Quantifying spatial correlation in the turbulent solar wind flow using mutual information: simultaneous in-situ spacecraft observations from WIND and ACE.

R. T. Wicks, S. C. Chapman, R. O. Dendy

Center for Fusion, Space and Astrophysics
University of Warwick
Coventry
UK
r.wicks@warwick.ac.uk

Thanks to the WIND and ACE magnetometer and SWE teams

Introduction

Developed by Shannon in 1949

Mutual information (MI) is a measure of shared entropy between two signals (X and Y)

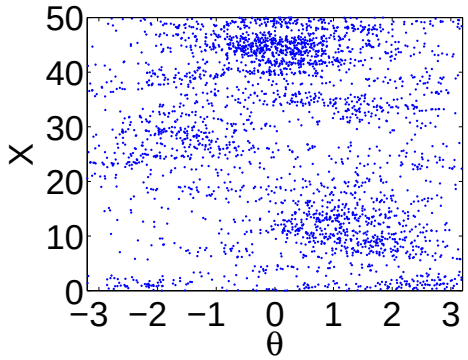
$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

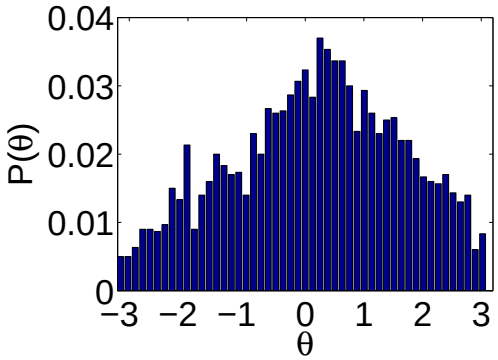
$$H(X) = -\sum_i P(X_i) log_2(P(X_i))$$

Histograms

Probabilities $P(X)$, $P(\Theta)$ and $P(X, \Theta)$ estimated by frequency of occurrence

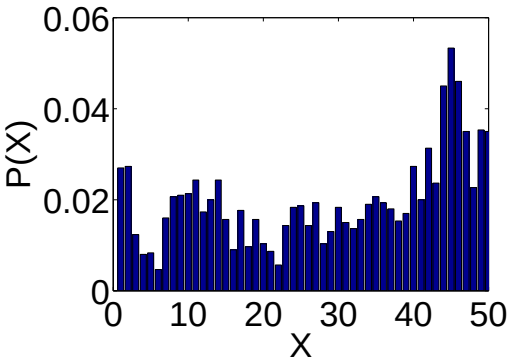
Raw data

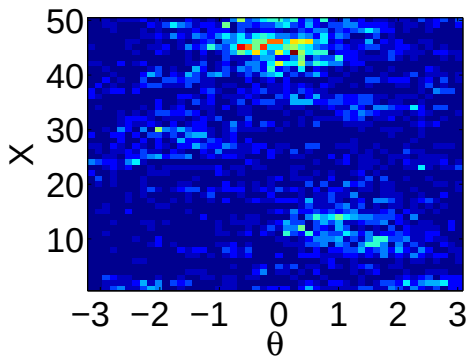




Θ value histogram

X position histogram





Joint probability distribution $P(X, \Theta)$

Vicsek Model Rules

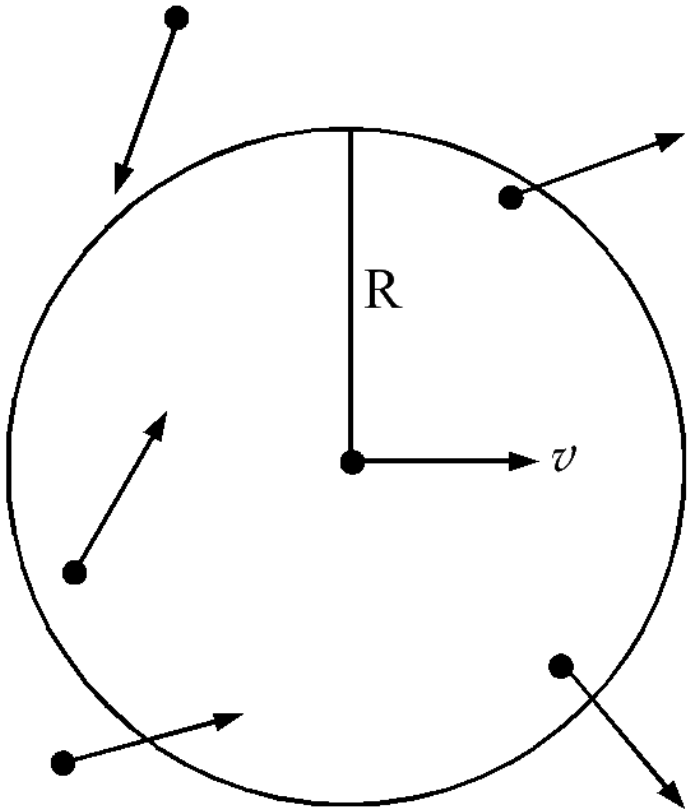
A model for co-orienting self-propelled particles

