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A sandpile model with dual scaling regimes for laboratory, space and astrophysical plasmas

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There is increasing evidence that the Earth's magnetosphere, like other macroscopic confined plasma systems (magnetic fusion plasmas, astrophysical accretion discs), displays sandpile-type phenomenology so that energy dissipation is by means of avalanches which do not have an intrinsic scale. This may in turn imply that these systems evolve via self-organized criticality (SOC). For example, the power law dependence of the power spectrum of auroral indices, and *in situ* magnetic field observations in the Earth's geotail, indicate that the coupled solar wind-magnetospheric system can to some extent be described by an avalanche model. However, substorm statistics exhibit probability distributions with characteristic scales. In this paper a simple sandpile model is discussed which yields for energy discharges due to internal reorganization a probability distribution that is a power law, implying SOC, whereas systemwide discharges (flow of ''sand'' out of the system) form a distinct group whose probability distribution has a well defined mean. When the model is analyzed over its full dynamic range, two regimes having different inverse power law statistics emerge. These correspond to reconfigurations on two distinct length scales: short length scales sensitive to the discrete nature of the sandpile model, and long length scales up to the system size which correspond to the continuous limit of the model. These are anticipated to correspond regimes accessible to both laboratory and astrophysical plasmas. The relevance of the emergence of distinct self-organized confinement regimes in space, astrophysical, and magnetic fusion plasmas is discussed. Since the energy inflow may be highly variable, the response of the sandpile model is examined under strong or variable loading. [S1070-664X(99)03011-6]

I. INTRODUCTION

There is growing interest in relating the observed characteristics of global energy transport in space, astrophysical and fusion plasmas to ''sandpile'' models that dissipate energy by means of avalanches. In such models the probability distributions of energy released by avalanches, and of avalanche length and duration, should display scale free, inverse power law statistics that are a characteristic of self-organized criticality (SOC),[1,2,3] see also Ref. 4 and references therein. The corresponding power spectra may also have a characteristic inverse power law signature over a given range.

In the case of magnetic fusion plasmas, there is now extensive experimental evidence to show that transport phenomena can display elements of nonlocality, rapid self-organization, critical gradients, avalanching, and inverse power law statistics and spectra, with generic similarities to sandpiles; for reviews, we refer to the work of Carreras, Diamond, and co-workers[5,6,7] and Dendy and Helander.[8] Among the phenomena originally cited in this context are profile resilience in low-confinement (L mode) and Ohmic discharges[9,10,11] which may be linked to the principle of profile consistency;[12,13,14] evidence for the importance of disparate lengthscales (ion Larmor radius to minor radius) in L-mode confinement;[15,16,17] the possibility of universal indexes for measured broadband fluctuation spectra;[18] fast time scale energy propagation effects[19,20,21] which appear to be nonlocal nondiffusive; close relations between marginally stable and experimentally measured radial profiles,[22] and extremely rapid global changes of transport and confinement properties.[23,24,25] To these can be added: Berk–Breizman dynamics[26] of fusion alpha-particle populations, see Ref. 8; and recent large-scale numerical calculations of, first, transport processes arising from ion temperature gradient-driven turbulence[27] and, second, of particle transport due to edge turbulence in the tokamak scrape-off layer.[28]

Astrophysical accretion disks are formed and fueled by matter flowing, with finite angular momentum, toward compact objects such as black holes in active galactic nuclei and neutron stars in binary x-ray sources. The disks (which are topologically toroidal) are typically modeled as rotating, turbulent, viscous, resistive magnetohydrodynamic (MHD) systems. X-ray signals provide an observational indicator of the nature of the transport processes occurring as matter is transferred onto and across the disk, and eventually onward to the compact object.

The possible role of sandpile and SOC phenomenology in astrophysical accretion discs was raised explicitly by Mineshige *et al.*,[29] and strongly implicitly by Bak *et al.*[2] It was subsequently taken further in a range of studies motivated both by observations of inverse power law spectra in a variety of accreting astrophysical objects, and by *a priori* considerations of accretion discs as fuelled, driven-dissipative, magnetized confined plasmas (for a recent review see Ref. 30). Flickering signals with inverse power law power spectra have been observed in the x-ray variability of active galactic nuclei, specifically 0.05–2 keV x-rays from the Seyfert galaxy NGC4051[31] and 2–7 keV x-rays from the Seyfert galaxy NGC5506,[32] and also for 10–140 keV x-rays from the massive compact binary Cyg X-1.[33] Similar spectra for x-ray variability from other binary accreting systems were noted by Mineshige *et al.*[29] (neutron stars, Makashima[34]) and by Geertsema and Achterberg[35] (cataclysmic variables and dwarf novae, Wade and Ward[36]). Manmoto *et al.*[37], Takeuchi *et al.*[38], and Ptak *et al.*[39] considered x-ray fluctuations from locally unstable advection-dominated disks, for example in active galactic nuclei, and identified a number of relevant features. An observational study of nonlinear x-ray variability from the broad-line radio galaxy 3C 390.3 by Leighly and O'Brien[40] also suggests the possibility of interpretation in terms of SOC. Cataclysmic variables, and in particular dwarf novae, are also of interest in the latter context (see Ref. 30 and references therein).

