - 1$$

- NMI takes a value of 1 for identical, random signals, and a value of 0 for uncorrelated signals.
- We use a histogram method to calculate the probabilities $P(a_i)$, $P(b_i)$ and $P(a_i;b_i)$.
- Histogram bins are of width ½ σ, the standard deviation of the data, data outside ±5σ is omitted.

$$b_z = B_z - \langle B_z \rangle_{24\,hours} \qquad V_z = v_z - \langle v_z \rangle_{24\,hours}$$

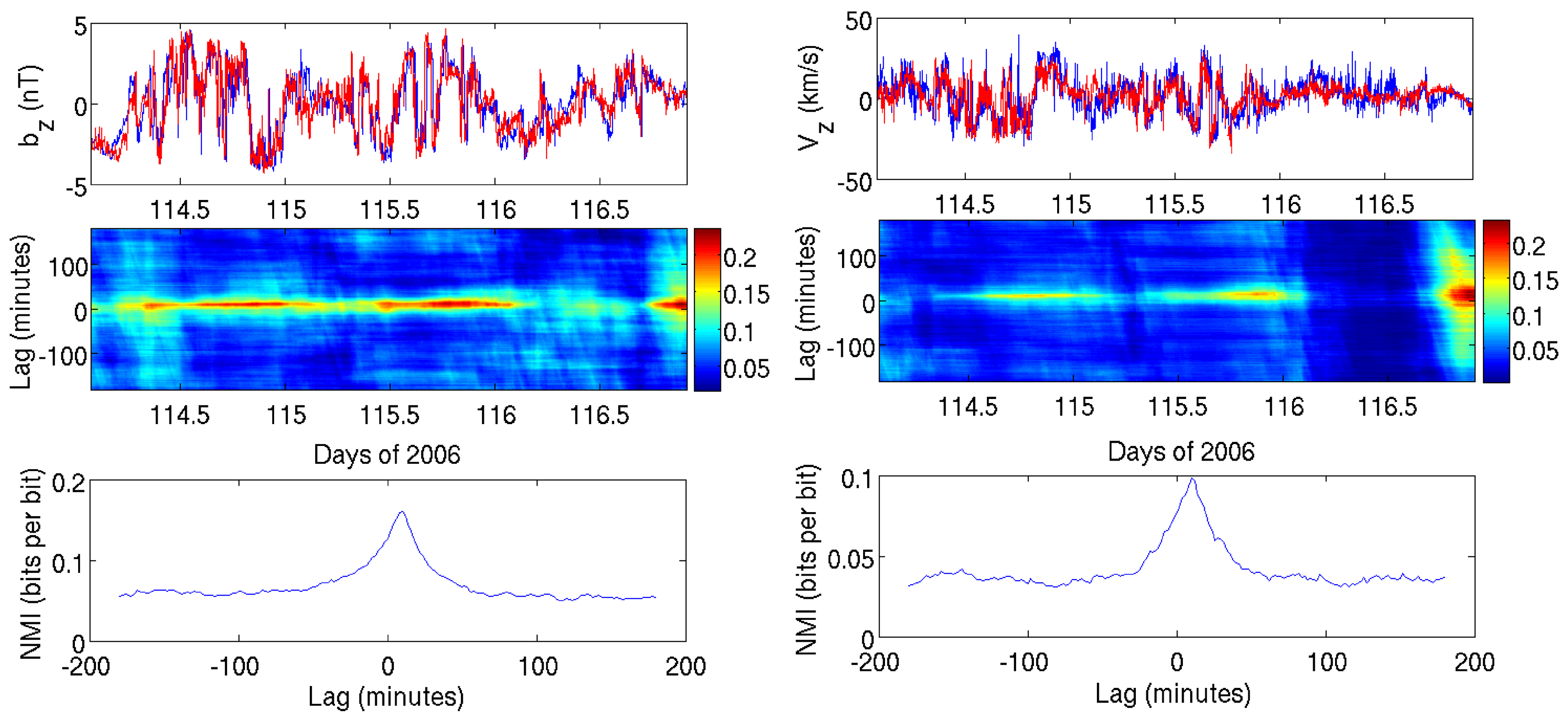


FIG 1: Example of the use of NMI on solar wind data. The top two panels show magnetic field data and velocity data averaged over 2 minute intervals, taken from the ACE and WIND spacecraft during 2006, a 24 hour average is subtracted to remove the largest scale variations. The middle panels show the result of NMI calculations on periods of 480 points at different time lags. The bottom panels show the average NMI with different time lags over this whole time period.