Particles align within radius R

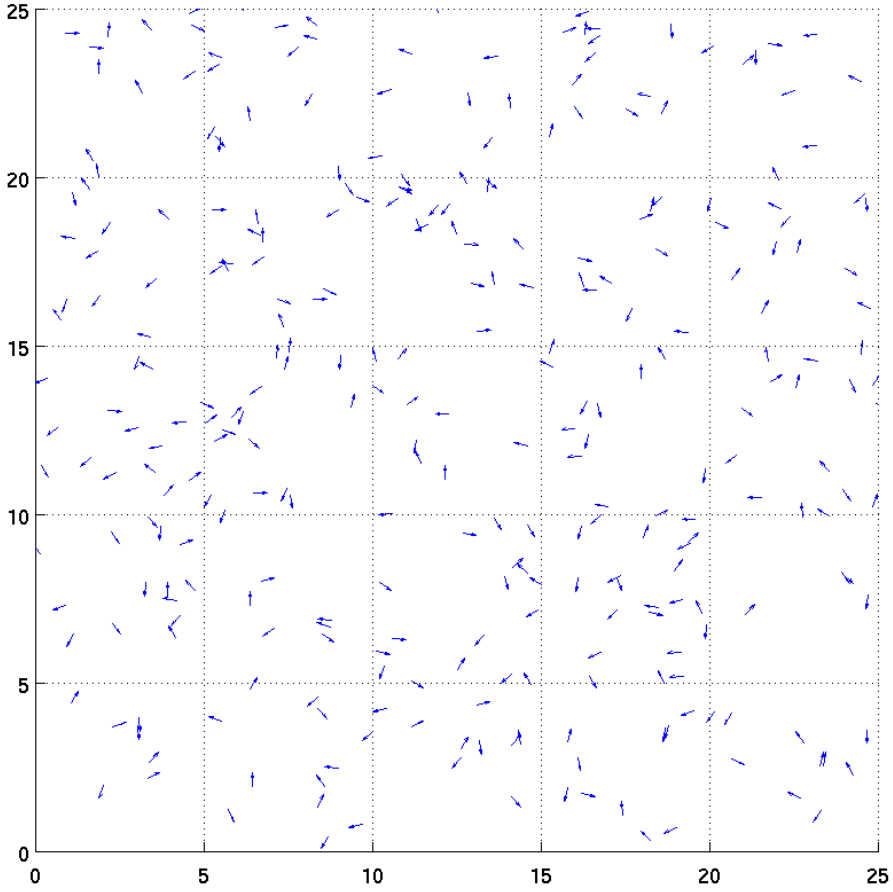
$$\begin{aligned} \underline{x}^i_{n+1} &= \underline{x}^i_n + \underline{v}^i_n \, \delta t \\ \underline{v}^i_n &= v_0 \left(\cos\theta^i_n \underline{\hat{x}} + \sin\theta^i_n \underline{\hat{y}} \right) \\ \theta^i_{n+1} &= \langle \theta^{N_R}_n \rangle + \delta\theta^i_n \end{aligned}$$