In the case of the solar wind–magnetosphere–ionosphere system, exploration of avalanche models has followed the suggestion by Chang[41–43] that the magnetosphere is in an SOC state. Observational motivation is provided by the sporadic nature of energy release events within the magnetotail,[44] a power law spectrum in the magnetic field measured *in situ*,[45] and power law features of magnetospheric index data, notably AE which is an indicator of energy dissipated by the magnetosphere into the ionosphere. Tsurutani *et al.*[46] described a broken power law AE spectrum; this evidence is indicative but not conclusive. Consolini used AE data taken over a ten year period to construct the distribution $D(s)$ of a burst measure s (see Ref. 47 and references therein) extending the result obtained for one year.[48] This work strongly suggests that inverse power law statistics are a robust feature of the data. (Care should be taken however since AE is a compound index.[47]) Given that the global disruptions of the magnetotail (substorm events) appear to have occurrence statistics with a well defined mean, a key issue in this context[49] is the application of a group of sandpile models[50,51] which yield systemwide avalanches where the statistics have a well-defined mean (intrinsic scale), whereas the internal avalanche statistics are scale free.

A further issue arises in space and astrophysical plasma systems where observations, rather than controlled experiments, are the means by which the conjecture of scale free statistics can be verified. Ideally we require the probability distributions of energy dissipated, length scales and duration of avalanches to be power law as evidence for SOC[4] in a slowly driven sandpile. A generic feature of this type of statistical experimental evidence is that long runs of data are required. In the magnetospheric system it is unavoidable that both the instantaneous value and the recent mean of the loading rate (the solar wind) will have strong variation. Another class of evidence is derived from *in situ* data where a given region in the magnetosphere is observed repeatedly under different conditions. Variability in the loading rate will again be an issue when constructing a statistical ensemble of a large number of such observations. In many astrophysical and laboratory plasma systems, the loading is not expected to be constant over time and variation in loading may not be directly measurable. Examples include mass flows to the outer regions of accretion discs and situations where instability within some zone of a magnetic fusion experiment transfers energy or particles to a potentially sandpile-like outer region.

In this paper we establish the robust features of the avalanche statistics of an avalanche model[50,49] that are required for application to laboratory, space, and astrophysical plasma data as discussed above. We investigate the extent to which the model can under ''ideal'' slow loading yield inverse power law avalanche statistics, and establish how these statistics are modified under strong or variable loading. We shall also see that two distinct regimes of energy confinement, both having power law avalanche statistics, emerge from the sandpile algorithm, depending on the size of the system.

II. A TWO-REGIME AVALANCHE MODEL

A. The algorithm

Typically sandpile algorithms incorporate the following: an array of nodes, at each of which there is a variable amount (height) of sand; a critical gradient (difference in height between neighboring nodes) which, if exceeded by the actual gradient, triggers local redistribution of sand; and algorithms for redistribution and fuelling. Interest focusses on the statistics of the emergent avalanche distribution. We refer to Ref. 52 for an early classification of such models, whose linkage to experimental sandpiles, and the ideal concept of SOC remain topics of active research (see, for example, Ref. 4, also Refs. 8 and 50).

The sandpile cellular automaton used here is described in more detail in Refs. 50, 49, and 53. The sandpile is represented by a one-dimensional grid of N equally spaced cells one unit apart, each with sand at height h_j and local gradient $z_j = h_j - h_{j+1}$. There is a repose gradient z_R (an ''angle of repose'') below which the sandpile is always stable, (the heights h_j and the gradients z_j are measured relative to the values at the angle of repose). Each cell is assigned a critical gradient z_{cj}; if the local gradient exceeds this, the sand is redistributed to neighbouring cells and iteration produces an avalanche. The critical gradients on each of the N nodes are selected from a top hat probability distribution, that is, $P(z_{cj})$ is generated by choosing the z_{cj} at random with uniform probability from the range $[a,b]$, and the integral of $P(z_{cj})$ over all z_{cj} is unity.

Sand is added to this edge driven sandpile at cell 1 at a rate g, and we normalize length and time to the mean loading rate. As soon as the critical gradient is exceeded at cell 1,

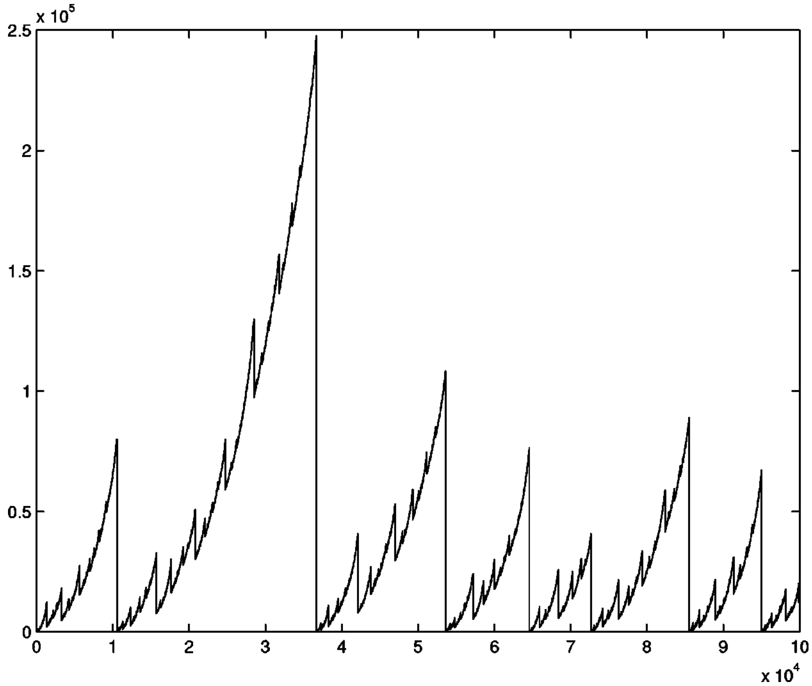

FIG. 1. The time evolution of the energy in a 5000 cell sandpile, with fuelling $g=0.001$ and a probability distribution for the critical gradients that is top hat in the range [0.5,1.5].