References:

[1] W. H. Matthaeus *et al* PRL **95** 231101 (2005)
[2] T. K. March *et al*., *Geophys. Res. Lett.* **32**, L04101 (2005)
[3] T. K. March *et al*.,Physica D **200**, 171 (2005)
[4] C. E. Shannon, Bell System Tech. Journal **27** 379-423 (1948)
[5] C. Studholme *et al* Pattern Recognition **32** 71-86 (1999)
[6] K. T. Osman *et al* Astrophys. J. **654** 103-106 (2007)

Solar Wind Data

- Data is taken from periods of lowest solar activity (1998, 2005, 2006).
- 24 hour average is subtracted from data to remove the longest timescale changes caused by spacecraft position, solar rotation etc.
- Magnetic field **B**, ion density ρ, average velocity **v** and Elsässer variables $\mathbf{Z}^{\pm}$ are compared between ACE and WIND using NMI and linear correlation.

$$Z^{\pm} = v \pm \frac{B}{\sqrt{4\pi\rho}}$$

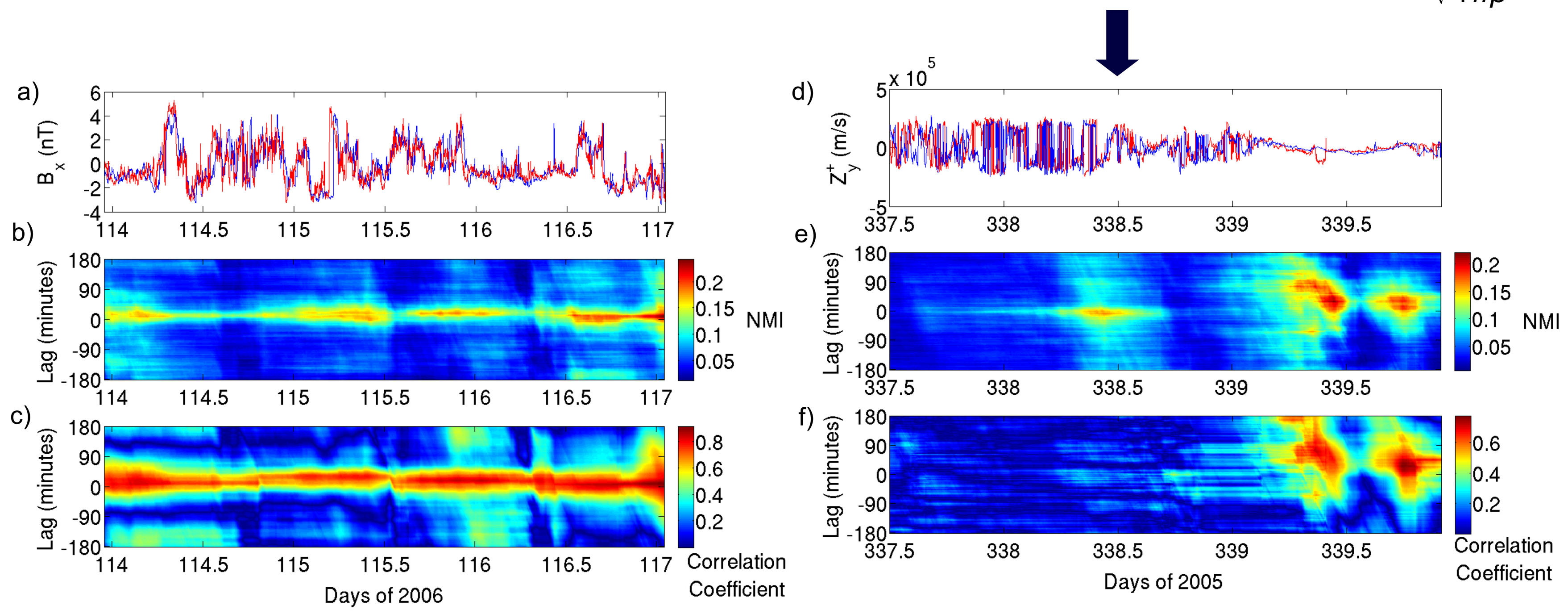


FIG 2: a) X component of magnetic field (GSE coordinates) measured by ACE and WIND for 3 days in 2006. b) NMI measured between these two signals for periods of 960 minutes with a time lag between -180 and 180 minutes. c) Correlation coefficient for the same data. d) Y components of Elsässer variable $\mathbf{Z}^{+}$ (GSE coordinates) calculated for a period in 2005. e) NMI calculated between these variables using the same method as above. f) Correlation coefficient between these data.

- NMI measurements of the same variable, for example B_x, show similar results for NMI and correlation coefficient.
- NMI measurements between Elsässer variables and ion density have regions where NMI and linear correlation give significantly different results, as indicated by the blue arrow in FIG 2.
- NMI and linear correlation are also calculated between different variables measured by a single spacecraft, in this case WIND.
- The highest correlation between magnetic field and velocity components occurs when there is little correlation between the magnetic field magnitude and the density, as shown by the blue arrows in FIG 3. This is typical of all the data analysed from solar minimum so far.