$$\delta\theta^i_n \in [-\eta, \eta]$$

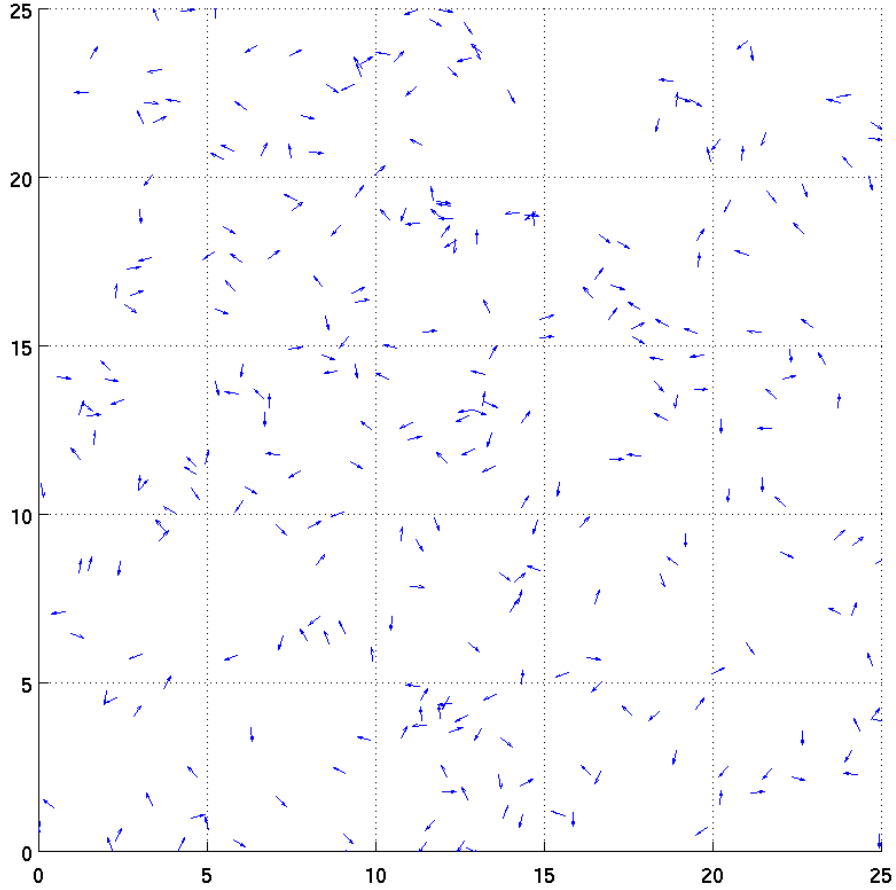
Vicsek et al 1995.