the sand is redistributed. The redistribution rule (see Ref. 50) is conservative and instantaneous: sand will propagate to cell 2 and if the local critical gradient there is exceeded, to cell 3 and so on. Within this avalanche, the sand is instantaneously ''flattened'' back to the angle of repose at which the sandpile is always stable. The propagation of an ongoing avalanche from one cell (k) to the next ($k+1$) thus occurs if

$$h_k - h_{k+1} > z_{ck}. \tag{1}$$

This results in a quantity of sand Δ being deposited on the next cell:

$$h^*_{k+1} = h_{k+1} + \Delta \tag{2}$$

such that the gradient at k relaxes to the angle of repose (here normalized to zero)

$$h^*_k - h^*_{k+1} = z_R = 0. \tag{3}$$

Also, since all cells within the ongoing avalanche $1,2,\ldots,k$ are at the angle of repose following this conservative redistribution of sand to the $k+1$ cell, we require the heights of all these cells to become:

$$h^*_{1\cdots k} = h_{1\cdots k} - \frac{\Delta}{k}. \tag{4}$$

This iterative procedure (in which superscript * denotes an intermediate step) is repeated until the avalanche reaches a cell where the gradient is below critical. The critical gradients at cells within the flattened post-avalanche region are then rerandomized as above, and more sand is added at cell 1 until it again becomes unstable, triggering another avalanche. An avalanche may be entirely an internal rearrangement of sand or may continue until it spreads across all N cells of the pile (a systemwide discharge) in which case the entire sandpile is emptied and returns to the angle of repose.

A major feature of this relaxation rule is that it allows propagation of information (correlation) across the avalanche and therefore potentially on all length scales in the sandpile. This permits the possibility of scale free self-organizing behavior in a one dimensional system, whereas traditional reorganization rules[1,4] generally require two dimensions.

The total energy dissipated by an avalanche (internal or systemwide) is just the difference in the potential energy in the entire sandpile before and after the avalanche

$$d\epsilon = \sum_{j=1}^{N} h_j^2 \bigg|_{\text{after}} - \sum_{j=1}^{N} h_j^2 \bigg|_{\text{before}}. \tag{5}$$

A typical time series for the energy is shown in Fig. 1. The 5000 cell sandpile was loaded slowly ($g=0.001$) with respect to the mean value of the z_{cj}, which are uniformly and randomly distributed in the range [0.5,1.5]. With the angle of repose normalized to zero, the time evolution is characterized by systematic growth as sand is added, interspersed with systemwide avalanches where the energy falls back to zero, and internal avalanches where the energy is reduced to some nonzero value.

The statistics of the energy released in internal and systemwide avalanches for a longer run of this sandpile are shown in Fig. 2, with the normalized probability distribution $P(d\epsilon)$ for each class of avalanche plotted separately to ensure good statistics in both populations. As in all sandpile runs in this paper, the populations comprise 5×10^5 internal and 2×10^4 systemwide avalanches. The systemwide avalanches cluster around a well defined mean, whereas the internal avalanches show two distinct inverse power law regions with a turndown at small $d\epsilon$. The distinct behavior of the systemwide avalanches is a necessary condition for applicability to the magnetosphere.[49] In astrophysical and laboratory plasmas this behavior implies that brief energy or

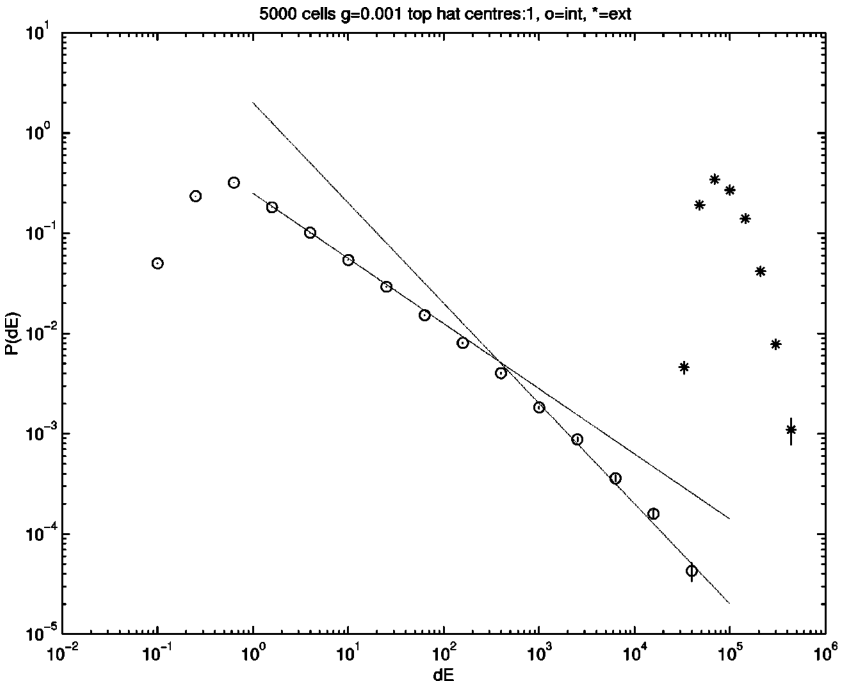

FIG. 2. The probability density of all internal (○) and systemwide (*) avalanches for a 5000 cell sandpile with constant fuelling $g=0.001$ and probability distribution for the critical gradients that is top hat in the range [0.5,1.5].