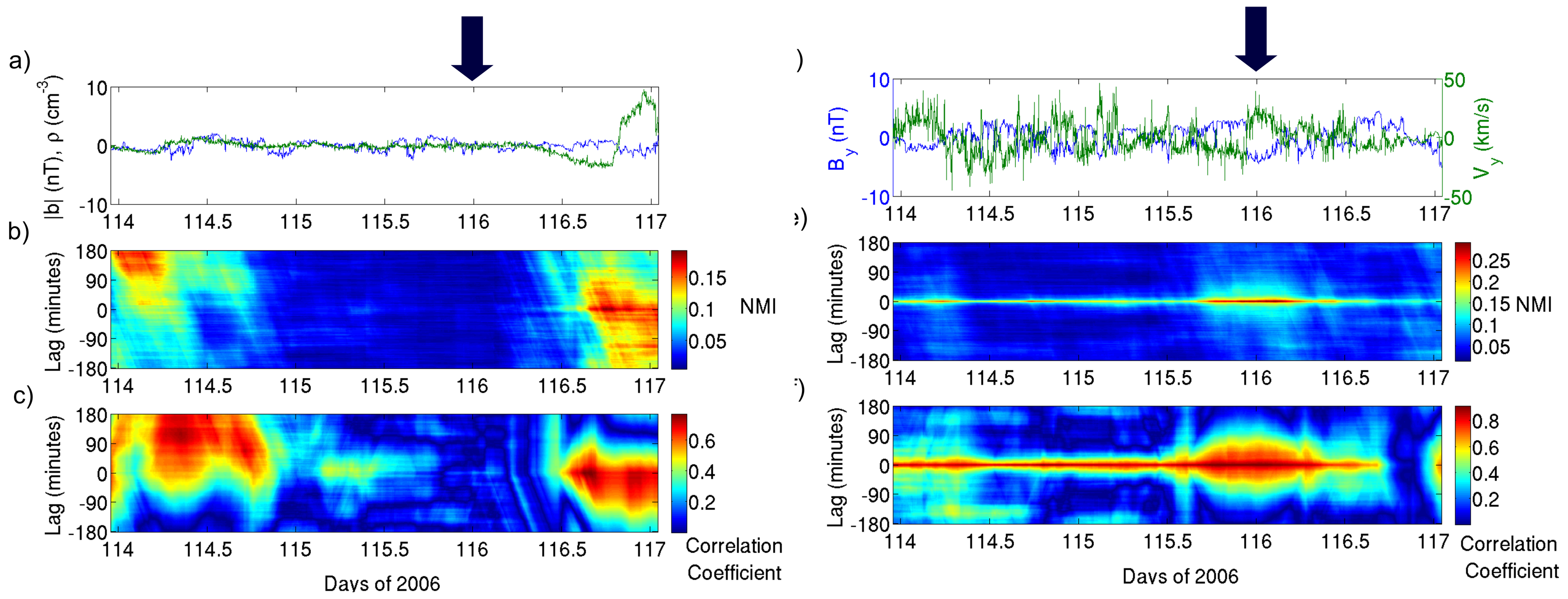
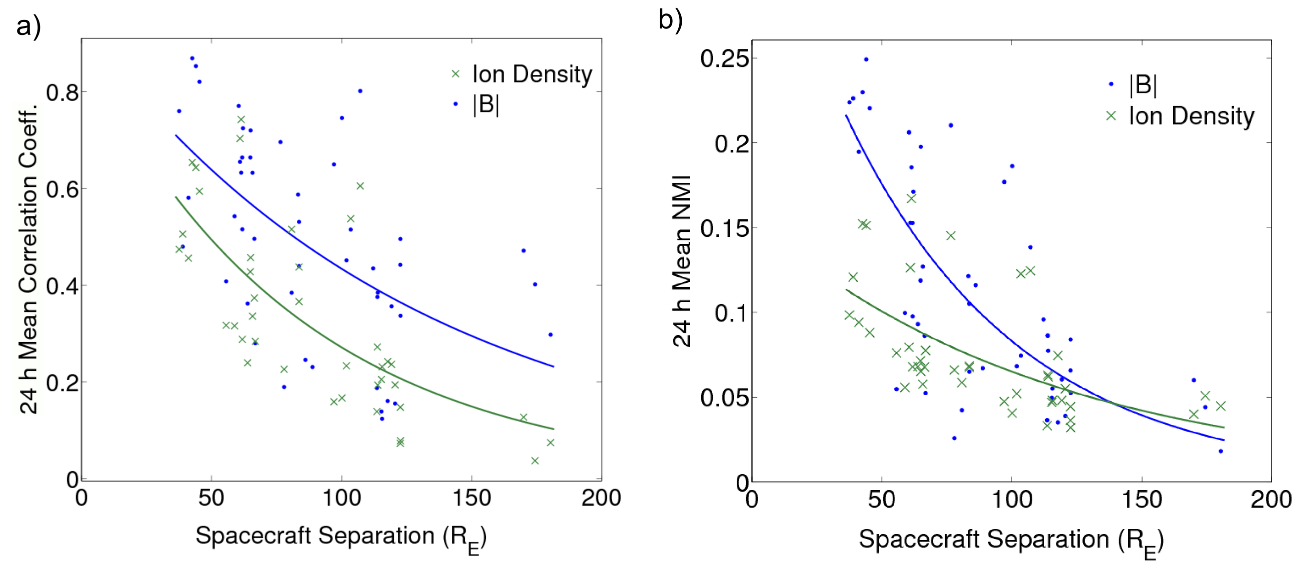


FIG 3: a) Magnetic field magnitude (blue) and ion density (green) as measured by WIND. b) NMI calculated between these data as in FIG 2. c) Correlation coefficient for the same data. d) Magnetic field component (blue) and solar wind velocity (green) in the y direction (GSE coordinates). Calculated NMI e) and correlation coefficient f).

Spacecraft Separation

- By making measurements of NMI and correlation coefficient at different times the decrease in these values can be plotted with distance and field alignment.
- Figure 4 shows the decrease of both correlation coefficient and NMI with distance for ion density and |B|.
- In both plots ion density correlation decreases faster than the magnetic field.