Behaviour



Low noise $\longrightarrow$ ordered motion



High noise $\longrightarrow$ disorder

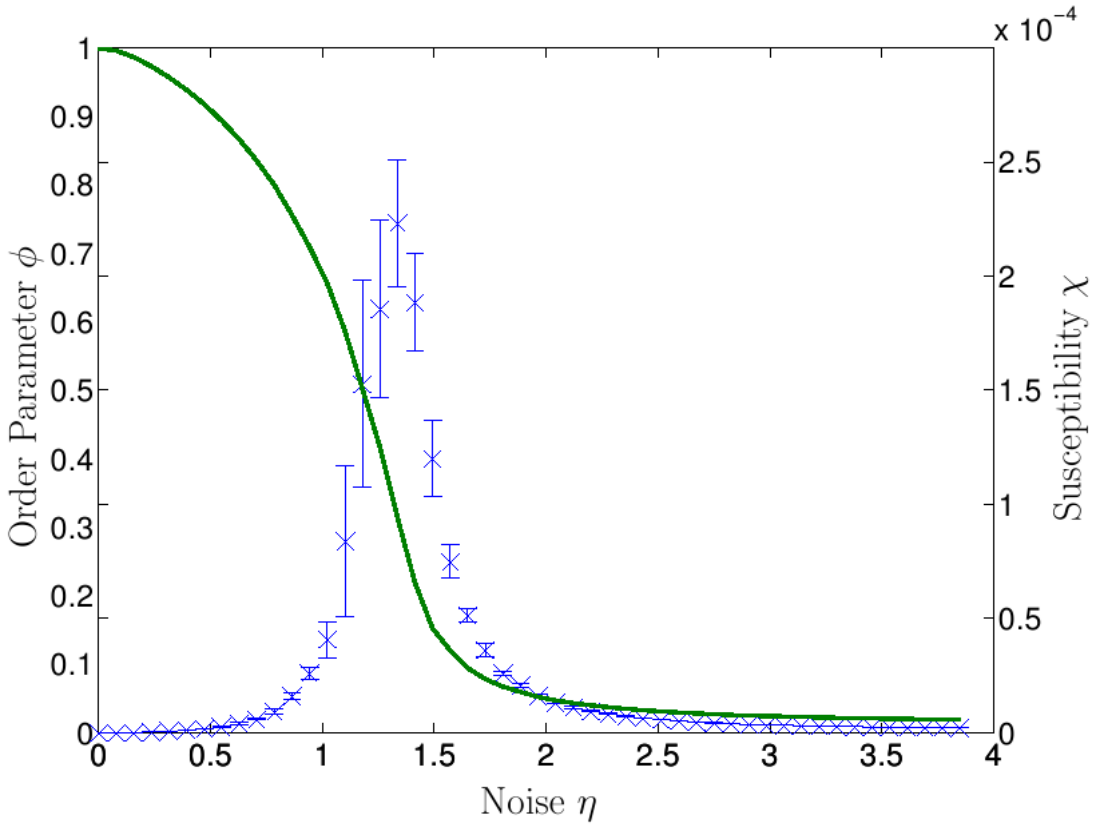
Order Parameters

Order parameter:

$$\phi = \frac{1}{Nv_0}\left|\sum_{i=1}^{N}\underline{v}^i\right|$$

Susceptibility:

$$\chi = \frac{1}{N}\left(\langle\phi^2\rangle - \langle\phi\rangle^2\right)$$



Wicks et al PRE 2007 (in press)

Vicsek Model MI

MI is calculated between position (X, Y) and angle of motion Θ using a histogram method

$$I(X,\Theta) = \sum_{i,j} P(X_i,\Theta_j)\log_2\left(\frac{P(X_i,\Theta_j)}{P(X_i)P(\Theta_j)}\right)$$

$$I(Y,\Theta) = \sum_{i,j} P(Y_i,\Theta_j)\log_2\left(\frac{P(Y_i,\Theta_j)}{P(Y_i)P(\Theta_j)}\right)$$