particle release events with a well defined mean (for example, those associated with edge-localized modes in tokamaks) are still compatible with SOC. Thus the scope of SOC in macroscopic plasma systems extends far beyond the set of phenomena where power law distributions of avalanches are directly observed. We now discuss the internal avalanches in more detail.

B. System scales and power law index

Straight lines $\alpha\, d\epsilon^{-\gamma}$ and $\beta\, d\epsilon^{-1}$ are drawn on Fig. 2 (and all subsequent figures). The values $\alpha=0.25$, $\gamma=0.65$, and $\beta=2$ are an approximate best fit to the points. The sandpile exhibits two distinct regimes of energy transport and confinement. In the case where $P(z_{cj})=\delta(z-a)$, (a any constant) the sandpile evolution with time can be obtained analytically and, if the system is normalized to have total length unity, it can be shown that $P(d\epsilon)=d\epsilon^{-1}$.[53] A region of power law index -1 in $P(d\epsilon)$ is to be expected in a sandpile with $P(z_{cj})$ of finite width, and we might also anticipate that as the width of $P(z_{cj})$ is decreased more of the total range of $P(d\epsilon)$ would be characterized by a power law index -1. Intriguingly this is not the case. The $P(d\epsilon)$ for four sandpile runs are overplotted in Fig. 3, each differing only by the choice of $P(z_{cj})$. For three of these the same mean $\langle z_{cj}\rangle=1$ but three different widths (0.01,0.1,1) have been used. The fourth run also has $P(z_{cj})$ with width 0.1, but

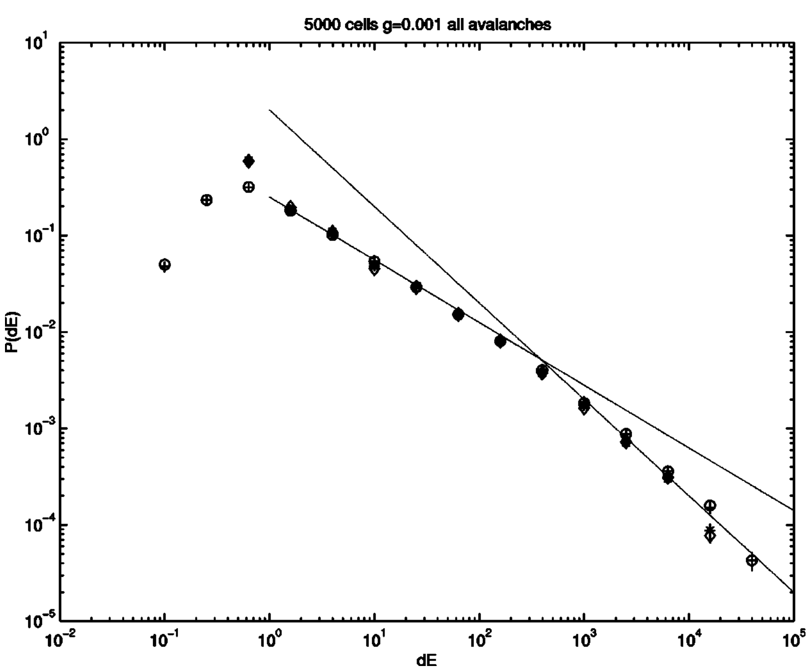

FIG. 3. The probability density of internal avalanches for a 5000 cell sandpile with constant fuelling $g=0.001$ and four different runs with probability distributions for the critical gradients that are top hat: ○≡[0.5,1.5], *≡[0.95,1.05], ◇≡[0.995,1.005] and (with rescaling $d\epsilon\rightarrow d\epsilon\times 100$) +≡[0.05,0.15].