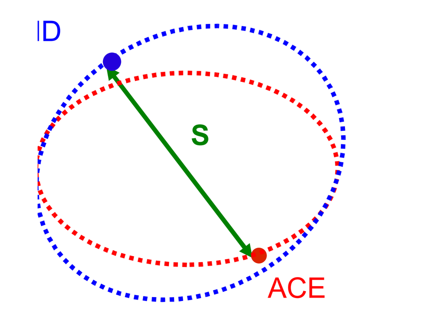


FIG 4: a) Correlation coefficient averaged over 24 hours of data as measured for |B| (blue) and ion density (green), plotted against the average ACE-WIND separation |S|. Lines are exponentials fitted to this data. b) NMI calculated for the same data with exponential fits.

- The lines in FIG 4 a) are exponential fits of the form A exp (-x/λ). These fits give values of λ = 130 R_E for |B| and λ = 80 R_E for ion density.
- NMI fitting gives λ = 115 R_E for |B| and λ = 70 R_E for ion density.
- By splitting separation, **S**, into field parallel, $r_{\parallel}$ and perpendicular $r_{\perp}$, directionally resolved measurements of magnetic field NMI can be made.
- There is an indication of higher correlation in the $r_{\parallel}$ direction at lower separation.
- In particular the y and z components show anisotropy.

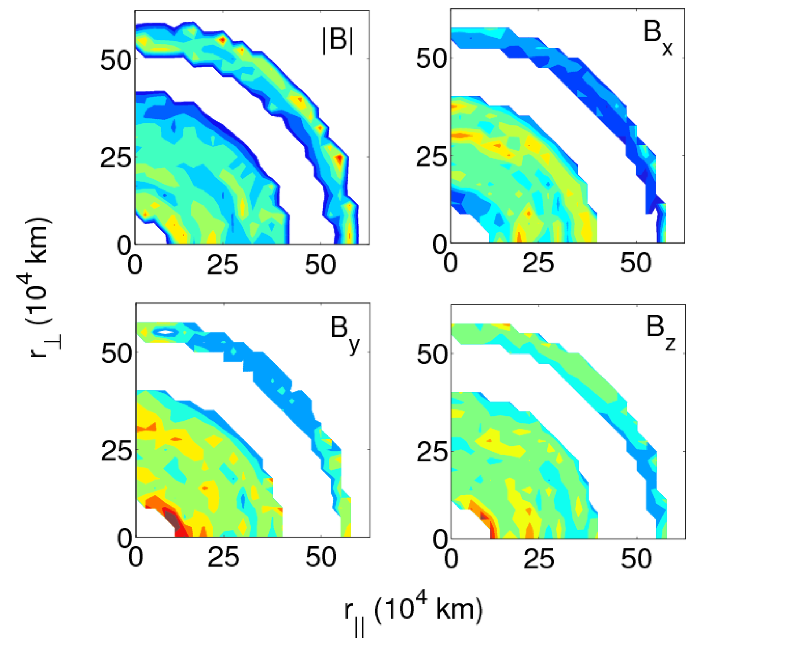


FIG 5: Directionally resolved NMI measurements of the magnetic field show signs of anisotropy.

Conclusions

- Single spacecraft measurements have been used to calculate the correlation between magnetic field and velocity components, and field magnitude and ion density. **These results indicate a separation of behaviours, Alfvénic wave behaviour being strongest away from more pressure dominated density fluctuations.**
- The variation of these parameters with spacecraft separation has been measured and a clear **difference** between the **magnetic field** and **density** correlation has been shown. The measured magnetic field correlation scale length λ is slightly **smaller** than recent measurements [1], however the cause may be that all data used in this analysis were taken from periods of **low solar activity**.
- We expect that further analysis of this data will provide insight into why the nonlinear spatial correlation properties of the density and the magnetic field do not decline with distance at the same rate. **This bears on the fundamental question of the extent to which structures in the solar wind plasma are of solar origin, or are really turbulent.**
- If structures of solar origin increase the correlation length, this should vary with the solar cycle. This is important for future studies, as it is difficult to find long time periods in the ecliptic which do not contain such structures.

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