$$I = \frac{I(X,\Theta)+I(Y,\Theta)}{2}$$

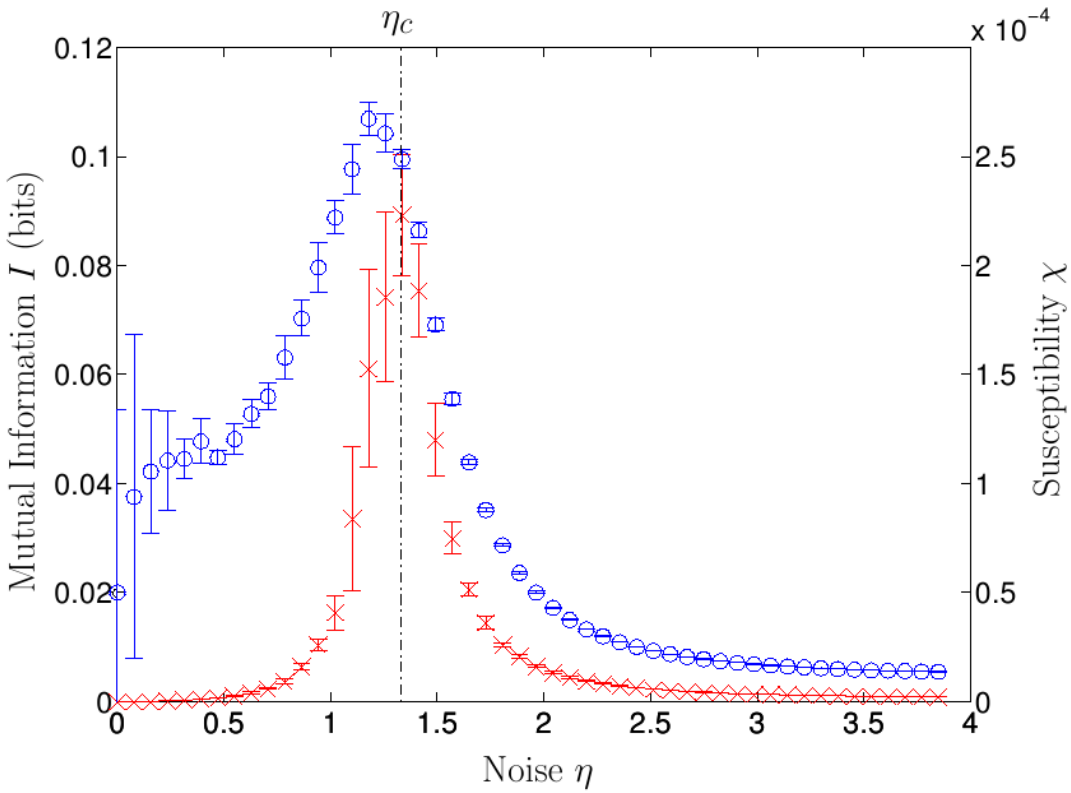
Vicsek Model MI

MI and χ are calculated on snapshots of the system

η_c defined by peak χ

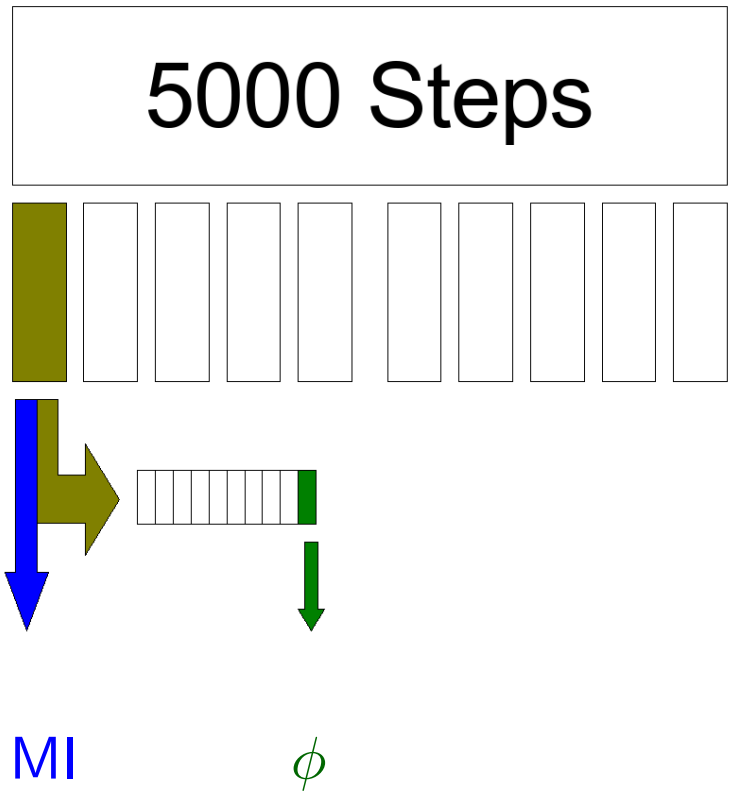
Error on MI much smaller than on χ around peak