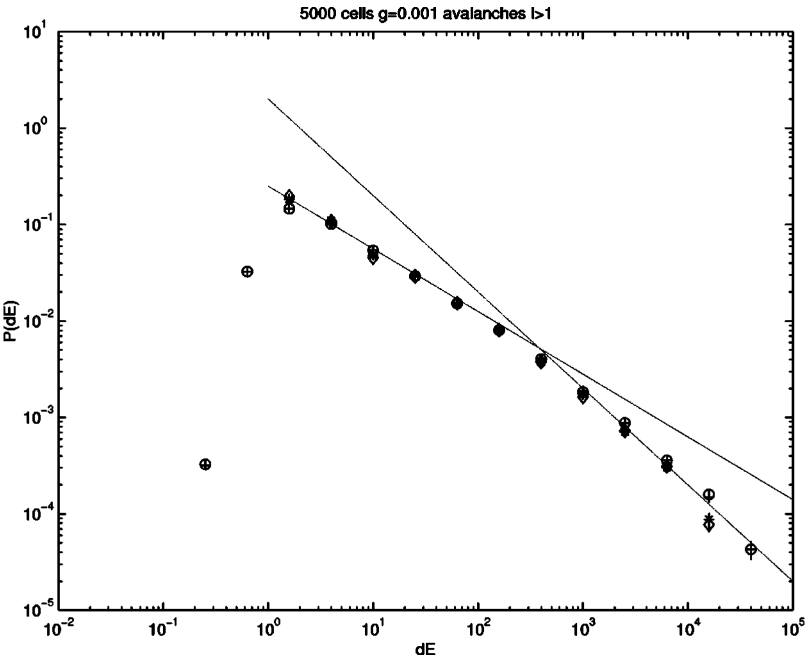

FIG. 4. The probability density of internal avalanches of length greater than 1 for a 5000 cell sandpile with constant fuelling $g=0.001$ and four different runs with probability distributions for the critical gradients as in Fig. 3.

has a different mean $\langle z_{cj}\rangle = 0.1$. In this latter case we have rescaled $d\epsilon \rightarrow d\epsilon \times 100$ since, on average, the heights of sand needed for instability will be smaller by an order of magnitude, so that (5) will yield values of $d\epsilon$ that are on average smaller by two orders of magnitude. Figure 3 demonstrates that all features of the probability distribution are robust against the choice of $P(z_{cj})$, which effectively represents the local condition for instability. Randomness in $P(z_{cj})$ is, however, sufficient to produce a system that appears to differ fundamentally from the analytical $P(z_{cj}) = \delta(z-a)$ case, since the statistics are robust against progressively decreasing the width of $P(z_{cj})$ over two orders of magnitude.

Avalanches dissipating smaller amounts of energy might be expected to extend over smaller length scales. In Figs. 4–7 we replot the data shown in Fig. 3, showing only the contribution from successively longer avalanches (avalanches with lengths >1, 8, 32, 64, respectively, are shown).

Independent of the details of $P(z_{cj})$ we see that the power law index ~ -0.65 corresponds to avalanches that extend over less than ~ 64 cells. Also, Fig. 4 then shows that the drop at $d\epsilon \sim 1$ in Fig. 3 corresponds to avalanches that are one cell in length.

The sandpile thus has three distinct regimes in its statistics: single cell avalanches that (as one might expect) are not power law; avalanches smaller than ~ 64 cells, with power law index ~ -0.65, which may in some sense reflect the discrete nature of the grid; and avalanches longer than ~ 64 cells and up to the system size, with power law index -1,

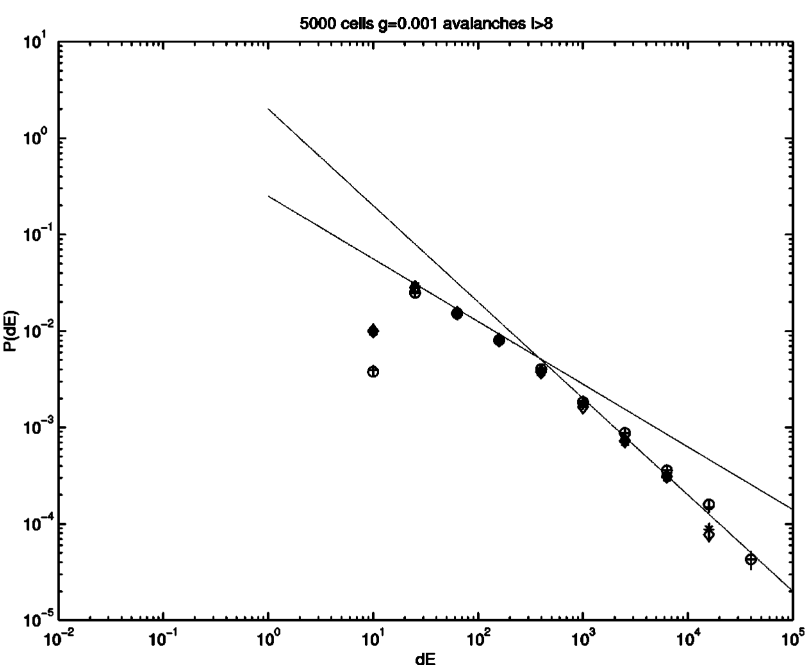

FIG. 5. The probability density of internal avalanches of length greater than 8 for a 5000 cell sandpile with constant fuelling $g=0.001$ and four different runs with probability distributions for the critical gradients as in Fig. 3.

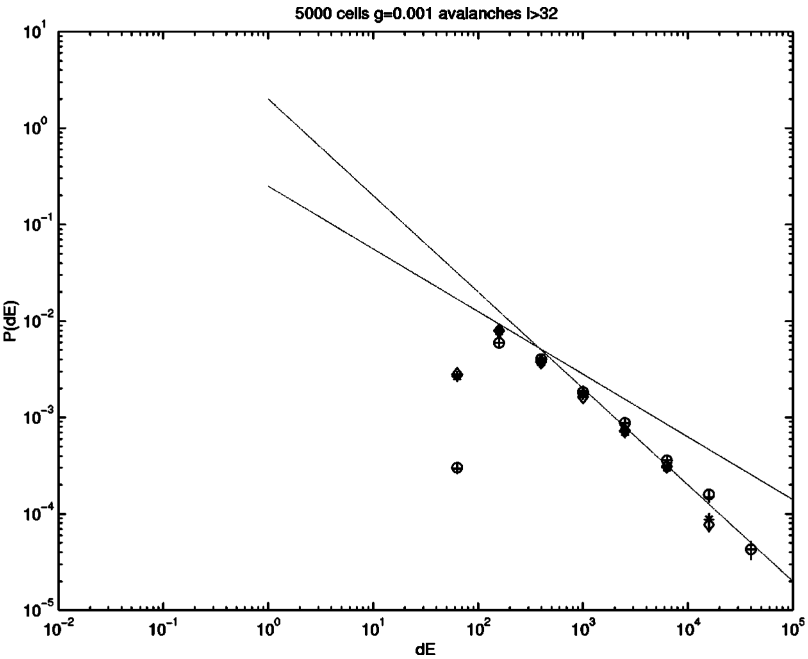

FIG. 6. The probability density of internal avalanches of length greater than 32 for a 5000 cell sandpile with constant fuelling $g=0.001$ and four different runs with probability distributions as in Fig. 3.

which may approach a continuous limit for the system. Astrophysical plasmas might be expected to extend well into the large scale regime, whereas in magnetic fusion plasmas the behavior on small scales could dominate. Anomalous energy transport in tokamaks is believed to arise from interacting nonlinearly saturated modes, each of which has finite spatial extent and is typically localized about a surface with rational toroidal winding number (''safety factor'' $q=m/n$, with m,n low integers). The number of such modes that play a significant role depends *inter alia* on the radial size of the tokamak, via the range of rational q-values present and their spatial separation. Depending on experimental conditions this number could be either large or small compared to 64. These modes would correspond to the cells in a sandpile model of tokamak phenomenology. The emergence, through self organization, of two distinct power law confinement regimes is in itself of interest for potential application to tokamaks, as it apparently represents a step towards a widely sought objective, namely ''a sandpile with an H mode'' (e.g., a heuristic approach to localized regions of sheared flow in a sandpile is described in Ref. 5).

Finally the question arises as to the robustness of these results against differing system size. The sandpile rules that we use have been implemented for systems of length 50 (Ref. 50), 500 (Ref. 49) and here, 5000 cells. The results for the smaller systems reveal only the first power law range of index approximately -0.65, whereas larger systems extend into the power law index -1 regime, suggesting robustness

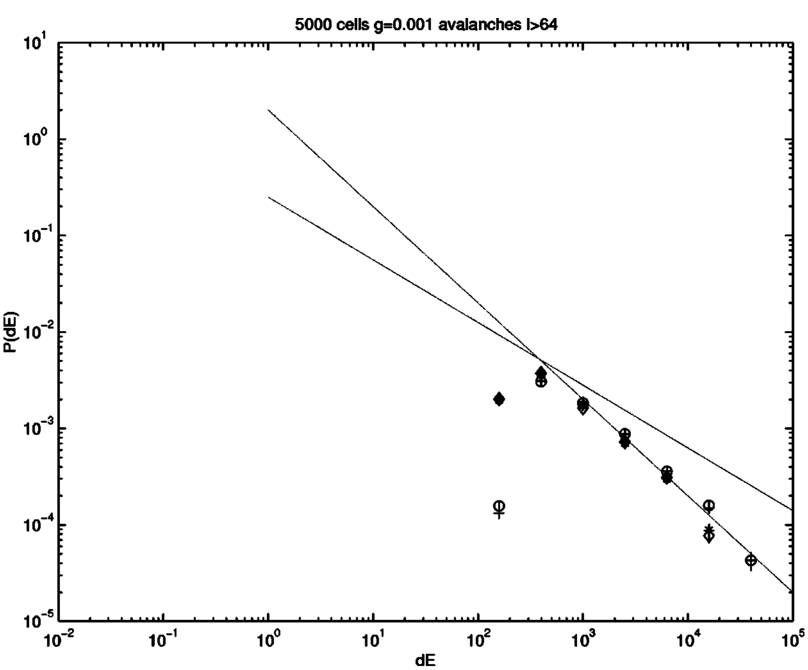

FIG. 7. The probability density of internal avalanches of length greater than 64 for a 5000 cell sandpile with constant fuelling $g=0.001$ and four different runs with probability distributions as in Fig. 3.

of these features. Given the requirement for good statistics over the entire range of avalanche size (range of $d\epsilon$) a system of significantly larger size is currently computationally prohibitive.

III. ''NONIDEAL'' LOADING

Let us now briefly estimate the effect of breaking the constraint of slow loading $g \ll 1$ (see Ref. 47 for details).

The contribution to a given avalanche of recently added sand can be assessed as follows. If h_j is the height at cell j immediately before the avalanche, and $\langle g \rangle$ is the mean (ensemble average) fueling rate, the amount of sand transferred to the second cell can be written

$$\alpha_1(h_1 - h_2 + \langle g \rangle), \tag{6}$$

where we assume the total height at each cell increases so that the (model dependent) fraction transferred $\alpha_1 < 1$ by construction. Similarly, an amount