Matsuda et al Int. J. Theor. Phys. 1996



Limited Data

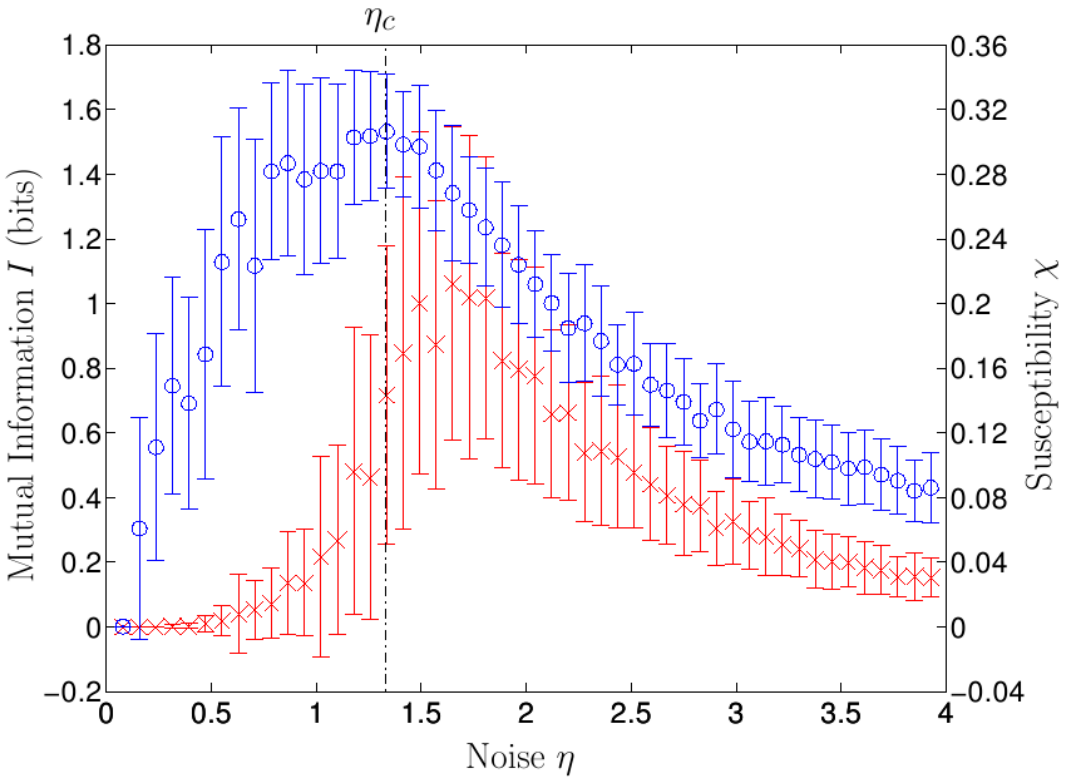
Real world measurements often measure local or reduced dimensional data from a many degree of freedom system



- Take timeseries data from only 10 particles from 3000
- Divide into 10 smaller sections
- Calculate MI

$$\chi = \frac{1}{N}\left(\langle\phi^2\rangle - \langle\phi\rangle^2\right)$$

Limited Data



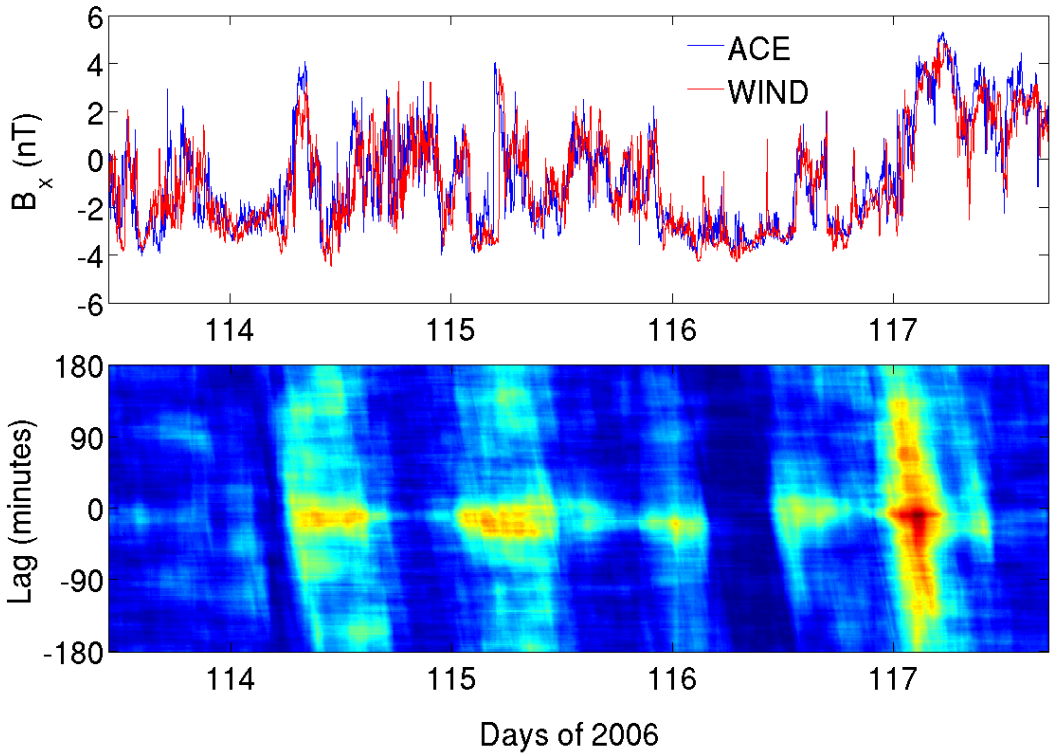
Error on χ is very large

Peak approximately correct for MI

χ cannot identify η_c

Wicks et al PRE 2007 (in press)

Solar Wind MI

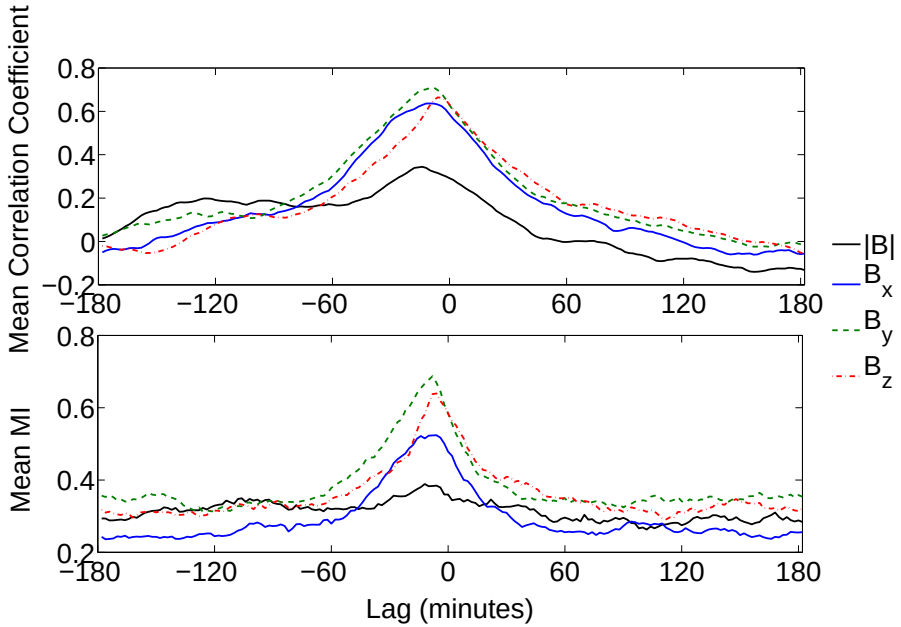


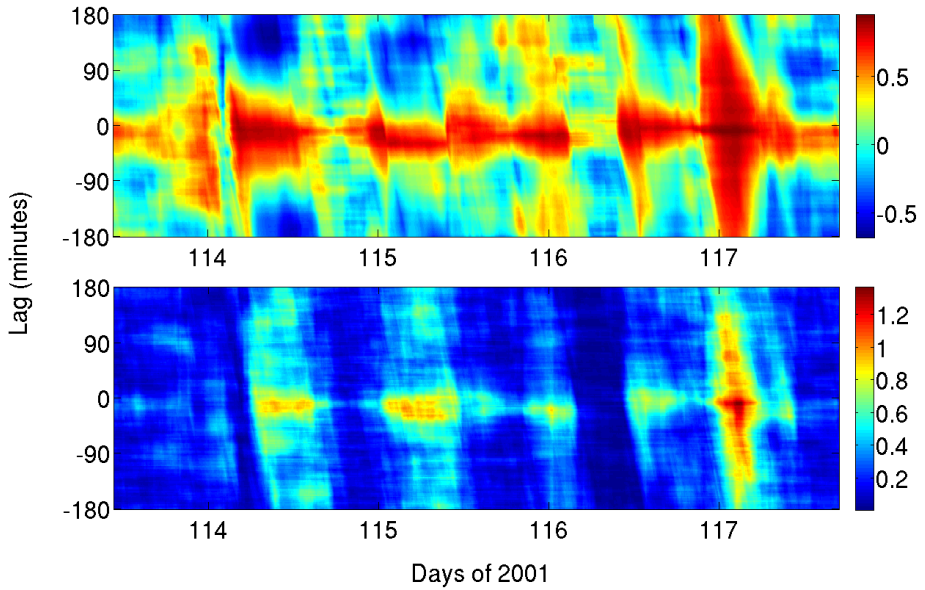
ACE and WIND data

MI calculated with lag
Min MI 0.0001
Max MI 1.42

MI gives indication of typical correlation length within solar wind

Correlation vs. MI





MI and correlation agree on time lag and ordering of the data.