$$\alpha_2\{h_2 - h_3 + \alpha_1(h_1 - h_2 + \langle g \rangle)\} \tag{7}$$

is transferred to the third cell, where $\alpha_2 < 1$, and so on. For the final transfer to the jth cell, it follows that $\langle g \rangle$ is multiplied by $\Pi_{i=1}^{j-1}\alpha_i$, whereas the nearest-neighbor height difference $h_{j-1} - h_j$ is multiplied by α_{j-1}, the next nearest neighbor is multiplied by $\alpha_{j-2}\alpha_{j-1}$, and so on. Since $\alpha_1\alpha_2 \cdots \alpha_k \leq \alpha_1\alpha_2 \cdots \alpha_{k-1} \leq \alpha_1\alpha_2 \cdots \alpha_{k-2}$ and so on, it follows that the relative contribution of $\langle g \rangle$ to avalanche dynamics diminishes as the scale j of the avalanche increases. We restrict attention to the class of sandpile models yielding scale free power law avalanche statistics in the limit of small but nonzero $\langle g \rangle$. This requires that $\Pi_{i=1}^{j-1}\alpha_i\langle g \rangle$ must decline sufficiently rapidly with j that small oscillations in g (with $\langle g \rangle$ small) are effectively damped by the sandpile.

We can now consider the effect of fast, or strong, loading. There are three characteristic time scales implicit in any sandpile algorithm: the relaxation time τ_r over which an avalanche takes place; the average time required, following an avalanche, for instability to recur at cell 1, τ_u; and the iteration timestep Δt. The amount of sand added at cell 1 per timestep is again g, which may be constant, or modulated over time, or be drawn from some random distribution. It is clear that $\tau_u \sim \langle z_c \rangle / \langle g \rangle$ and hence, for instantaneously relaxing sandpile models ($\tau_r \ll \Delta t$) we identify slow and fast loading regimes:

$$\text{slow:} \quad \tau_r \ll \Delta t \ll \tau_u, \tag{8}$$

$$\text{fast:} \quad \tau_r \ll \tau_u \sim \Delta t, \tag{9}$$

respectively. In the latter case, instability is likely to be triggered at each timestep. Normalizing Δt to unity, that is one timestep occurs in unit time, the fast loading condition becomes $\langle g \rangle \sim \langle z_c \rangle$. For any loading rate there is an effective minimum avalanche length required to dissipate the energy associated with the sand added in each Δt. In the slow limit this corresponds to less than one cell, in the fast limit to many. If the mean fuelling rate $\langle g \rangle$ is increased towards and beyond $\langle z_c \rangle$, the smallest scale avalanches will be increasingly eliminated. This will be reflected in the lower bound of any range of ''power law'' event statistics for energy release in the system.

The above is also an indicator of how far information concerning the modulation or fluctuation of g is transmitted in avalanches, and whether its consequences may become visible in the event statistics (''the sandpile acts as a filter''). Insofar as fluctuations in g take the sandpile intermittently into the fast driving regime, the preceding comments apply. There will exist a maximum avalanche scale (number of cells n_f) beyond which the effect of fluctuations in g becomes vanishingly small in the avalanche statistics. We then have two possible cases: (i) where n_f is sufficiently large that there is a detectable signature in the internal statistics (disruption of the power law) but is much smaller than the system size and (ii) where n_f is of order the system size, in which case the event statistics of both internal and external (or systemwide) events reveal a signature reflecting both the mean level and the fluctuation spectrum on g. In the latter case we expect a sandpile model to yield good correlation between fluctuations in the inflow and in the external (systemwide) time series in the extremely strongly driven limit. In practice, we can only obtain an estimate of n_f numerically.

Figures 8 and 9 show the behavior of a 5000 cell sandpile with z_{cj} in the range [0.5,1.5] as before. Figure 8 shows the result with constant $g = 10$, and Fig. 9 with mean $\langle g \rangle = 10$ and with each value of g drawn from a uniform random distribution, in the range [5,15], hence $\langle g \rangle \gg \langle z_{cj} \rangle$. The same line $\beta / d\epsilon$ from the previous figures is drawn in Figs. 8 and 9.

The systemwide event statistics remain unaffected by the fast loading, and the internal avalanches at large $d\epsilon$ still follow an inverse power law slope -1, so that at this value of $\langle g \rangle$ the statistics of large avalanches are not sensitive to details of the fuelling as expected from the arguments above. The smaller events are supressed in comparison to the results for small $\langle g \rangle$ and as a consequence the slope -1 region of the normalized probability distribution is displaced w.r.t. that for the small $\langle g \rangle$ results (Figs. 2–7). In the case of constant loading there is a sharp cutoff, events below this size are of low probability. The effect of fluctuations in g is to broaden the peak in the probability distribution. Finally, the statistics of the largest internal avalanches show a roll off as the corresponding values of $d\epsilon$ begin to overlap with that of the systemwide population. This is not inconsistent with the small $\langle g \rangle$ results and is due to the improved statistics of the large avalanches resulting from the elimination of avalanches at small $d\epsilon$ as we increase $\langle g \rangle$ (all plots are comprised of the same number of internal avalanches).