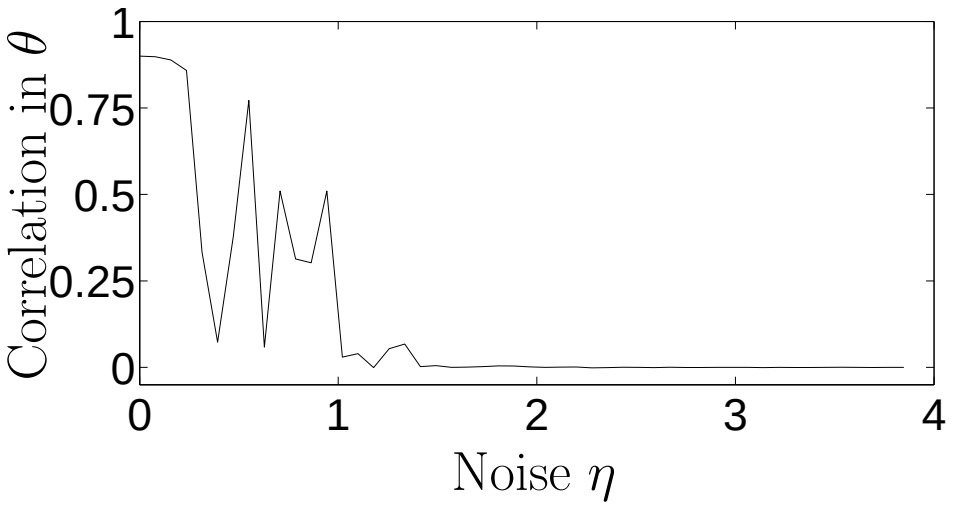
MI peaks sharper than correlation.

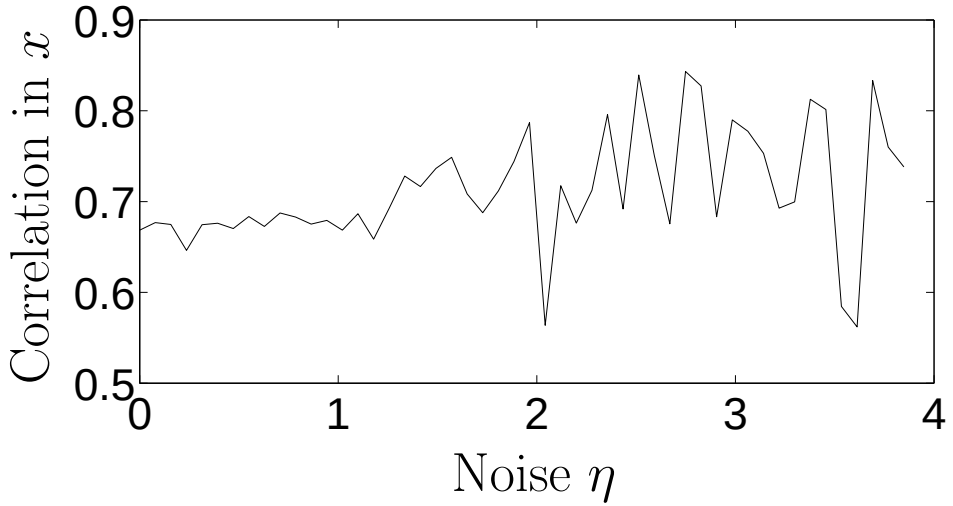
Linear Correlation identifies the convecting structures. Distinct signature in MI.

- We used a simple model with dynamics and a phase transition to compare susceptibility and MI.
- Mutual information is able to identify the phase transition accurately.
- When knowledge of the system is limited, MI performs better than susceptibility at identifying the phase transition.
- MI demonstrated on 'real world' dataset of multiple spacecraft measurements of the solar wind.
- Both linear correlation and MI detect convecting structures.
- Result from MI and linear correlation are distinct. Reflects presence of both linearly convecting structures and evolving turbulence in the flow.

Wicks et al PRE 2007 (in press)

Linear Correlation





Linear correlation of Vicsek model

No indication of phase transition