IV. CONCLUSIONS

A simple one dimensional sandpile model has been developed which exhibits two distinct characteristics in the probability distribution of energy discharges. For discharge which corresponds to internal reorganization there are two distinct inverse power law regimes, while for systemwide discharges (flow of sand out of the system) the probability distribution has a well-defined mean. The model may be ap-

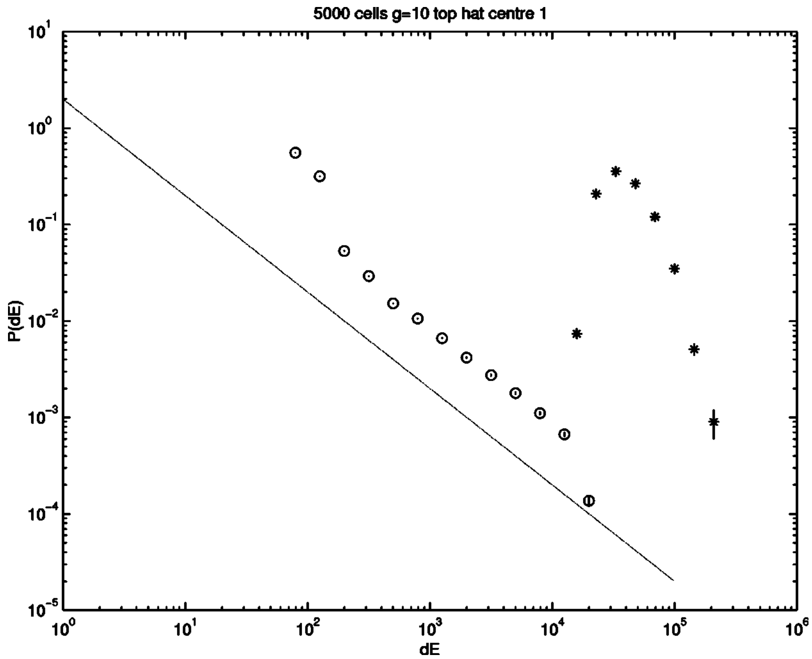

FIG. 8. The probability density of all (○) and system-wide (∗) avalanches for a 5000 cell sandpile with constant fuelling $g=10$.

plied to magnetospheric dynamics,[49] for example in reconciling power law indexes in internal dynamics with well-defined substorm event statistics. The model substantiates the underlying theoretical point[50]—which may also apply to astrophysical accretion systems and magnetic fusion plasmas—that external event statistics with a well-defined mean can be entirely compatible with internal avalanche dynamics governed by SOC.

The sandpile exhibits two confinement regimes which have inverse power law statistics of index ~ -0.65 and -1. These correspond to reconfigurations on distinct length scales. Short length scales arise that may be sensitive to the discrete nature of the grid, and longer scales, up to the system size, that effectively approach a continuous limit of the model. The transition between these regimes occurs at avalanche lengths of about 64 cells. This focuses attention on the number of participating modes (each corresponding to a cell in the sandpile) that are expected to determine the sandpile aspects of a macroscopic plasma system. We anticipate that very large plasma systems such as Earth's magnetosphere and astrophysical accretion discs are more likely to operate well into the large scale (continuous) limit, whereas magnetic fusion plasmas access both regimes due partly to the mode discretization provided by nested toroidal flux surfaces and to their smaller scale.

For space and astrophysical plasma systems observations taken over long periods are required to test for possible inverse power law statistics. The loading of the system (e.g., in

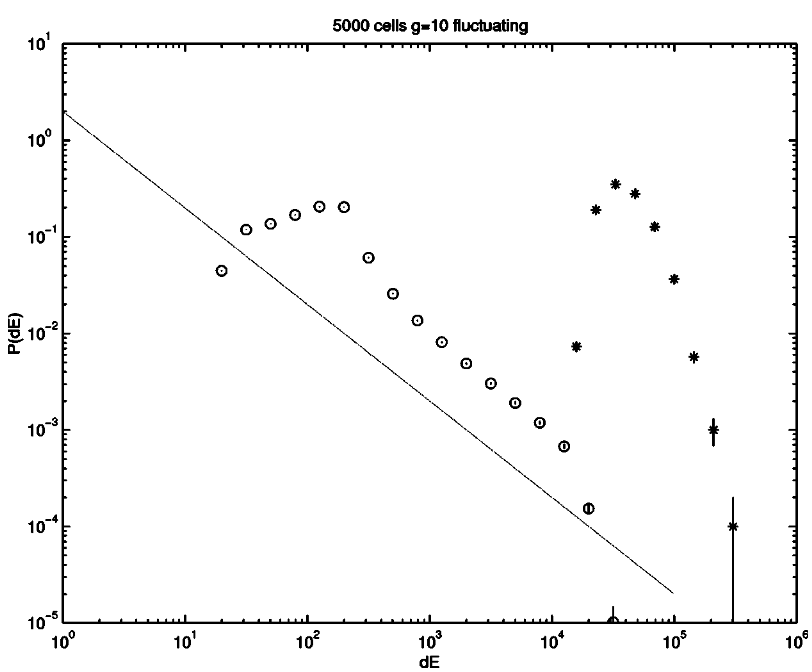

FIG. 9. The probability density of all (○) and system-wide (∗) avalanches for a 5000 cell sandpile with randomly fluctuating fuelling in the range [13,8] and $\langle g\rangle = 10$.

the case of the magnetosphere, the solar wind, in laboratory plasmas, auxiliary heating) is often characterized by both strong variability about a mean, and a large dynamic range of mean energy input. The inverse power law form of the sandpile statistics has been shown to be robust under fast loading. The effect of large loading rates is to exclude events which dissipate small amounts of energy, hence the sandpile tends to yield a single inverse power law regime with downturn at lower energies. We would thus expect inverse power law avalanche distributions to be a persistent feature in long runs of data that include ''fast'' inflow conditions if the underlying system is governed by SOC.

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